

# Chapter 2

## Amplitude Modulation (AM)

### Topics

- Introduction
- Amplitude Modulation (AM)
- AM Spectrum and Bandwidth
- Double Side Band Suppressed Carrier (DSBSC)
- DSBSC Spectrum and Bandwidth
- Single Side Band (SSB) Carrier
- SSB Spectrum and Bandwidth

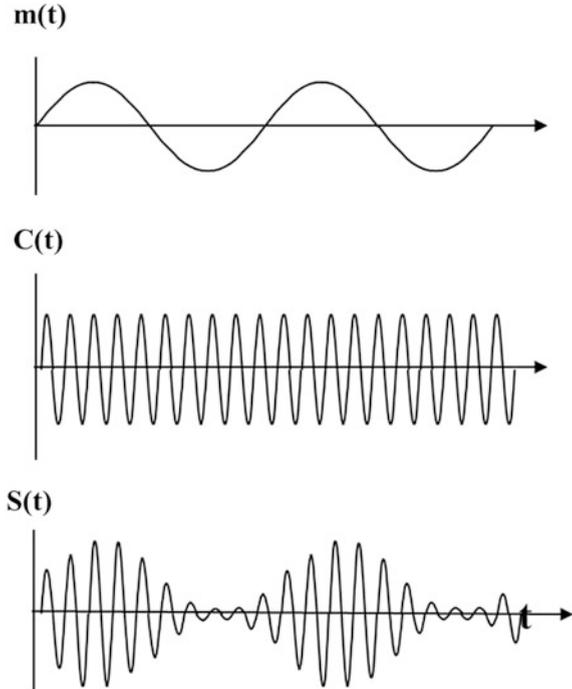
### 2.1 Introduction

By inventing the wireless transmitter or radio in 1897, the Italian physicist Tomaso Guglielmo Marconi added a new dimension to the world of communications [1, 2]. This enabled the transmission of the human voice through space without wires. For this epoch-making invention, this illustrious scientist was honored with the Nobel Prize for Physics in 1909. Even today, students of wireless or radio technologies remember this distinguished physicist with reverence. A new era began in Radio Communications.

The classical Marconi radio used a modulation technique known today as “Amplitude Modulation” or just AM, which is the main topic of this chapter. In AM, the amplitude of the carrier changes in accordance with the input analog signal, while the frequency of the carrier remains the same. This is shown in Fig. 2.1, where

- $m(t)$  is the input modulating audio signal,
- $C(t)$  is the carrier frequency, and
- $S(t)$  is the AM-modulated carrier frequency.

**Fig. 2.1** AM waveforms.  
The amplitude of the carrier changes in accordance with the input analog signal. The frequency of the carrier remains the same



As shown in the figure, the audio waveform changes the amplitude of the carrier to determine the envelope of the modulated carrier. This enables the receiver to extract the audio signal by demodulation. Notice that the amplitude of the carrier changes in accordance with the input signal, while the frequency of the carrier does not change after modulation. However, it can be shown that the modulated carrier  $S(t)$  contains several spectral components, requiring frequency domain analysis. In an effort to examine this, this chapter will present the following topics:

- Amplitude Modulation (AM) and AM spectrum
- Double Sideband-suppressed carrier (DSBSC) and DSBSC spectrum
- Single sideband (SSB) carrier and SSB spectrum

In the following sections, the above disciplines in AM modulation will be presented along with the respective spectrum and bandwidth. These materials have been augmented by diagrams and associated waveforms to make them easier for readers to grasp.

## 2.2 Amplitude Modulation

Amplitude Modulation (AM) is a method of analog modulation that utilizes amplitude variations of the relative amplitude of the carrier frequency [3–5]. The signal to be modulated and transmitted is analog. This is referred to as AM, where the amplitude of the carrier changes in accordance with the input signal.

Figure 2.2 shows a functional diagram of a typical AM modulator for a single tone. Here,  $m(t)$  is the input analog signal we want to transmit,  $C(t)$  is the carrier frequency without modulation, and  $S(t)$  is the output AM-modulated carrier frequency. These parameters are described below:

- $m(t) = A_m \cos(2\pi f_m t)$
- $C(t) = A_c \cos(2\pi f_c t) f_c \gg f_m$
- $S(t) = [1 + m(t)]C(t)$
- $\quad = C(t) + m(t)C(t)$

(2.1)

Therefore,

When  $m(t) = 0$ :

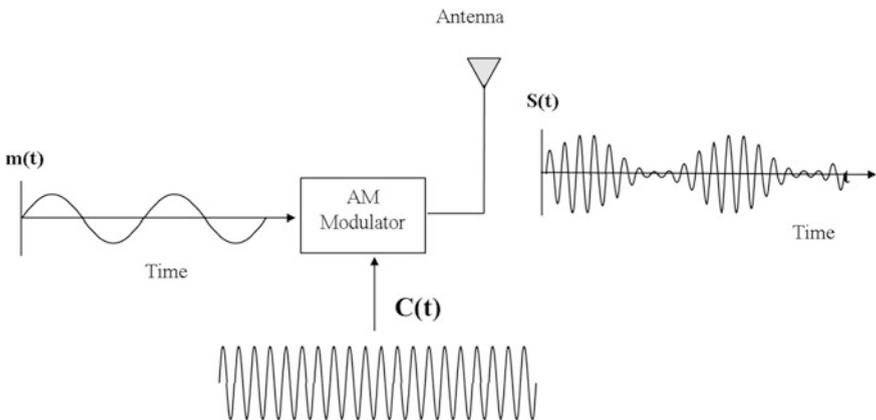
- $S(t) = A_c \cos(2\pi f_c t)$

(2.2)

When  $m(t) = A_m \cos(2\pi f_m t)$ :

- $S(t) = A_c \cos(2\pi f_c t) + A_c A_m \cos(2\pi f_m t) \cos(2\pi f_c t)$

(2.3)



**Fig. 2.2** Illustration of amplitude modulation. The amplitude of the carrier  $C(t)$  changes in accordance with the input modulating signal  $m(t)$ .  $S(t)$  is the modulated waveform which is transmitted by the antenna

In the above equation, we see that:

- The first term is the carrier only, which does not have information
- The second **term** contains **the** information, which has several spectral components, requiring further analysis to quantify them.

## 2.3 AM Spectrum and Bandwidth

In wireless communications, the scarcity of RF spectrum is well known. For this reason, we have to be vigilant about using transmission bandwidth and modulation. The transmission bandwidth depends on the following:

- Spectral response of the input modulating signal
- Spectral response of the carrier frequency and
- Modulation type (AM, FM ASK, FSK, PSK, etc.)

Let us take a closer look!

### 2.3.1 Spectral Response of the Input Modulating Signal

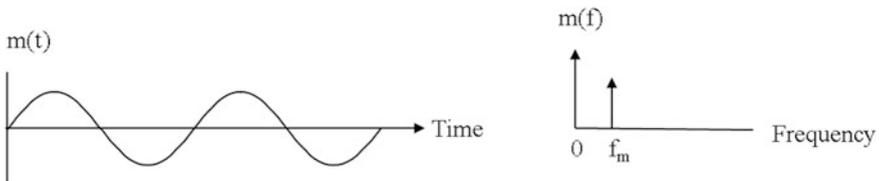
In AM, the input modulating signal is a continuous time low-frequency analog signal. For simplicity, we use a sinusoidal waveform, which is periodic and continuous with respect to time. It has one frequency component. For example, the sine wave is described by the following time domain equation:

$$V(t) = V_p \sin(\omega_m t) \quad (2.4)$$

where

- $V_p$  = Peak voltage
- $\omega_m = 2\pi f_m$
- $f_m$  = Input modulating frequency in Hz

Figure 2.3 shows the characteristics of a sine wave and its spectral response. Since the frequency is constant, its spectral response is located in the horizontal axis at  $f_m$ , and the peak voltage is shown in the vertical axis. The corresponding bandwidth is zero.



**Fig. 2.3** A low-frequency sine wave and its frequency response

### 2.3.2 Spectral Response of the Carrier Frequency

A carrier frequency ( $f_c$ ) is essentially a sinusoidal waveform, which is periodic and continuous with respect to time. It has one frequency component, which is much higher than the input modulating frequency ( $f_c \gg f_m$ ). For example, the sine wave is described by the following time domain equation:

$$V(t) = V_p \sin(\omega_c t) \tag{2.5}$$

where

$$V_p = \text{Peak voltage}$$

- $\omega_c = 2\pi f_c$
- $f_c = \text{Carrier frequency in Hz}$

Figure 2.4 shows the characteristics of a sine wave and its spectral response. Since the frequency is constant, its spectral response is located in the horizontal axis at  $f_c$  and the peak voltage is shown in the vertical axis. The corresponding bandwidth is zero.

### 2.3.3 AM Spectrum and Bandwidth

Let us consider the AM signal again, which was derived earlier:

$$S(t) = A_c \cos(2\pi f_c t) + A_c A_m \cos(2\pi f_m t) \cos(2\pi f_c t) \tag{2.6}$$

Using the following trigonometric identity:

$$\cos A \cos B = 1/2 \cos(A + B) + 1/2 \cos(A - B) \tag{2.7}$$

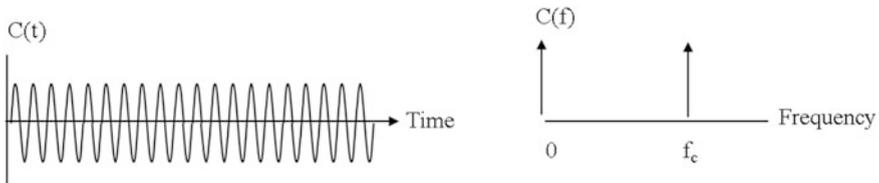


Fig. 2.4 A high-frequency sine wave and its frequency response

where

- $A = 2\pi (f_c + f_m)t$
- $B = 2\pi (f_c - f_m)t$

we get

$$S(t) = A_c \cos(2\pi f_c t) + (1/2)A_c A_m \cos[2\pi(f_c + f_m)t] + (1/2)A_c A_m \cos[2\pi(f_c - f_m)t] \tag{2.8}$$

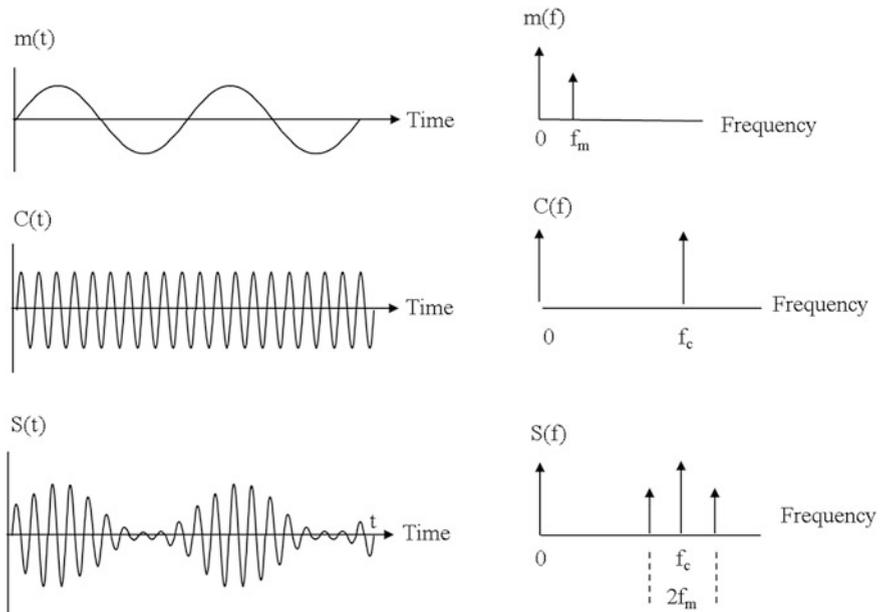
This is the spectral response of the AM-modulated signal. It has three spectral components:

- The carrier:  $f_c$
- Upper sideband:  $f_c + f_m$
- Lower sideband:  $f_c - f_m$

where  $f_c$  is the carrier frequency and  $f_m$  is the input modulating frequency. This is shown in Fig. 2.5. The AM bandwidth (BW) is given by

$$BW = 2f_m \tag{2.9}$$

Notice that the power is distributed among the sidebands and the carrier, where the carrier does not contain any information. Only the sidebands contain the information. Therefore, AM is inefficient in power usage.



**Fig. 2.5** AM spectrum. The bandwidth is given by  $2 f_m$

### 2.3.4 AM Response Due to Low and High Modulating Signals

If  $m(t)$  has a peak positive value of less than +1 and a peak negative value of higher than -1, then the modulation is less than 100 %. This is shown in Fig. 2.7.

On the other hand, if  $m(t)$  has a peak positive value of +1 and a peak negative value of -1, then the modulation is 100 % [3–5]. Therefore,

- For  $m(t) = -1$  :  $S(t) = A_c[1 - 1] \cos(2\pi c t) = 0$  (2.10)

- For  $m(t) = +1$  :  $S(t) = A_c[1 + 1] \cos(2\pi f_c t)$   
 $= 2A_c \cos(2\pi f_c t)$  (2.11)

This is called 100 % modulation, as shown in Fig. 2.6. The percent modulation is described by the following equation:

The overall modulation percentage is:

$$\begin{aligned} \% \text{ Overall Modulation} &= \frac{A_{\max} - A_{\min}}{A_c} \times 100 \\ &= \frac{\max[m(t)] - \min[m(t)]}{2A_c} \times 100. \end{aligned} \tag{2.12}$$

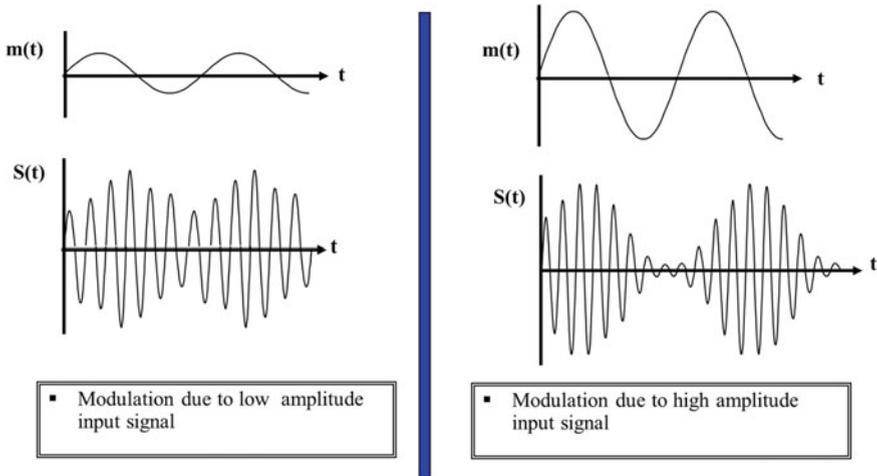
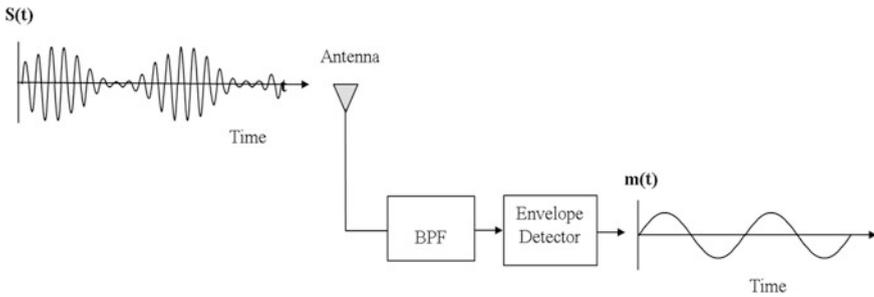


Fig. 2.6 Amplitude modulation due to low and high modulating signals



**Fig. 2.7** AM demodulation technique. As the signal enters the receiver, it passes through the band-pass filter, which is tuned to the carrier frequency  $f_0$ . Next, the recovered signal is passed through an envelope detector to recover the original signal that was transmitted

### 2.3.5 AM Demodulation

Once the modulated analog signal has been transmitted, it needs to be received and demodulated. This is accomplished by the use of a band-pass filter that is tuned to the appropriate carrier frequency. Figure 2.7 shows the conceptual model of the AM receiver. As the signal enters the receiver, it passes through the band-pass filter, which is tuned to the carrier frequency  $f_0$ . Next, the recovered signal is passed through an envelope detector to recover the original signal that was transmitted.

### 2.3.6 Drawbacks in AM

- The modulated signal contains the carrier; carrier takes power and it does not have the information
- Therefore, AM is inefficient in power usage
- Moreover, there are two sidebands, containing the same information
- It is bandwidth inefficient
- AM is also susceptible to interference, since it affects the amplitude of the carrier.

Therefore, a solution is needed to improve bandwidth and power efficiency.

**Problem 2.1** Given:

- Input modulating frequency  $f_m = 10$  kHz
- Carrier frequency  $f_c = 400$  kHz

Find

- Spectral components
- Bandwidth

**Solution**

Spectral Components:

- $f_c = 400$  kHz
- $f_c + f_m = 400$  kHz + 10 kHz = 410 kHz
- $f_c - f_m = 400$  kHz - 10 kHz = 390 kHz

Bandwidth

- $BW = 2 f_m = 2 \times 10$  kHz = 20 kHz.

**2.4 Double Sideband-Suppressed Carrier (DSBSC)****2.4.1 DSBSC Modulation**

Double sideband-suppressed carrier (DSBSC), also known as product modulator, is an AM signal that has a suppressed carrier [3–5]. Let us take the original AM signal once again, as given below:

$$S(t) = A_c \cos(2\pi f_c t) + (1/2)A_c A_m \cos[2\pi(f_c + f_m)t] + (1/2)A_c A_m \cos[2\pi(f_c - f_m)t] \quad (2.13)$$

Notice that there are three spectral components:

- The first term is the carrier only, which does not have any information
- The second and third terms contain information.

In DSBSC, we suppress the carrier, which is the first term that does not have any information. Therefore, by suppressing the first term we obtain the following:

$$S(t) = (1/2)A_c A_m \cos[2\pi(f_c + f_m)t] + (1/2)A_c A_m \cos[2\pi(f_c - f_m)t] \quad (2.14)$$

Next, we use the following trigonometric identities:

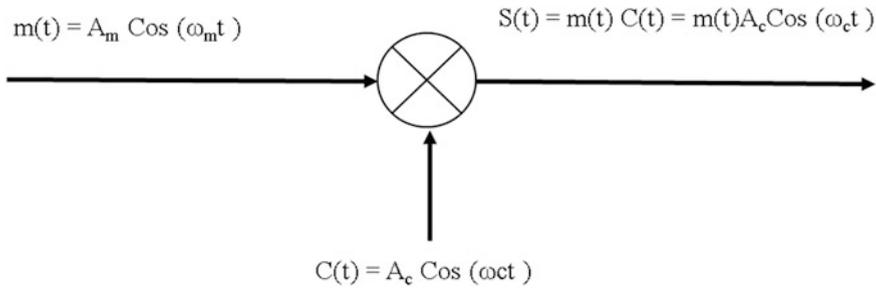
- $\cos(A + B) = \cos A \cos B - \sin A \sin B$
- $\cos(A - B) = \cos A \cos B + \sin A \sin B$

With  $A = 2\pi f_m t = \omega_m t$  and  $B = 2\pi f_c t = \omega_c t$ , we obtain:

$$S(t) = A_c A_m \cos(\omega_m t) \cos(\omega_c t) \quad (2.15)$$

Now, define

- $m(t) = A_m \cos(\omega_m t)$
- $C(t) = A_c \cos(\omega_c t)$



**Fig. 2.8** Symbolic representation of DSBSC, also known as product modulator

Then, we can write the above equations as:

$$S(t) = m(t) C(t) \tag{2.16}$$

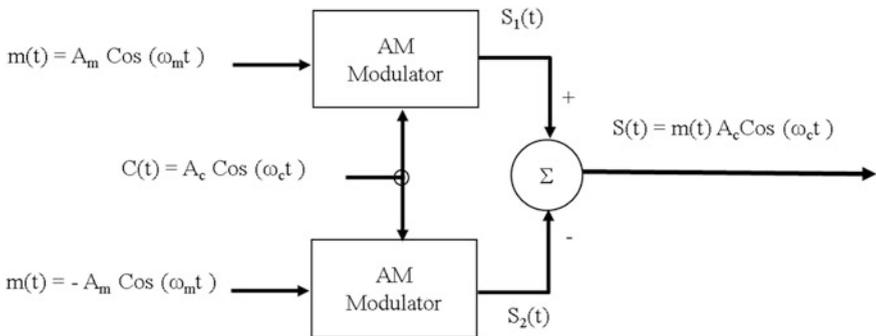
This is the DSBSC waveform. Since the output is the product of two signals, it is also known as product modulator. The symbolic representation is given in Fig. 2.8, where  $m(t)$  is the input modulating signal and  $C(t)$  is the carrier frequency.

### 2.4.2 Generation of DSBSC Signal

A DSBSC signal can be generated using two AM modulators arranged in a balanced configuration as shown in Fig. 2.9 [3–5]. The outcome is a cancellation of the discrete carrier. Also, the output is the product of two inputs:  $S(t) = m(t) C(t)$ . This is why it is called “product modulator.”

#### Proof of DSBSC

Consider the DSBSC modulator as shown in Fig. 2.9. Here, the AM modulators generate  $S_1(t)$  and  $S_2(t)$ , which are given by:



**Fig. 2.9** Construction of DSBSC modulator. The output is the product of two signals

$$S_1(t) = A_c[1 + m(t)] \cos(\omega_c t) \quad (2.17)$$

$$S_2(t) = A_c[1 - m(t)] \cos(\omega_c t)$$

Subtracting  $S_2(t)$  from  $S_1(t)$ , we essentially cancel the carrier to obtain:

$$\begin{aligned} S(t) &= S_1(t) - S_2(t) \\ &= 2m(t)A_c \cos(\omega_c t) \end{aligned} \quad (2.18)$$

Therefore, except for the scaling factor 2, the above equation is exactly the same as the desired DSBSC waveform shown earlier, which does not have the carrier. In other words, the carrier has been suppressed, hence the name double sideband-suppressed carrier (DSBSC).

### 2.4.3 DSBSC Spectrum and Bandwidth

We begin with the DSBSC-modulated signal:

$$S(t) = 2m(t)A_c \cos(\omega_c t) \quad (2.19)$$

where

$$m(t) = A_m \cos(\omega_m t)$$

Therefore,

$$S(t) = 2A_c A_m \cos(\omega_m t) \cos(\omega_c t) \quad (2.20)$$

This is the desired DSBSC waveform for spectral analysis.

Now, use the trigonometric identity:

- $\cos A \cos B = 1/2 \cos(A + B) + 1/2 \cos(A - B)$  (2.21)

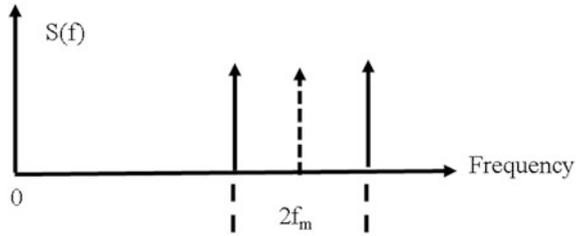
where

$$A = \omega_m t = 2\pi f_m t \text{ and } B = \omega_c t = 2\pi f_c t$$

Therefore,

$$\begin{aligned} S(t) &= A_c[\cos(\omega_c + \omega_m)t + \cos(\omega_c - \omega_m)t] \\ &= A_c[\cos 2\pi(f_c + f_m)t + \cos 2\pi(f_c - f_m)t] \end{aligned} \quad (2.22)$$

**Fig. 2.10** DSBSC spectrum where the carrier frequency is suppressed. The bandwidth is given by  $2 f_m$



Notice that the carrier power is distributed among the sidebands (Fig. 2.10). Therefore, it is more efficient. The bandwidth is given by:

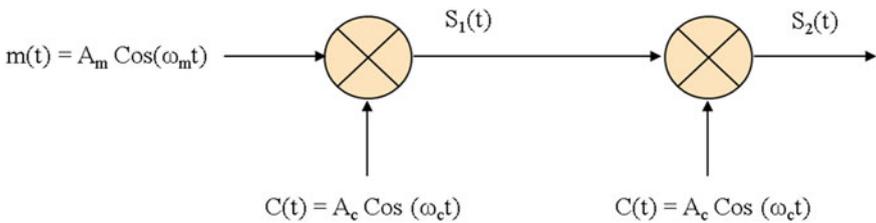
$$BW = 2f_m \tag{2.23}$$

### 2.4.4 DSBSC Drawbacks

- There are two identical sidebands.
- Each sideband contains the same information
- Bandwidth is  $2 f_m$
- Unnecessary power usage

Therefore, a solution is needed to improve bandwidth efficiency.

**Problem 2.2** Given: Two product modulators using identical carriers are connected in a series, as shown below:



Find

- (a) The output waveform  $S_2(t)$
- (b) What is the function of this circuit?

**Solution**

- (a)  $S_1(t) = m(t) C(t)$   
 $S_2(t) = S_1(t) C(t) = m(t) C^2(t) = A_m \cos(\omega_m t) A_c^2 \cos 2(\omega_c t)$
- (b) The function of the circuit is to demodulate DSBSC signals, where the carrier frequency is filtered out.

## 2.5 Single Sideband (SSB) Modulation

### 2.5.1 Why SSB Modulation?

- The basic AM has a carrier which does not carry information—*Inefficient power usage*
- The basic AM has two sidebands contain the same information—*Additional loss of power*
- DSBSC has two sidebands, containing the same information—*Loss of power*
- Therefore, the basic AM and DSBSC are bandwidth and power inefficient
- SSB is bandwidth and power efficient.

### 2.5.2 Generation of SSB-Modulated Signal

Single sideband (SSB) modulation uses two product modulators as shown in Fig. 2.11 [3–5], where

$$m(t) = A_m \cos(\omega_m t) \tag{2.24}$$

$$m(t)^* = A_m \sin(\omega_m t) - \text{(Hilbert Transform)} \tag{2.25}$$

$$C(t) = A_c \cos(\omega_c t) \tag{2.26}$$

$$C(t)^* = A_c \sin(\omega_c t) - \text{(Hilbert Transform)} \tag{2.27}$$

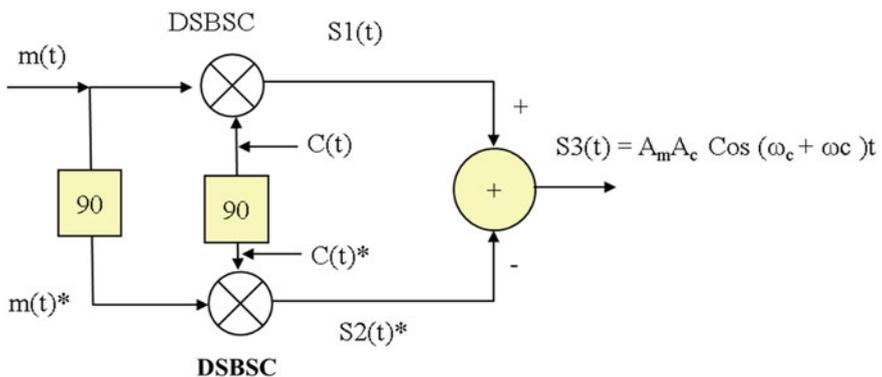


Fig. 2.11 Generation of SSB signal

Solving for  $S_1$ ,  $S_2$ , and  $S_3$ , we obtain:

$$\bullet S_1(t) = A_c A_m \cos(\omega_m t) \cos(\omega_c t) \quad (2.28)$$

$$\bullet S_2(t) = A_c A_m \sin(\omega_m t) \sin(\omega_c t) \quad (2.29)$$

$$\begin{aligned} S_3(t) &= S_1(t) - S_2(t) \\ &= A_c A_m \cos(\omega_m t) \cos(\omega_c t) - A_c A_m \sin(\omega_m t) \sin(\omega_c t) \end{aligned} \quad (2.30)$$

Using the following formula:

- $\cos A \cos B = 1/2 \cos(A + B) + 1/2 \cos(A - B)$
- $\sin A \sin B = 1/2 \cos(A - B) - 1/2 \cos(A + B)$

Solving for  $S_3$ , we get:

$$S_3 = A_c A_m \cos(\omega_c + \omega_m)t \quad (2.31)$$

In the above equation,  $S_3(t)$  is the desired SSB signal, which is the upper sideband only.

### 2.5.3 SSB Spectrum and Bandwidth

Let us consider the SSB signal again:

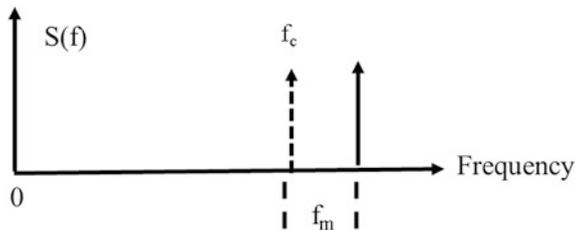
$$\begin{aligned} S_3 &= A_c A_m \cos(\omega_c + \omega_m)t \\ &= A_c A_m \cos 2\pi(f_c + f_m)t \end{aligned} \quad (2.32)$$

Here, we see that the SSB spectrum contains only one sideband. Therefore, it is more efficient. The SSB bandwidth is given by:

$$\text{SSB BW} = f_m \quad (2.33)$$

Figure 2.12 displays the SSB spectrum.

**Fig. 2.12** SSB spectrum showing the upper sideband. The SSB bandwidth is  $f_m$



**Problem 2.3**

Given:

- $m(t) = A_m \cos(\omega_m t)$
- $m(t)^* = A_m \sin(\omega_m t)$  – (Hilbert Transform)
- $C(t) = A_c \cos(\omega_c t)$
- $C(t)^* = A_c \sin(\omega_c t)$  – (Hilbert Transform)

Design an SSB modulator to realize the lower sideband. Sketch the spectral response.

**Solution**

Solving for  $S_1$  and  $S_2$ , we obtain:

- $S_1(t) = A_c A_m \cos(\omega_m t) \cos(\omega_c t)$
- $S_2(t) = A_c A_m \sin(\omega_m t) \sin(\omega_c t)$

Obtain  $S_3$  as:

$$S_3(t) = S_1(t) + S_2(t) \\ = A_c A_m \cos(\omega_m t) \cos(\omega_c t) + A_c A_m \sin(\omega_m t) \sin(\omega_c t)$$

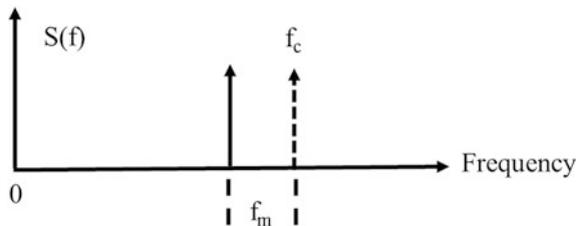
Using the following formula:

- $\cos A \cos B = 1/2\cos(A + B) + 1/2\cos(A - B)$
- $\sin A \sin B = 1/2\cos(A - B) - 1/2\cos(A + B)$

Solving for  $S_3$ , we get:

$$S_3 = A_c A_m \cos(\omega_m - \omega_c)t \\ = A_c A_m \cos 2\pi(f_c - f_m)t$$

In the above equation,  $S_3(t)$  is the desired SSB signal, which is the lower sideband only. The spectral response, showing the lower sideband, is presented below.



## 2.6 Conclusions

This chapter presents the key concepts and underlying principles of Amplitude Modulation. It was shown how the audio waveform changes the amplitude of the carrier to determine the envelope of the modulated carrier. It was also shown that the modulated carrier contains several spectral components that lead to DSBSC and SSB modulation techniques. In particular, the following topics were presented in this chapter:

- Amplitude Modulation (AM)
- AM spectrum and bandwidth
- Double sideband-suppressed carrier (DSBSC)
- DSBSC spectrum and bandwidth
- Single sideband (SSB)
- SSB spectrum and bandwidth

These materials have been augmented by diagrams and associated waveforms to make them easier for readers to grasp.

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Radio Frequency Modulation Made Easy

Faruque, S.

2017, IX, 104 p. 68 illus., 36 illus. in color., Softcover

ISBN: 978-3-319-41200-9