Reduction of Low Frequency Acoustical Resonances Inside Bounded Space Using Eigenvalue Problem Solutions and Topology Optimization

Andrzej Błażejewski

Abstract The chapter deals with the problem of a space with an acoustical source, which forms a field of some values. All apply to an acoustic field characterized by an acoustic pressure p. In the low-frequency range and high values of boundary impedance, the modal approach is successfully applied. In this case, the field variation in all points of space is described by a specific time-dependent variable $w(t)$. The field shape is related to eigenfunctions $\Psi(r)$, which are the solution of the eigenvalue problem. Eventually, the acoustic pressure distribution $p(r, t)$ is defined by a sum over a set of a space's eigenfunctions $\Psi(r)$ and time components $w(t)$. Each $w(t)$ contains the source factor Q, which is an integral of the strength source multiplied by the related eigenfunction values in points where the source is located. Thereafter, if the integration is calculated over a region, where the value of the eigenfunction Ψ_m is zero, the source factor Q is zero as well. Considering the above, the aim of this research is to obtain the space where as many points as possible exist, where eigenfunction Ψ_m values are equal to zero, for as many eigenfrequency ω_m as possible. In order to find the specific configuration of the topology, an optimization problem is formulated. The eigenfunctions are considered as design variables. A minimum of multiobjective functions, based on eigenvalue problem solutions is searched. As the result of the optimization, the shape of space and point locations is obtained. The specified point is a possible source location, which guarantees reduction of resonances in a particular frequency range.

1 Introduction

This chapter deals with the problem of reduction of acoustic resonances that may occur when a source is placed inside the domain. This kind of problem appears in room acoustics, where locations of the source inside the enclosure are preferable, to avoid the situation when speech becomes unintelligible or unclear. In the case of devices, an improper location makes their work more oppressive. According to the

A. Błażejewski (✉)
Koszalin University of Technology, Śniadeckich 2, 75-453 Koszalin, Poland
e-mail: andrzej.blazejewski@tu.koszalin.pl

J. Awrejcewicz (ed.), *Dynamical Systems: Theoretical and Experimental Analysis*, Springer Proceedings in Mathematics & Statistics 182, DOI 10.1007/978-3-319-42408-8_2

aim of the room acoustics research, the best acoustic properties of the room are the most common problem in the area of interest [9]. Some works deal with the room size and determination of the room ratio, in particular, the optimal enclosure ratio and shape, which make rooms suitable for music listening or conference rooms audience friendly [19]. More generally, the rooms' shapes and irregularity influence reverberation time and time decay of particular modes, which is investigated in the works of [6, 8, 23]. The problem of the best geometry of the room, in order to create the best acoustic properties, is solved by Cox, D'Antonio, and Avis [4]. It is achieved by optimization methods exclusively for a simple enclosure at a low frequency. Another group of the investigations is generally focused on a modification of existing boundary conditions. In practice it is investigating a proper distribution of absorbing material in the room. Dühring, Jensen, and Sigmund designed the rooms by using topology optimization. Their work shows how to reduce a noise by choosing the best configuration of a reflecting material in the design domain without changing its size [5]. In their paper there is also the review of how other researchers achieve improvement of speech intelligibility by an absorbing material distribution in the room. This method reduces the amplitude response from the loudspeaker and time reverberation. The space/enclosure boundary modification by locating specific acoustic structures, in the form of resonators or wall shaping, is another method that can be applied to affect the acoustic field [27]. Boundary shaping and distributing reflecting and/or absorbing materials, together with optimization methods (i.e., topology optimization) are applied by other authors in the case of small devices design [10, 25]. The values that describe the acoustic field are considered separately in the area of interest. The first is acoustic pressure, as the scalar value is commonly used. Because of the ease measurements of sound pressure, this quantity is suitable in many cases. The intensity is the vector quantity which is more complicated to measure [22], but gives the energetic assessment of the acoustic problem. Pan analyzes the enclosed spaces from the energetic point of view and modal approach [16–18]. These works show that intensity prediction by using the mode model needs mode coupling consideration. At the same time in their work, Franzoni and Bliss [7], show this problem. Simultaneously, the existence of the intensity vortices and a set of vortex modes with eigenfrequencies, which form a harmonic series, predicted by Waterhouse [26], are confirmed. The numerical studies based on the mode model, which is presented by Meissner, show those properties of the intensity field [11]. In this work, the active and reactive components of intensity inside an L-shaped room are investigated. His previous and following works [13] indicate that vortices of active intensity are strongly related to zeros of eigenfunctions. This feature is applied in this chapter as a criterion of resonance reduction, which appears in closed space. In the case of optimization, or generally in a control of the acoustic field inside an enclosure, many factors should be considered, including area of boundaries, their configuration, location, the acoustic properties of the cover materials, and sound/noise source position. Genetic algorithms (GAs) successfully calculate the minimum of objective functions, considering a large number of design variables, such as room surfaces with their acoustic impedance [2, 3]. The authors show how to minimize the level of acoustic pressure inside the whole enclosure, using properties of modal amplitudes (time components in modal expansion in steady states of an acoustic pressure) by applying a specific configuration of a boundary condition. The configuration is found without changing the size of the enclosure. This chapter deals with the problem of reduction, or more generally control, of the acoustic field. It is realized considering two aspects. On the one hand the source, which is located inside the domain on the specific location, may generate lower values of acoustic pressure than sources that occupy other places. However, those areas are dependent on the domain shape. Those features are introduced in the optimization criteria.

2 The Modal Approach to Acoustic Field Description in a Bounded Space

An acoustic field in an enclosure is a specific case of acoustic wave propagation. The sound source generates an acoustic signal, which is usually partly absorbed and reflected by boundaries. If the source is permanently active the acoustic energy absorbed on the boundaries is equalized in the short term by the energy from the source. After the transient period, the steady-state acoustic field dominating in an enclosure is attained. In order to describe the acoustic field distribution inside a room, the modal approach can be applied under several restrictions [15]. One of them is a low-frequency range of signals generated by a source, which is limited by the Schroeder frequency [20, 21]. This kind of signal guarantees the sparsely distributed acoustic modes, and in the case of high impedance on boundaries, mode uncoupling can be applied. The modal approach assumes that the acoustic field distribution inside an enclosure is dependent on its normal modes (eigenfunctions). The modes are obtained by the solution of the Helmholtz equation for a domain bounded by perfectly rigid walls. It is defined by Neumann's boundary condition equal to zero. After that, eigenfunctions $\Psi_m(r)$, in all enclosure points with coordinates $r(x, y, z)$, together with eigenfrequencies ω_m are determined. Applying eigenfunction $\Psi_m(r)$ in the modal expansion leads to the sum in the form:

$$p(r,t) = \sum_{m=0}^{\infty} w_m(t)\Psi_m(r), \quad (1)$$

where orthogonality and normalization of eigenfunctions are required [12]. Additionally, low frequencies and a narrow band of excitation allow reducing an infinite sum to a finite number N of factors. The first factor in modal expansion, the time components $w(t)$, describe acoustic pressure variation in time, during increasing and decreasing sound, when a source starts and becomes mute. In a steady-state field condition, $w(t)$ represents the magnitude of acoustic pressure in a particular point of the domain. If the sum 1 describes the acoustic field inside the room with a source, it satisfies the linear, inhomogeneous wave equation and the specific boundary conditions. Most often, the conditions are determined by the acoustic impedance. Modal expansion is introduced in the following wave equation [14],

$$-\nabla^2 p(r,t) + \frac{1}{c^2}\frac{\partial^2 p(r,t)}{\partial t^2} = f(r,t), \quad (2)$$

where c is the sound velocity in air and function f defines the source power or outflow. Both the function, $p(r,t)$, and each eigenfunction $\Psi_n(r)$, satisfy the Green theorem. Finally it leads to the solution represented by a set of ordinary differential equations of time components $w_n(t)$. Denoting upper dots as time derivates and omitting independent variables r and t, the equations take the form:

$$\ddot{w}_n + \omega_n w_n + (\rho c^2 \int_S \frac{\Psi_n^2}{Z} dS)\dot{w}_n = -\frac{c^2}{\sqrt{V}} \int_V f\Psi_n dV. \quad (3)$$

Equation 3 shows a form of a second-order linear differential equation with constant coefficients, in the case where the mod coupling is neglected. Coefficient ρ represents a density of medium inside the volume V, bounded by surface S characterized by acoustic impedance Z. The general solutions are presented in [1]. If a source is harmonic and described by the function f in the form $f = q(r)e^{j\omega t}$, the solutions can be obtained after some algebraic calculations. The amplitude of harmonic time components $w_n(t)$ is given by formulae:

$$w_{n\omega} = \frac{Q_n}{(\omega_n^2 - \omega^2) + 2j\alpha_n\omega} \quad \text{and} \quad w_{0\omega} = -\frac{Q_0}{2j\alpha_0\omega - \omega^2}, \quad (4)$$

where index $n\omega$ means a solution for a particular frequency, 0ω a time component, which is the solution of Eq. 3 in the case $\omega_n = 0$; that is, eigenvalue $\lambda_n = \sqrt{\frac{\omega_n^2}{c^2}} = 0$. The coefficients α and Q are defined by the integrals:

$$\alpha_n = \frac{1}{2}\rho c^2 \int_S \frac{\Psi_n^2}{Z} dS \quad \text{and} \quad Q_n = -\frac{c^2}{\sqrt{V}} \int_V q\Psi_n dV, \quad (5)$$

Generally coefficients α and Q5 are described in Eq. 3: damping in the system caused by the impedance Z of the boundaries S and a source component. The source component in each equation is an integral of a specific eigenfunction multiplied by a source magnitude $q(r)$ in points, where the source is located. Outside the source location the integral becomes zero. If eigenfunction Ψ_n in the source location has values close to zero, the whole source component in Eq. 5 and consequently related time component $w_{n\omega}$ in Eq. 4 have minimal absolute values. Moreover, in the case of harmonic source, the time components for eigenfrequencies ω_n, significantly different from ω, tend to zero. Consequently, the solution of the wave equation 2 in the form of sum 1, which contains time components 4, gets minimum.

Therefore, the question arises about a space geometry configuration with the interior region, where an active source without regard for damping in the acoustic system and the strength of a source guarantee minimal values of acoustic pressure for particular frequencies.

3 Optimization Problem

In view of these aspects, the optimization problem is formulated. The result of the optimization is a shape of the enclosure and the interior area, where points of eigenfunctions with value equal to zero or close to zero are located. In the case of arbitrary shapes of the enclosure those points are in different locations for different eigenfunctions. There are shapes of closed spaces that own this feature. One of them is a circle in a two-dimensional or sphere in a three-dimensional domain. In their centers are the points, where many eigenfunctions have zero values. Therefore, the enclosure is searched among N different shapes of different k dimensions. These dimensions are considered as design variables, called X_k and $k \in \mathcal{N}_1$. Each set of design variables $\{X_k\}$ is related to a set of eigenfunctions of the exact shape of the enclosure. Thereby, some following set $\{\Psi_0, \Psi_1, \Psi_2, \ldots \Psi_n, \ldots \Psi_m\}_N \equiv \{\Psi_n\}_N \equiv \{X_k\}_N$ of potential solutions is considered, where m represents the limit of the number of following functions considered for each shape. The expected solution is the enclosure, where many zero points are located, as close to each other as possible. It means that two configurations are possible: first, when the zero points for eigenfunctions coincide with others and create one spot, and second, when the zero points for some eigenfunctions overlie an area of enclosure. There are three criteria $C1$, $C2$, and $C3$ that express the above cases:

$$C1 = \left| r_i^* | \Psi_n(r_i^*) = 0, n \in \langle 0, m \rangle \in \mathcal{N}_0, i \in \mathcal{N}_1 \right| \rightarrow max \tag{6}$$

$$C2 = \left| \omega_n^* | \Psi_n(r_j^*) = 0, n \in \langle 0, m \rangle \in \mathcal{N}_0, j \in \mathcal{N}_1 \right| \rightarrow max \tag{7}$$

$$C3 = \sum_{i=1}^{C_1} \sqrt{r_0 - r_i^*} \rightarrow min \tag{8}$$

In Eq. 8 r_0 is a coordinate of an assumed source location inside the enclosure. In the case of an arbitrary enclosure, some eigenfunctions are possible that have very small values (but with not many zero points) in the whole space, whereas others are characterized by significant variation from negatives to positives. The first are preferable in the case of reduction of acoustic pressure. Therefore, it is a suitable approach to look for not zeros but some minimal values. It is proposed to concern 1 % of the mean of all absolute values of all eigenfunctions in the considered range $\langle 0, N \rangle$ as a reference, instead of zero. It generates modification in (6) and (7) $\Psi_n(r_{i,j}) = 0.01 \cdot mean(\{\Psi_n\}_N)$. On the basis of the criteria the multicriteria objective function (F_{obj}) is created in the form:

$$\min_{\{\Psi_n\}_N} F_{obj}(\{\Psi_n\}_N) = [-C1, -C2, C3]^T \tag{9}$$
$$subject \quad to:$$
$$\wedge \quad X_{1min} < X_1 < X_{1max}$$
$$\wedge \quad \cdots$$
$$\wedge \quad X_{kmin} < X_k < X_{kmax}$$

3.1 Solution for an Example 2D Problem

The genetic algorithm is implemented in order to find solutions. The GA uses procedures of nondominated selection of solutions, so-called Pareto solutions. During the selection, which process repeats iteratively, the fitness values of F_{obj} are calculated. The fitness values determine the potential solutions. In each iteration, the chosen set of solutions, in a so-called Pareto set, is compared by GA and nondominated individuals are chosen. Here, the fitness values are calculated, using the modified criteria $C1$ and $C2$, which are taken as negatives and values of criterion $C3$ is taken directly, bearing in mind that GA searches the minimum. As an example the shape represented by a two-dimensional object shown in Fig. 1 is optimized. The optimized object is created as a union of three squares (the big one and two small on the sides) and a circle. The four characteristic dimensions, which vary during the optimization, are indicated as design variables X_k defining $\{\Psi_n\}_N$. There are: X_1, side of the big square; X_2, X_3, sides of the small squares; and X_4, radius of the circle. The constraints are defined as $X_{1min} = 10$, $X_{1max} = 12$, $X_{2min} = 5$, $X_{2max} = 6$, $X_{3min} = 5$, $X_{3max} = 6$, $X_{4min} = 4$, and $X_{4max} = 6$. Additionally, the point indicated in criterion $C3$ with coordinates $r_0(0, 0)$ is chosen. Two shapes determined by dimensions related to optimization constraints, the solutions, that is, the points distributed

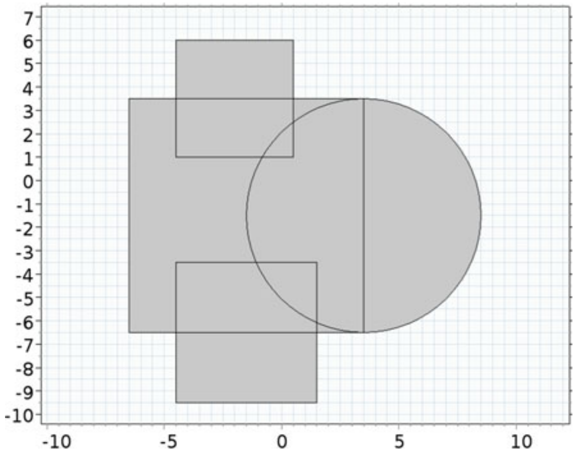

Fig. 1 The design variables: X_1—side of the big *square*, X_2, X_3—sides of the smalls *squares* and X_4—radius of the *circle*

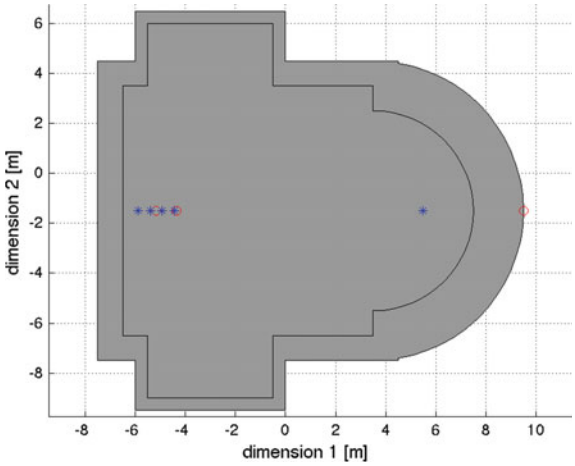

Fig. 2 Points distribution inside examined object in the case of design variable $\{X_k max\}$ and criteria values $C1 = -3, C2 = -48, C3 = 19.5902$ (*circles* o), and criteria values $C1 = -5, C2 = -47, C3 = 27.2038$ (*asterisks* $*$)

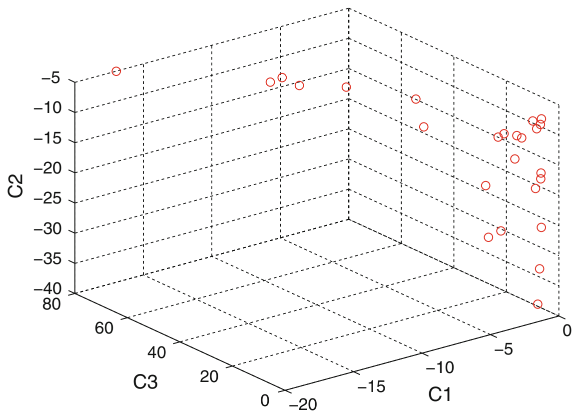

Fig. 3 Pareto optimal solutions found by genetic algorithm in case of 10 Iteration for 30 Individuals in Pareto set

inside the objects, satisfying modified criteria values, are shown in Fig. 2. It seen in this figure, that in both cases the points are located far from point r_0. As mentioned in the previous section, these points can be considered as the area, where a located sound source shall be damped in the range of frequencies $\omega \to \omega_n^*$. The GA after 10 iterations, during each of them comparing the Pareto set $N = 30$ indicates Pareto optimal solutions shown in Fig. 3. Values of criteria are shown on the proper axes. These points lie on a 2-dimensional hyperplane in 3-dimensional criteria space. These three criteria, according to Eq. 9, are related to a set of design variables X_k, which describe a particular space. In order to give the background for the general optimization solution, in Figs. 4, 5 and 6 some of the chosen spaces related to the specific criteria values from an optimal set are shown. In the Fig. 4 there is a space, where at the point shown is the place where an emitted signal is strongly damped for 17 frequencies equal to proper eigenfrequecies. Figure 5 indicates the space with the closest point to the point $(0, 0)$. But at this point the number of damped frequen-

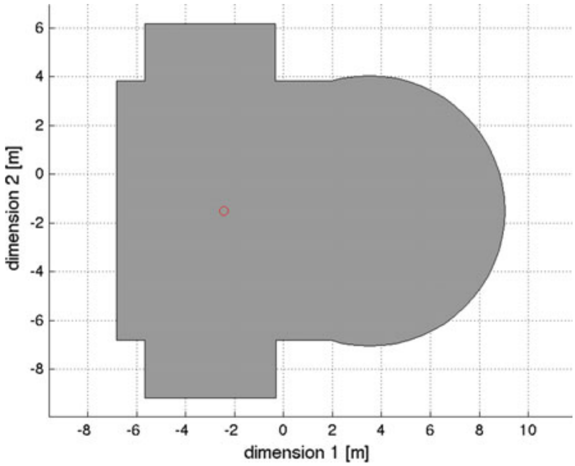

Fig. 4 The space related to the optimal solution found in the case of minimal value of criterion $C2 = -17$ ($C1 = -1; C3 = 2.8774$). The *circle* o, indicates the point, where 17 different frequencies are damped

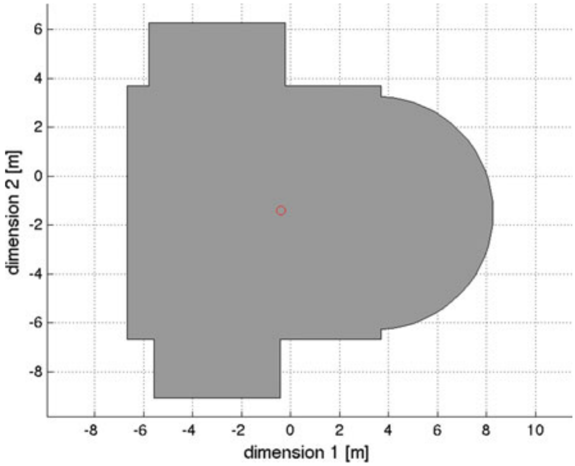

Fig. 5 The space related to the optimal solution found in the case of minimal value of criterion $C3 = 1.4954$ ($C1 = -1; C2 = -7$). The *circle* o, indicates the point the closest to point (0, 0) where 7 different frequencies are damped

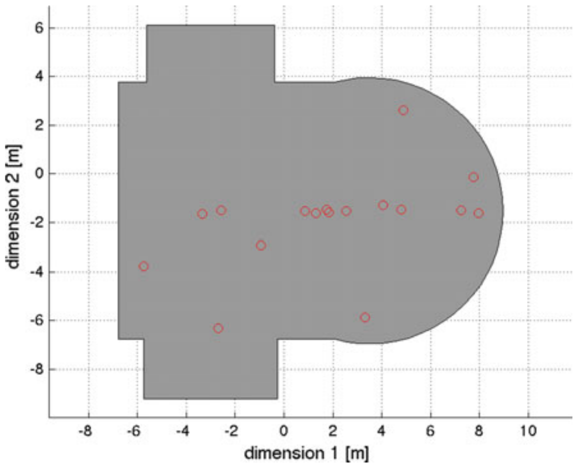

Fig. 6 The space related to the optimal solution found in the case of minimal value of criterion $C1 = -38$ ($C2 = -1; C3 = 79.9091$). The *circles* o, indicate the points where only one frequency is damped

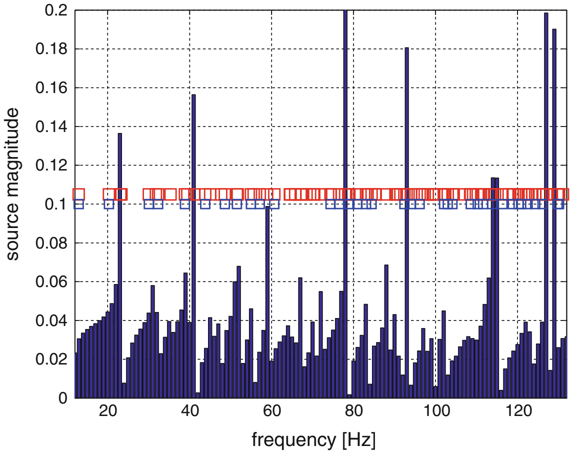

Fig. 7 The magnitude of the source at the point, found by optimization. *Red square* symbols indicate eigenfrequencies of the space. *Blue square* symbols indicate the eignfrequencies, which does not excite resonance, for the source at this point

cies equals 7. Subsequently Fig. 6 presents the space with the maximal number of points, which are found and fulfill the optimization criteria. Thirty-eight points are indicated, where only one frequency is damped.

4 Conlusions

The results presented are related to the work by [11, 13] that deals with the problem from energetic aspects. It is stated there that the vortex of acoustic intensity is characterized by zero pressure at its center. The null pressure is reached when all eigenfunctions get a zero value at the vortex center: the completely reduced acoustic pressure is in the case when all eigenfunctions get zero values in a particular point. As was stated, it is nearly impossible in reality. Therefore, the optimal solution is search and it is stated that it is possible to find the space where there are the point(s) which guarantee that the source located at these point(s) does not excite the acoustic resonance in the chosen frequency range. The field is generated by the point sound source, which by the pressure or volume impact of some magnitude influences the acoustic field. The character of the created field is well described when the modal approach is used to solve the problem in a low-frequency range and weak sound damping. The modal amplitudes (time components) can be reduced or “literally vanished” if the sound source is located in a proper point(s) inside the space. The example shows how to find the maximal field reduction for a specific source location, by “optimal shaping” the space. In this case two main approaches can be distinguished to gain a limited area with points, where the sound source is damped in a wide frequency range or many points, where the source with small dimension is damped at many possible locations, but in a narrow frequency range for each. This feature is shown for the analyzed example space. At some point, found by using optimization, the source is damped in a significant way. The simulation data in Fig. 7 illustrate this property. The square red symbols in this figure indicate eigenfrequencies of this space. The harmonic source of these or very close frequencies may excite acoustic resonance. Square blue symbols indicate the eigenfrequencies that do not excite resonances for the source at this point.

References

1. Błażejewski A.: Modal approach application and significance analysis inside bounded space in a steady state acoustic field condition. International Journal of Dynamics and Control **3** 50–57 (2015)
2. Błażejewski, A. Krzyżyński, T.: Application of genetic algorithms in multi-objective optimization in room acoustics. Logistyka **6**, 281–289 (2010)
3. Błażejewski, A. Krzyżyński, T.: Multi-objective optimization of the acoustic impedance distribution for room steady state sound field condition. in: Cempel C., Dobry W. (eds.) Vibration in Physical Systems 24 pp. 57–62 (2010)
4. Cox, T.J. D'Antonio, P. A. M.: Room sizing and optimization at low frequencies. Journal of the Audio Engineering Society **52(6)**, 640–651 (2004)
5. Dühring, M.B. Jensen, J. S. Sigmund O.: Acoustic design by topology optimization. Journal of Sound and Vibration **317**, 557–575 (2008)
6. Easwaran V, Craggs, A.: An application of acoustic finite element models to finding the reverberation times of irregular rooms. Acta Acustica united with Acustica **82**, 54–64 (1996)
7. Franzoni, L.P. Bliss, D.: A discussion of modal uncoupling and an approximate closed-formsolution for weakly coupled systems with application to acoustics. Journal of the Acoustical Society of America **103**, 1923–1932 (1998)
8. Gerretsen, E.: Estimation methods for sound levels and reverberation time in a room with irregular shape or absorption distribution. Acta Acustica united with Acustica **92**, 797–806 (2006)
9. Kutruff, H.: *Room Acoustics, fifth edition*. Taylor & Francis Group, New York (2009)
10. Luo, J. Gea, H. Optimal stiffener design for interior sound reduction using a topology optimization based approach. Journal of Vibration and Acoustics **125**, 267–273 (2003)
11. Meissner, M.: Analytical and numerical study of acoustic intensity field in irregularly shaped room. Applied Acoustics **74**, 661–668 (2013)
12. Meissner, M.: Influence of wall absorption on low-frequency dependence of reverberation time in room of irregular shape. Applied Acoustics **69**, 583–590 (2008)
13. Meissner, M.: Numerical investigation of acoustic field in enclosures: Evaluation of active and reactive components of sound intensity. Journal of Sound and Vibration **338**, 154–168 (2015)
14. Morse, P.M., B. R. Sound waves in rooms. Reviews of Modern Physics **16**, 69–150 (1994)
15. Morse, P.M., I. K.: Theoretical acoustics. Mc Graw-Hill, New York (1968)
16. Pan, J.: A note on the prediction of sound intensity. Journal of the Acoustical Society of America **93**, 1641–1644 (1993)
17. Pan, J.: A second note on the prediction of sound intensity. Journal of the Acoustical Society of America **97**, 691–694 (1995)
18. Pan, J.: A third note on the prediction of sound intensity. Journal of the Acoustical Society of America **105**, 560–562 (1999)
19. Rayna, A.L. Sancho, J.: Technical note: the influence of a room shape on speech intelligibility in rooms with varying ambient noise levels. Noise Control Engineering Journal **31**, 173–179 (1988)
20. Schroeder, M.: Reverberation: Theory and measurement. Journal of the Acoustical Society of America. Proceedings Wallace Clement Sabine Centennial Symposium (1994)

21. Schroeder, M.: The „ Schroeder frequency" revisited. Journal of the Acoustical Society of America **99(5)**, 3240–3241 (1996)
22. Schultz, T.J. Smith, P. M. Malme, C.I.: Measurement of acoustic intensity in reactive sound field. Journal of the Acoustical Society of America **57** 1263–1268 (1975)
23. Sum, K. Pan, J.: Geometrical perturbation of an inclined wall on decay times of acoustic modes in a trapezoidal cavity with an impedance surface. Journal of the Acoustical Society of America **120**, 3730–3743 (2006)
24. Vito, A.: Thesis title Thesis title Thesis title Thesis title Thesis title. PhD thesis, University/School. A sentence about the Supervisor (2015)
25. Wadbro, E. Berggren, M.: Topology optimization of an acoustic horn. Computer Methods in Applied Mechanics and Engineering **196** 420–436 (2006)
26. Waterhouse, R.: Vortex modes in rooms. Journal of the Acoustical Society of America **82**, 1782–1791 (1987)
27. Zhu, X. Zhu, Z. C. Cheng, J.: Using optimized surface modifications to improve low frequency response in a room. Applied Acoustics **65**, 841–860 (2004)

Springer

http://www.springer.com/978-3-319-42407-1

Dynamical Systems: Theoretical and Experimental Analysis
Łódź, Poland, December 7-10, 2015
Awrejcewicz, J. (Ed.)
2016, XIV, 421 p. 242 illus., 114 illus. in color., Hardcover
ISBN: 978-3-319-42407-1