Preface

This textbook presents what Joseph-Louis Lagrange called *Analytical Mechanics*. Historically this was a great advance beyond the methods of Euclidean geometry employed by Isaac Newton in the *Philosophiae Naturalis Principia Mathematica*. With the methods of Lagrange and Leonhard Euler, we could actually perform calculations. Lagrange and Euler used the calculus and did not require the formidable expertise in the use of geometry that Newton possessed.

The step introduced by William Rowan Hamilton simplified the formulation. Hamilton's ideas also represent a great step forward in our understanding of the meaning of Analytical Mechanics. This, coupled with the simplification added by Carl Gustav Jacobi, provided us with a pathway to the more modern uses of Analytical Mechanics including applications to astrophysics, complex systems, and chaos.

Our approach will introduce a modern version of what was done in the 18th and 19th centuries. We will follow essentially the historical development because the ideas unfold most logically if we do so. We will, however, pay more attention to the development of Analytical Mechanics as a valuable tool than to a historical study.

Our final step will be the relativistic formulation of Analytical Mechanics. That is an absolute necessity in any complete study of Analytical Mechanics.

Logically we begin this text with a chapter on the history of mechanics. Many texts include brief historical comments or even added pages outlining individual contributions. That is certainly an improvement on the anecdotes that our professors often passed on to us without citation. Those anecdotes piqued our interest and added flavor. But they lacked a continuity of thought and that all-important accuracy that we prize. Analytical Mechanics is the oldest of the sciences. And the history stretches from the beginnings of philosophy in Miletus in 600 BCE to the advances in scientific thought introduced in the Prussian Academy and in Great Britain. I have sincerely endeavored to shorten this, as any serious student will easily recognize. But I still worry about the length.

Because my own understanding of science has been greatly enriched by studies in history, I cannot recommend that a professor ignore the first chapter completely. The student should understand something of the interesting and tortured individual Newton was. And we cannot really comprehend the origins of the ideas that gave birth to Analytical Mechanics without encountering the work of Pierre Maupertuis, Johann Bernoulli,[1] Euler, and Lagrange. The sections of Hamilton and Jacobi may be held until after the students have gained an appreciation for the methods of Analytical Mechanics. But those sections will be of interest to students as they encounter the chapter on the Hamilton-Jacobi approach. They should see the simplicity of what Jacobi brought and his great respect for the ideas of Hamilton. Then to emphasize the importance these ideas, I include an outline of Erwin Schrödinger's original published derivation of his wave equation from the Hamilton-Jacobi equation. With the caveat surrounding a second variation, the quantum theory is buried in the theory of Hamilton and Jacobi.

In Chap. 1, I have not included the historical events leading to Albert Einstein's development of the Special Theory of Relativity in 1905. Some of this I have placed in the final chapter. The historical importance of Einstein's contributions is more easily understood by a reader who has a general grasp of the classical theory of fields, which is not our primary topic.

Beyond the history, the primary part of the text, in which I present the basis and applications of Analytical Mechanics, begins with Chap. 2 on Lagrangian Mechanics. There the issue is the Euler-Lagrange equation and the variational problem, which is solved by the Euler-Lagrange equation. This I follow by a chapter on Hamiltonian Mechanics, which, through the Legendre transformation, is a logical next step from the approach of Euler and Lagrange. The canonical equations were actually obtained by Hamilton in his papers of 1834 and 1835 with another goal in mind. But the procedure was the Legendre transformation. With these chapters, we have Analytical Mechanics essentially in place.

Then I introduce the Hamilton-Jacobi approach. I do not present a method to be memorized and applied because doing so obscures the logic and the simplicity. I follow in spirit, but not in precise detail, the ideas of Jacobi. The generator of a canonical transformation will take center stage, as it did for him. The final method does not follow a head-down approach, but one with finesse.

In all texts there are final chapters. And all courses are of finite duration. Therefore, there will always be parts of the student's experience that will become lost in the fuzziness of the final days. In this text, those final chapters contain studies of complex systems, chaos, and relativistic mechanics. Each of these chapters deals with subjects of entire courses at many institutions. I have written the chapters on complex systems and on chaos as introductions to these very interesting topics. They may then be treated as windows opening onto studies that may occupy

[1] Johann was the original name given by his parents. Jean or John appears sometimes, depending upon whether the author is French or English. Johann Bernoulli was born and died in Basel, Switzerland.

the students' interests completely at a later time. They may even provide interest for the last weeks of a semester. But the final chapter on special relativity is not of the same character.

I elected deliberately to make the final chapter on the Special Theory of Relativity an almost self-contained unit. The reader who is not completely familiar with the theory of classical fields will be able to pass over a portion of the chapter in which we develop the field strength tensor and electromagnetic force. However, the approach to relativistic mechanics and finally to relativistic Analytical Mechanics should be considered carefully by the serious reader. There I have followed some of the classic sources, such as Wolfgang Pauli, Peter G. Bergmann, and Wolgang Rindler. The principal product of this work is the Hamiltonian and the canonical equations for relativistic motion in the electromagnetic field. We required the nonrelativistic approximation to these results for our treatment of this motion in a previous chapter.

I cannot expect that all students will be stirred, as some of mine have been, when they see the connections among the ideas common to theoretical physics. But I hope they are.

I am deeply indebted to generations of students who have gone through this intellectual adventure with me during the past forty years. I am thankful that I have been part of their intellectual pathways and for the questions with which they continued to press me. They have seen me grow in understanding and love for the ideas I try to express here.

I am also very thankful to my teachers who introduced me to the beauty and power of Analytical Mechanics. Isaac Greber particularly stands out. He presented us with remarkably inspired and almost impossibly difficult problems to which I devoted all of my energies on many cold winter nights in Cleveland. Isaac has been a friend and an inspiration.

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