

Contents

1	Introduction	1
1.1	Micro from Macro	1
1.2	Outline of the Topics	4
1.3	About Citations	6
1.4	Conventions	6
1.5	Measure Units	7
	References	8
2	Hot Big Bang Model	13
2.1	Cosmic Expansion and Cosmological Principle	14
2.1.1	The Universe at Large Scales	14
2.1.2	Friedmann–Lemaître–Robertson–Walker Background	19
2.2	Einstein and Continuity Equations	24
2.2.1	Energy Conditions	26
2.3	Perfect Fluid	27
2.3.1	Scalar Field	30
2.4	Friedmann Equations	31
2.5	Content of the Universe	33
2.5.1	Dust and Radiation	33
2.5.2	Hot Big Bang and the Big-Bang Problem	36
2.5.3	Dark Energy and the cosmological Constant Problem	39
2.5.4	Spatial Curvature and Topology	43
2.6	An Obscure Big Picture	44
2.7	Problems and Solutions	46
	References	58
3	Cosmological Perturbations	63
3.1	Metric Perturbations	64
3.1.1	Linearized Einstein Equations	64
3.1.2	Gauge Invariance and Gauge Fixing	66

3.1.3	Cosmological Horizons	66
3.1.4	Separate Universe Approach	68
3.2	Linear Tensor Perturbations	70
3.2.1	Transverse-Traceless Gauge	70
3.2.2	Equation of Motion	71
3.2.3	Mukhanov–Sasaki Equation and Solution	71
3.2.4	Discovery of Gravitational Waves	74
3.3	Scalar Perturbations	76
3.3.1	Non-linear Perturbations	76
3.3.2	Non-linear Perturbations at Large Scales	77
3.3.3	Linear Perturbations at Large Scales	78
3.4	Gaussian Random Fields	81
3.4.1	Power Spectrum	82
3.4.2	Bispectrum and Trispectrum	83
3.5	Problems and Solutions	84
	References	88
4	Cosmic Microwave Background	91
4.1	Cosmic Background Radiation	93
4.1.1	Boltzmann Equation and Spectral Distortions	94
4.1.2	Last-Scattering Surface	97
4.2	Temperature Anisotropies: Formalism	97
4.2.1	Spherical Harmonics	97
4.2.2	Gaussian Spectrum	99
4.2.3	Ergodic Hypothesis and Cosmic Variance	102
4.3	Temperature Power Spectrum	104
4.3.1	What we Observe	104
4.3.2	Angular Scales	106
4.3.3	Sachs–Wolfe Plateau ($\ell \lesssim 60$)	110
4.3.4	Acoustic Peaks ($60 \lesssim \ell \lesssim 1000$)	114
4.3.5	Damping Tail ($\ell \gtrsim 1000$)	117
4.3.6	Secondary Anisotropies	118
4.4	Cosmological Parameters and Observational Constraints	119
4.4.1	Shape of the Angular Spectrum and Parameters	119
4.4.2	Primordial Spectra	120
4.5	Polarization	126
4.5.1	Formalism	128
4.5.2	Spectra	130
4.5.3	What we Observe	131
4.6	Non-Gaussianity	135
4.6.1	Bispectrum	135
4.6.2	Trispectrum	137

4.6.3	Physical Origin	138
4.6.4	Current Estimates	141
4.7	Problems and Solutions	142
	References	146
5	Inflation	153
5.1	Problems of the Hot Big Bang Model	154
5.1.1	Planck and GUT Scale	155
5.1.2	Flatness Problem	156
5.1.3	Horizon Problem	157
5.1.4	Monopole Problem	158
5.1.5	Primordial Seeds Problem	159
5.2	Inflationary Mechanism	159
5.2.1	Solution of the Flatness Problem	160
5.2.2	Solution of the Horizon Problem	161
5.2.3	Solution of the Monopole Problem	163
5.2.4	Solution of the Primordial Seeds Problem	163
5.3	Cold Big Bang	163
5.3.1	Equation of State	164
5.3.2	Chaotic Inflation	165
5.3.3	Reheating	167
5.3.4	Observable Inflation	169
5.3.5	Timeline of the Early Universe	170
5.4	Scalar Field: Background Dynamics	171
5.4.1	Hamilton–Jacobi Formalism	171
5.4.2	Slow-Roll Parameters	172
5.4.3	Inflationary Attractor	176
5.5	Models of Inflation	178
5.5.1	Large-Field Models	179
5.5.2	Small-Field Models	183
5.5.3	Multi-field Inflation	185
5.6	First Glimpse of the Quantum Universe	187
5.6.1	Decoherence	188
5.6.2	From Quantum Fields to Classical Spectra	189
5.6.3	Choice of Vacuum	191
5.6.4	Mukhanov–Sasaki Equation Revisited	193
5.6.5	Eternal Inflation	197
5.7	Cosmological Spectra	198
5.7.1	Gaussianity	198
5.7.2	Linear Tensor Perturbations	199
5.7.3	Linear Scalar Perturbations	200
5.7.4	Consistency Relations and Lyth Bound	204
5.8	Non-Gaussianity	205
5.8.1	Stochastic Inflation	206
5.8.2	Multi-field Non-Gaussianity	213

5.9	Observational Constraints on Inflation	213
5.9.1	Temperature Spectra	213
5.9.2	Polarization	216
5.9.3	Non-Gaussianity	217
5.10	Unsolved and New Problems	218
5.10.1	Graceful Entry Problem	218
5.10.2	Graceful Exit Problem	219
5.10.3	Trans-Planckian Problem	219
5.10.4	Naturalness or Model-Building Problem	220
5.11	The Inflaton and Particle Physics	221
5.11.1	Not Only Scalars	221
5.11.2	Higgs Inflation	222
5.12	Supersymmetry and Supergravity	224
5.12.1	Global Supersymmetry	225
5.12.2	Supergravity	228
5.12.3	η -problem	229
5.12.4	Inflation in Supergravity	230
5.13	Problems and Solutions	237
	References	239
6	Big-Bang Problem	261
6.1	Spacetimes and Singularities	262
6.1.1	Globally Hyperbolic Spacetimes	262
6.1.2	Focusing Theorems	265
6.1.3	Classifications of Singularities	267
6.2	Singularity Theorems	269
6.2.1	Hawking–Penrose Theorems	269
6.2.2	Borde–Vilenkin Theorems	272
6.2.3	Borde–Guth–Vilenkin Theorem	273
6.2.4	An Undecided Issue	275
6.3	BKL Singularity	278
6.3.1	Tetrads and Bianchi Models	278
6.3.2	Kasner Metric	280
6.3.3	Generalized Kasner Metric	282
6.3.4	Mixmaster Dynamics	284
6.3.5	BKL Conjecture	291
6.4	Problems and Solutions	293
	References	295
7	Cosmological Constant Problem	301
7.1	The Problem in Field Theory	302
7.1.1	Spontaneous Symmetry Breaking and Dynamical Λ	302
7.1.2	Zero-Point Energy and Higher Loops	305
7.1.3	Supersymmetry and Supergravity	308

- 7.2 Other Versions of the Problem and Strategies 310
 - 7.2.1 Broken Symmetries 310
 - 7.2.2 The 4π Puzzle 311
 - 7.2.3 UV or IR Problem? Strategies for a Solution 313
- 7.3 Quintessence 315
 - 7.3.1 Tracking, Freezing and Thawing 316
 - 7.3.2 Periodic and Power-Law Potentials 321
 - 7.3.3 Exponential and Hyperbolic Potentials 322
 - 7.3.4 Inverse Power-Law Potential 323
 - 7.3.5 Other Potentials 324
 - 7.3.6 Quintessence and the Inflaton 325
 - 7.3.7 Summary 326
- 7.4 Scalar-Tensor Theories 327
 - 7.4.1 Motivations 328
 - 7.4.2 Conformal Transformations 328
 - 7.4.3 Perturbations, Quantum Theory
and Extended Inflation 332
 - 7.4.4 Cosmological Constant Problem 333
 - 7.4.5 Experimental Bounds and Chameleon Mechanism 334
- 7.5 Higher-Order and Higher-Derivative Gravity Models 336
 - 7.5.1 Motivation and Ghosts 336
 - 7.5.2 General $f(R)$ Action 338
 - 7.5.3 Palatini Formulation 340
 - 7.5.4 Form of $f(R)$ 340
 - 7.5.5 Horndeski Theory and Extensions 342
- 7.6 Other Approaches 343
 - 7.6.1 Varying Couplings 343
 - 7.6.2 Void Models 344
 - 7.6.3 Unimodular Gravity 346
 - 7.6.4 Analogue Gravity and Condensates 348
- 7.7 Emergent Gravity 351
 - 7.7.1 Rindler Observer and Null Congruences 352
 - 7.7.2 Dynamics 355
 - 7.7.3 Holographic Equipartition 356
 - 7.7.4 Cosmological Constant Problem 360
- 7.8 Problems and Solutions 361
- References 369
- 8 The Problem of Quantum Gravity 389**
 - 8.1 Do We Need to Quantize Gravity? 390
 - 8.2 Perturbative Quantum Gravity 392
 - 8.2.1 Supergravity 397
 - 8.2.2 Effective Field Theory 397
 - 8.2.3 Resummed Quantum Gravity 398
 - 8.3 Approaches to Quantum Gravity 400

8.4	Problems and Solutions	402
	References	402
9	Canonical Quantum Gravity	407
9.1	Canonical Variables in General Relativity	408
9.1.1	First-Order Formalism and Parity	408
9.1.2	Hamiltonian Analysis	414
9.1.3	Ashtekar–Barbero Variables	424
9.1.4	ADM Variables	426
9.2	Wheeler–DeWitt Equation	428
9.2.1	Superspace and Quantization	429
9.2.2	Semi-classical States	432
9.2.3	Boundary Conditions	437
9.3	Some Features of Loop Quantum Gravity	439
9.4	Cosmological Constant Problem	442
9.4.1	Chern–Simons State	443
9.4.2	Λ as a Condensate?	447
9.5	Problems and Solutions	451
	References	459
10	Canonical Quantum Cosmology	467
10.1	Mini-superspace	468
10.1.1	Classical FLRW Hamiltonian	469
10.2	Wheeler–DeWitt Quantum Cosmology	471
10.2.1	de Sitter Solutions and Probability of Inflation	473
10.2.2	Massless Scalar Field and Group Averaging	476
10.2.3	Quantum Singularity	480
10.2.4	Cosmological Constant and the Multiverse	482
10.2.5	Perturbations and Inflationary Observables	484
10.3	Loop Quantum Cosmology	489
10.3.1	Classical FLRW Variables and Constraints	490
10.3.2	Quantization and Inverse-Volume Spectrum	493
10.3.3	Mini-superspace Parametrization	495
10.3.4	Quantum Hamiltonian Constraint	496
10.3.5	Models with Curvature or a Cosmological Constant ...	500
10.3.6	Homogeneous Effective Dynamics	501
10.3.7	Singularity Resolved?	508
10.3.8	Lattice Refinement: Quantum Corrections Revisited ...	510
10.3.9	Perturbations and Inflationary Observables	516
10.3.10	Inflation in Other Approaches	525
10.3.11	Is There a Bounce?	526
10.4	Problems and Solutions	528
	References	531

11	Cosmology of Quantum Gravities	543
11.1	Hausdorff and spectral dimension	545
11.2	Asymptotic Safety	547
11.2.1	Framework	548
11.2.2	Cosmology	551
11.3	Causal Dynamical Triangulations	555
11.3.1	Framework	555
11.3.2	Cosmology	561
11.4	Spin Foams	563
11.4.1	Framework	563
11.4.2	Cosmology	564
11.5	Group Field Theory	569
11.5.1	Framework	569
11.5.2	Cosmology	572
11.6	Causal Sets	584
11.6.1	Framework	584
11.6.2	Cosmology	588
11.7	Non-commutative Spacetimes	591
11.7.1	Framework	591
11.7.2	Cosmology	593
11.8	Non-local Gravity	596
11.8.1	Non-locality	596
11.8.2	Framework	599
11.8.3	Cosmology	603
11.9	Comparison of Quantum-Gravity Models	604
	References	607
12	String Theory	625
12.1	Bosonic String	627
12.1.1	Classical Free Strings and Branes	627
12.1.2	D-Branes	631
12.1.3	Quantum Strings and Critical Dimension	632
12.1.4	Interactions	637
12.1.5	Low-Energy Limit	638
12.1.6	String Field Theory	640
12.2	Superstring	642
12.2.1	Action	643
12.2.2	Quantization	645
12.2.3	Type-I Superstring	646
12.2.4	Type-II Superstrings	647
12.2.5	Interactions and Anomaly Cancellation	648
12.2.6	Heterotic Superstrings	649
12.2.7	Massless Spectra and Low-Energy Limits	651
12.2.8	Branes	653
12.2.9	Superstring Field Theory	655

12.3	Compactification	656
12.3.1	T-Duality	656
12.3.2	Spontaneous Compactification	658
12.3.3	Calabi–Yau Spaces and Orbifolds	658
12.3.4	Cycles and Fluxes	662
12.3.5	Moduli	663
12.3.6	Stacking Branes	668
12.3.7	Flux Compactification	669
12.3.8	String Theory and the Standard Model	671
12.3.9	Anti-de Sitter Vacua	671
12.4	Dualities and M-Theory	678
12.5	Problems and Solutions	682
	References	683
13	String Cosmology	701
13.1	String Landscape	703
13.1.1	de Sitter Vacua	705
13.1.2	Cosmological Constant	709
13.1.3	Open Problems	714
13.2	Inflation in the Landscape	715
13.2.1	Single-Field Inflation	716
13.2.2	Large-Field Models and the Weak Gravity Conjecture	718
13.2.3	Multi-field Inflation	719
13.2.4	Moduli Problem and η -Problem	720
13.3	Size Moduli Inflation	720
13.3.1	Large-Volume Inflation	721
13.3.2	Volume-Modulus Inflation	725
13.3.3	Fluxless Inflation	727
13.4	Axion Inflation	728
13.4.1	Racetrack Axion Inflation	729
13.4.2	Axion Valley	730
13.4.3	N -flation	732
13.4.4	Aligned and Hierarchical Axion Inflation	734
13.4.5	Monodromy Inflation	735
13.4.6	Problems with Axion Inflation and Ways Out	740
13.5	Slow-Roll D-Brane Inflation	742
13.5.1	Early Brane-Inflation Models	742
13.5.2	Warped D-Brane Inflation and $\mathbb{K}L\mathbb{M}T$ Model	744
13.5.3	Cosmological $\mathbb{K}L\mathbb{M}T$ Dynamics	748
13.5.4	Refinements and Related Models	750
13.5.5	Why the Tensor Spectrum Is Small	753
13.6	DBI Inflation	754
13.6.1	Setting	754
13.6.2	UV Model	755
13.6.3	IR Model	758

- 13.7 Other Models 761
 - 13.7.1 Braneworld 761
 - 13.7.2 Cosmological Tachyon 765
 - 13.7.3 Modified Gravity 767
 - 13.7.4 Non-local Models 769
 - 13.7.5 Pre-Big-Bang and Dilaton Cosmology 770
 - 13.7.6 String-Gas Cosmology 772
 - 13.7.7 Cyclic Ekpyrotic Universe 776
- 13.8 Inflation and Alternatives: Compact Summary 783
- 13.9 Big-Bang Problem 786
 - 13.9.1 Big Bang in String Theory 786
 - 13.9.2 Classical Billiards: Living with the Singularity 788
 - 13.9.3 Quantum Billiards: Avoiding the Singularity? 792
- References 795
- 14 Perspective** 823
- References 826
- Index** 829



<http://www.springer.com/978-3-319-41125-5>

Classical and Quantum Cosmology

Calcagni, G.

2017, XV, 843 p. 80 illus., 52 illus. in color., Hardcover

ISBN: 978-3-319-41125-5