Chapter 2
The Standard Model of Particle Physics

This chapter will review the current theoretical model of elementary particle physics, based largely on references [1–4]. Section 2.1 will give an overview of the particle content of this Standard Model of particle physics as well as the interactions between them. The set of observed particles has recently been completed by the discovery of a particle which so far appears to be compatible with the long searched for Higgs-boson, which had been predicted as part of the mechanism generating masses of the fundamental particles via spontaneous symmetry breaking. The electroweak interaction and the Higgs-mechanism are discussed in Sect. 2.2, followed by a brief overview of the strong interaction in Sect. 2.3. Despite of being one of the most successful theories in the history of science, the Standard Model has a number of shortcomings that will be highlighted in Sect. 2.4, as one of them is the motivation for the analysis documented in this work.

Throughout this thesis, natural units will be used, i.e. $\hbar = c = 1$.

2.1 Survey of Fundamental Particles and Their Interactions

The Standard Model (SM) of particle physics describes the fundamental building blocks of matter and their interactions. All visible matter is made of two kinds of elementary particles (i.e. without any substructure): leptons and quarks. They are fermions, i.e. they carry half-integer spin, and they interact via three fundamental forces: the electromagnetic, the weak and the strong interaction, with the first two being unified in the electroweak force. The incorporation of the fourth fundamental force, gravitation, into the Standard Model is still an unresolved challenge. However, at the involved mass scales its strength is negligible compared to that of the other interactions. The interactions between quarks and leptons are mediated by the
Table 2.1  Overview of the three fundamental forces described by the Standard Model, the corresponding gauge bosons and charges

<table>
<thead>
<tr>
<th>Interaction</th>
<th>Gauge boson</th>
<th>Mass (GeV)</th>
<th>Charge</th>
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<tbody>
<tr>
<td>Strong</td>
<td>8 gluons ($g$)</td>
<td>0</td>
<td>Colour (r,g,b)</td>
</tr>
<tr>
<td>Electromagnetic</td>
<td>Photon ($\gamma$)</td>
<td>0</td>
<td>Electrical</td>
</tr>
<tr>
<td>Weak</td>
<td>$Z$</td>
<td>$\sim$91.2</td>
<td>Weak isospin</td>
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<tr>
<td></td>
<td>$W^{\pm}$</td>
<td>$\sim$80.4</td>
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exchange of particles with integer spin—the gauge bosons. There is one such boson for the electromagnetic interaction, the massless photon ($\gamma$), which couples to the electric charge but is itself uncharged. The weak interaction is mediated by three bosons, the electrically neutral $Z$-boson and the positively and negatively charged $W^{\pm}$-bosons, that each couple to the 3-component of the weak isospin. There are 8 electrically neutral and massless gluons ($g$) that mediate the strong force. The corresponding charge is called colour and comes in three variants (commonly labeled red, green and blue) and the corresponding anticolours. Gluons themselves carry colour charge, which allows them to interact with each other, leading to a short range for the strong force. The electromagnetic force, on the other hand, has infinite reach, since the photon is massless, while the weak interaction is short-ranged due to the mass of the $Z$ and $W$ bosons—roughly 91 and 80 GeV, respectively. Table 2.1 summarises the three interactions.

There are six leptons, grouped into three families, each family consisting of one (negatively) charged lepton and a neutrino which only carries weak charge. Each lepton has an anti-particle for which the additive quantum numbers have the opposite sign. The masses of the charged leptons—electron ($e$), muon ($\mu$) and tau ($\tau$)—increase in this order from approximately 511 keV over 105 MeV to 1.7 GeV [4]. Neutrinos are treated as massless in the SM. However, the observation of neutrino oscillations (cf. [5]) indicates that they have a non-vanishing mass. The current experimental upper bound is $m_{\nu} < 2$ eV [6, 7].

Quarks exist in six flavours and are also grouped into three families. The first family consists of the up ($u$)- and down ($d$)-quark, the names of which refer to their 3-component of the isospin, which is $+1/2$ for the up-quark and $-1/2$ for the down-quark. In analogy, the other families also comprise one up-type and one down-type quark. The up-type quarks have an electric charge of $2/3|e|$, the down-type quarks of $-1/3|e|$. As for the leptons, the quark masses increase throughout the families, the up-type quark of the third family, the top ($t$)-quark, being the heaviest fundamental particle with a mass of roughly 173 GeV [4]. All stable matter surrounding us is made up of fermions of the first family: atoms consist of electrons, proton and neutrons, the

1The isospin was originally introduced to treat neutron and proton as the same particles (nucleons) with different isospin orientation ($\pm 1/2$). In the quark model, the isospin of the nucleon results from the isospin of its constituents.
latter two being compositions of \( u \) - and \( d \)-quarks. The particles of the other families and compounds of them always decay into lighter particles.

Besides the electric and weak charge, quarks carry colour charge, i.e. they take part in the strong interaction. Again, there exists an anti-quark to each quark which carries anti-colour. Quarks do not exist as free particles in nature but occur only in bound states of two or three (anti-)quarks. Those composite particles are referred to as hadrons and can be classified into two main groups: *Mesons* consist of one quark and one anti-quark, *baryons* of three quarks (and anti-baryons of three anti-quarks). All observed hadrons appear to be colourless (*white*), i.e. colour singlet states, which is realised by combining either colour and anticolour for the mesons or red, green and blue (antired, antigreen, antiblue) for the baryons.

In Table 2.2, the fundamental fermions and some of the quantum numbers are listed. (The concept of weak isospin will be discussed in Sect. 2.2.)

Within the SM, the fundamental interactions are described in gauge theories, the underlying principle being that the corresponding Lagrangian density has to be invariant under certain local gauge transformations which define a symmetry. These transformations—or their representation as matrices, respectively—are the *generators* of the corresponding symmetry group. In order to have a global symmetry hold also locally, vector-boson fields, the *gauge fields*, have to be introduced, one for each generator of the symmetry group. This shall be illustrated here using the example of quantum electrodynamics (QED), the quantum field theory of electromagnetism.

The Dirac equation for a free particle with charge \( q \) and mass \( m \), described by a wave function \( \psi(x) \) is given by

\[
(i \gamma^\mu \partial_\mu - m) \psi(x) = 0.
\] (2.1)

Performing a local phase transformation of the form \( \psi'(x) = e^{i q \chi(x)} \psi(x) \) leads to

\[
(i \gamma^\mu \partial_\mu - m) \psi'(x) = e^{i q \chi(x)} (i \gamma^\mu \partial_\mu - m) \psi(x) - q \gamma^\mu (\partial_\mu \chi(x)) \psi'(x) = 0 = q \gamma^\mu A_\mu \psi'(x) \neq 0,
\] (2.2)

**Table 2.2** Overview over the fundamental fermions of the Standard Model and some of their quantum numbers: weak isospin \( I \), its third component \( I_3 \), electric charge \( Q_f \) and weak hypercharge \( Y \)

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<tr>
<td>Leptons &amp;</td>
<td>Fermions &amp;</td>
<td>( I ) &amp; ( I_3 ) &amp; ( Q_f ) [e] &amp; ( Y )</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>( (\nu_e)<em>L ) &amp; ( (\nu</em>\mu)<em>L ) &amp; ( (\nu</em>\tau)_L ) &amp; ( \frac{1}{2} ) &amp; ( -\frac{1}{2} ) &amp; ( 0 ) &amp; ( -1 ) &amp; ( -2 )</td>
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<tr>
<td></td>
<td>( e_R ) &amp; ( \mu_R ) &amp; ( \tau_R ) &amp; 0 &amp; 0 &amp; 0 &amp; 1 &amp; 2</td>
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<tr>
<td>Quarks &amp;</td>
<td>( (u)_L ) &amp; ( (c)_L ) &amp; ( (t)_L ) &amp; ( \frac{1}{2} ) &amp; ( -\frac{1}{2} ) &amp; ( \frac{2}{3} ) &amp; ( \frac{2}{3} ) &amp; ( \frac{1}{3} )</td>
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<tr>
<td>&amp; ( d_R ) &amp; ( s_R ) &amp; ( b_R ) &amp; 0 &amp; 0 &amp; ( \frac{1}{2} ) &amp; ( -\frac{1}{2} ) &amp; ( \frac{2}{3} )</td>
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with $A'_\mu(x) = -\partial_\mu \chi(x)$. This means, the transformed wave function does not fulfil the Dirac equation for a free particle, but for a particle in an electromagnetic field. To establish the invariance of the Dirac equation under a local phase transformation, the field has to be transformed as well, $A'_\mu(x) = A_\mu(x) - \partial_\mu \chi(x)$ and the derivative is replaced by the covariant derivative: $\partial_\mu \rightarrow D_\mu = \partial_\mu + iqA_\mu$. In this way,

$$(i\gamma^\mu D_\mu - m)\psi(x) = 0 \quad (2.3)$$

is rendered invariant under the simultaneous transformation of $\psi$ and the gauge field $A_\mu$.

Generalising this formalism, the Standard Model is described by a $SU(2)_L \otimes U(1)_Y \otimes SU(3)_C$ gauge symmetry. The $SU(3)_C$-term denotes the underlying symmetry of the strong interaction, with the three degrees of freedom of the colour charge (hence the index $C$). The $SU(3)$ has 8 generators, which are associated to eight gluons. The first two terms incorporate the gauge symmetry of the electroweak interaction, which has four generators. A local gauge symmetry forbids mass terms in the Lagrangian density, which means that the gauge bosons have to be massless. The $SU(3)$ of the strong interaction is an exact symmetry, and hence the gluons are massless. However, only one of the experimentally observed vector bosons of the electroweak interaction, the photon, is massless. $W$- and $Z$-bosons on the other hand are massive, indicating that the gauge symmetry is broken. The mechanism for this spontaneous symmetry breaking predicts the existence of another fundamental boson, which has to have spin 0. It is commonly referred to as the Higgs-boson, named after Peter Higgs who was one of the first ones to predict its existence [8–10]. Such a scalar boson was discovered by the ATLAS [11] and CMS [12] collaborations in 2012 and so far all measurements of its properties are consistent with those predicted for a Standard Model Higgs-boson. More details on the symmetry breaking mechanism will be discussed in Sect. 2.2.

### 2.2 Electroweak Interaction and Symmetry Breaking

Historically, the electromagnetic and weak interaction were considered two separate phenomena, until they were unified in the electroweak theory of Glashow, Salam and Weinberg [13–15], similar to the unification of electric and magnetic interactions by Maxwell [16].

A number of experimental observations on particle decays (especially $\beta$-decays) had to be incorporated when building a theory of the weak interaction. The short range of the interaction suggested, that the corresponding exchange particles had to be massive. For a long time, onlycharged currentinteractions were known, in which the charge of the leptons or quarks involved changes by ±1. Therefore, there should be at least two exchange particles, with charge +1 and −1, named $W^+$ and $W^-$, respectively. Assuming—in analogy to the electromagnetic interaction—that these particles have spin 1, the interaction in general can be described by a combination of a
vector (V) and an axial-vector (A) operator. The strength of the different contributions is described by coefficients $c_V$ and $c_A$, respectively, i.e. the interaction will contain a term of the form $\gamma^\mu (c_V + c_A \gamma^5)$. A parity-conserving interaction, which couples equally to left- and right-handed particles, can only be either purely vectorial ($c_A = 0$) or purely axial-vectorial ($c_V = 0$). If both coefficients have the same absolute value, parity is maximally violated.

Any spinor $u$ describing a fermion can be decomposed into a left-handed ($u_L$) and a right-handed ($u_R$) component in the following way:

$$u = u_L + u_R = \frac{1}{2}(\mathbb{1} - \gamma^5)u + \frac{1}{2}(\mathbb{1} + \gamma^5)u,$$

(2.4)

where $\mathbb{1}$ denotes the $4 \times 4$ unity matrix, and $P_{R/L} = \frac{1}{2}(\mathbb{1} \pm \gamma^5)$ are the helicity projection operators. It is experimentally found that only left-handed fermions participate in the charged currents, i.e. $c_V = 1$ and $c_A = 1$. Parity is maximally violated in these interactions, the theory is therefore also referred to as $V-A$-theory.

Moreover, it is found that the coupling strength is the same for all fermions. This is different from neutral currents, which do not change the electric charge of the participating fermions. They were first observed at the Gargamelle bubble chamber at CERN 1973 [17] and attributed to the exchange of a neutral vector boson, $Z^0$. It was subsequently found that the coupling strength depends on the charge of the fermions. The unified description of these phenomena within the electroweak theory is based on the introduction of a new quantum number, called weak isospin ($I$), and the consistent application of the isospin formalism. Left-handed fermions are grouped into doublets of weak isospin $I = 1/2$, with 3-component $\pm 1/2$, cf. Table 2.2. Right-handed fermions are weak isospin singlets, $I = I_3 = 0$, since they do not participate in charged current interactions.

Transitions between left-handed charged leptons and neutrinos or up- and down-type quarks are possible by emission of a charged $W^\pm$-boson. Since the 3-component of the weak isospin thereby changes by one unit, the $W$-bosons must have $I = 1$ and $I_3 = \pm 1$. To explain the transitions between different generations, the electroweak eigenstates of down-type quarks are interpreted not as the actual quark mass eigenstates ($d$, $s$, $b$) but mixtures of those, labelled $d'$, $s'$ and $b'$, according to the unitary Cabibbo–Kobayashi–Maskawa (CKM) matrix [18, 19]:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

(2.5)

The diagonal elements describe the transitions within one generation and are close to unity. Transitions between families are accordingly strongly suppressed.

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2 In case of the electromagnetic force it is a pure vector interaction.

3 A $V + A$—theory would describe an interaction only right-handed particles take part in.
Within the isospin formalism, there should be another boson with \( I_3 = 0 \) and the same couplings to fermions as \( W^\pm \), which does not change the 3-component, i.e. it mediates transitions that do not change the fermion flavour, just like the neutral currents. However, this boson cannot be identical to the \( Z^0 \), since the couplings of the latter are different for fermions with different electrical charge. To solve this problem, a fourth field is introduced, which is a weak isospin singlet, \( I = I_3 = 0 \), i.e. it couples to fermions without changing the 3-component of their isospin. Experimentally, indeed, two such bosons are observed: the photon and the \( Z \)-boson. The basic idea of electroweak unification is thus to express the observed bosons as mixtures of the two bosons with \( I_3 = 0 \). In the language of gauge theories this is expressed as follows.

The electric charge and the weak isospin are related via the Gell–Mann–Nishijima relation [20, 21]:

\[
Y = 2(I_3 + Q),
\]

(2.6)

where \( Y \) is called the weak hypercharge.

The symmetry group of the electroweak interaction is \( SU(2)_L \otimes U(1)_Y \). \( SU(2)_L \) is the weak isospin group describing transformations of the left-handed isospin doublets. \( U(1)_Y \) is the hypercharge group, which is essentially a phase transformation. To ensure local gauge invariance a triplet of vector fields, \( W^i_\mu \), \( i = 1, 2, 3 \), is introduced for the \( SU(2)_L \) and a single vector field, \( B_\mu \), for the \( U(1)_Y \).

The covariant derivative reads

\[
D^\mu = \partial^\mu + ig\vec{T} \cdot \vec{W}^\mu + ig' 2YB^\mu,
\]

(2.7)

with couplings \( g \) and \( g' \) for the \( SU(2)_L \) and \( U(1)_Y \), respectively. For left-handed fermions, \( \vec{T} \) is given by \( \vec{T} = \vec{\tau} \), where \( \tau_i \), \( i = 1, 2, 3 \) are the Pauli-matrices. \( T_i = \tau_i/2 \), \( i = 1, 2, 3 \) are the generators of \( SU(2)_L \). For right-handed fermions, \( \vec{T} = \vec{0} \). The generator of the hypercharge group is \( Y/2 \).

With this, the relations for the observable vector bosons are expressed as:

\[
W^\pm_\mu = \frac{1}{\sqrt{2}}(W^1_\mu \mp iW^2_\mu),
\]

(2.8)

\[
Z_\mu = W^3_\mu \cos \theta_W - B_\mu \sin \theta_W,
\]

(2.9)

\[
A_\mu = W^3_\mu \sin \theta_W + B_\mu \cos \theta_W.
\]

(2.10)

Here, the weak mixing angle \( \theta_W \), is related to the coupling constants in the following way:

\[
\sin \theta_W = \frac{g'}{\sqrt{g^2 + g'^2}}
\]

(2.11)
and has been measured to $\sin^2 \theta_W = 0.23119(14)$ [4]. As will be seen later, the mixing angle also relates the masses of the heavy gauge bosons as

$$\cos \theta_W = \frac{m_W}{m_Z}. \quad (2.12)$$

Moreover, there is a fundamental relation between the elementary charge $e$ and the coupling constants:

$$e = g' \cos \theta_W = g \sin \theta_W. \quad (2.13)$$

The couplings to the $W$ bosons are $g_W = g I_3$ for all fermions, the fermion dependent couplings to the $Z$-boson are given as

$$g_Z(f) = \frac{g}{\cos \theta_W} (I_3 - Q_f \sin^2(\theta_W)), \quad (2.14)$$

with the values of $I_3$ and $Q_f$ as given in Table 2.2. For the neutral currents the values of the coefficients for vector and axial-vector interaction are given by $c_V(f) = I_3 - 2 Q_f \sin^2(\theta_W)$ and $c_A(f) = I_3$. Accordingly, neutral current interactions are not maximally CP violating.

In 1983, the $W$- and $Z$-bosons were discovered at CERN [22, 23]. The bosons in Eqs. (2.8)–(2.10), however, are still massless, since they are linear combination of massless fields. In order to introduce a mechanism for the Gauge bosons to acquire mass, the Lagrangian including the interaction of the fields with fermions and the terms for kinetic energy is studied, which can be written as

$$\mathcal{L} = \mathcal{L}_F + \mathcal{L}_G = \sum_{f=1,2} f^\dagger \mathcal{D} f - \frac{1}{4} W_{\mu\nu}^i W^{i\mu\nu} - \frac{1}{4} B_{\mu\nu}^\mu B^{\mu\nu} \quad \text{with} \quad \mathcal{D} = \gamma^\mu D_\mu. \quad (2.15)$$

The field tensors are given as

$$W_{\mu\nu}^i = \partial_\mu W_{\nu}^i - \partial_\nu W_{\mu}^i - g \varepsilon_{ijk} W_{\mu}^j W_{\nu}^k \quad (2.16)$$

$$B_{\mu\nu} = \partial_\mu B_{\nu} - \partial_\nu B_{\mu}. \quad (2.17)$$

Writing the covariant derivative for left- and right-handed fermions ($f_L$ and $f_R$) explicitly, the Lagrangian reads

$$\mathcal{L} = f_L^\dagger \gamma^\mu \left( i \partial_\mu + g \frac{\tau^i}{2} W_{\mu}^i + \frac{g'}{2} Y B_{\mu} \right) f_L + f_R^\dagger \gamma^\mu \left( i \partial_\mu + \frac{g'}{2} Y B_{\mu} \right) f_R - \frac{1}{4} W_{\mu\nu}^i W^{i\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}. \quad (2.18)$$

To introduce the mass terms for the heavy bosons, the simplest extension is to introduce two complex scalar fields $\phi^+$ and $\phi^0$, that form an isospin doublet,
\[ \Phi = \left( \begin{array}{c} \phi^+ \\ \phi^0 \end{array} \right), \quad (I = 1/2, Y = 1). \quad (2.19) \]

According to Eq. (2.6), the above values of \( I \) and \( Y \) indeed yield charge +1 and 0. The Lagrangian for the Higgs-field is given by

\[ \mathcal{L}_{\text{Higgs}} = (\partial^\mu \Phi)^\dagger (\partial_\mu \Phi) - V(\Phi^\dagger, \Phi), \]
\[ V(\Phi^\dagger, \Phi) = m^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2, \quad m^2, \lambda \in \mathbb{R}. \quad (2.20) \]

The shape of the potential \( V(\Phi^\dagger, \Phi) \) depends on the choice of the parameters \( m \) and \( \lambda \). \( \lambda \) has to be greater than 0 to ensure stability of the vacuum. When in addition \( m^2 = -\mu^2 < 0 \) is chosen, the potential has a local maximum at the origin and degenerate minima on a circle around it. By adapting a particular ground state the symmetry is spontaneously broken. In particular, in the configuration where the expectation value of the charged Higgs-field vanishes, the ground state can be written as

\[ \Phi_0 \equiv \langle \Phi_0 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad v = \frac{\mu}{\sqrt{\lambda}}. \quad (2.21) \]

Considering a small excitation:

\[ \Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + \eta(x) \end{pmatrix}, \quad (2.22) \]

and inserting it into the Lagrangian yields

\[
\mathcal{L} = \left[ \frac{1}{2} (\partial^\mu \eta)(\partial_\mu \eta) - \mu^2 \eta^2 \right] - \frac{1}{4} W_{\mu\nu}^i W^{\mu\nu}_i \\
- \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \frac{1}{2} \cdot \frac{g^2 v^2}{4} (|W^+_{\mu}|^2 + |W^-_{\mu}|^2) + \frac{1}{2} \cdot \frac{v^2}{4} |g'B_\mu - gW^3_\mu|^2. \quad (2.23) \]

There is a real Goldstone boson, \( \eta \), with mass \( m_\eta = \sqrt{2}\mu \), which is identified with the Higgs-boson. In addition, the mass terms for the other bosons result from the Lagrangian as well:

\[ m_\gamma = 0, \quad (2.24) \]

since there is no masses for the electromagnetic four-potential. Moreover,

\[ m_W = \frac{1}{2} gv \quad (2.25) \]

and with \( g'B_\mu - gW^3_\mu = -\sqrt{g^2 + g'^2} Z_\mu \):

\[ m_Z = \frac{1}{2} v \sqrt{g^2 + g'^2}. \quad (2.26) \]
From the last two equations the relation Eq. (2.12) for the masses of $W$ and $Z$ bosons and the weak mixing angle is obtained. A measurement of all three parameters thus allows testing the SM predictions. The parameter $\mu$ which defines the Higgs mass cannot be predicted by the theory. The recently discovered Higgs-candidate particle has a mass of roughly 126 GeV, thus fixing the value of $\mu$.

The masses of the fundamental fermions can be generated by Yukawa couplings to the Higgs-field, adding another term to the Lagrangian:

$$L_{Yukawa} = -h_{dij} \bar{q}_{Li} \Phi d_{Rj} - h_{u_{ij}} \bar{q}_{Li} \tilde{\Phi} u_{Rj} - h_{lij} \bar{l}_{Li} \Phi e_{Rj} + h.c.$$  \hspace{1cm} (2.27)

with $\tilde{\Phi} = -i \sigma_2 \Phi^*$ and $q_L$ ($l_L$) and $u_R$, $d_R$ ($e_R$) being the quark (lepton) $SU(2)_L$ doublets and singlets. The mass of a fermion $f$ is given by

$$m_f = \frac{1}{\sqrt{2}} h_f v,$$  \hspace{1cm} (2.28)

i.e. the coupling $h_f$ is proportional to the fermion mass.

### 2.3 Quantum Chromodynamics

The quark model had initially been introduced by Gell-Mann [24] (and independently by Zweig) in 1964 to explain the multitude of observed hadrons as built up of fundamental constituents—the quarks. This hypothesis was corroborated experimentally by the results of deep inelastic scattering (DIS) experiments studying the structure of the proton, which indicated that the proton should consist of three charged constituents [25]. However, there remained scepticism about the model mainly due to two reasons: No free quarks were observed and states like the $\Delta^{++}$ baryon, hypothesised to consist of three $u$-quarks with the same spin, should not exist due to the Pauli principle. Already at the time the notion of confined quarks was brought up but lacked any form of explanation. A remedy for the dilemma of apparent violation of the Pauli principle had been proposed by Greenberg in 1964 [26]: He introduced a new quantum number which came to be known as colour. Nevertheless, only when the $J/\Psi$ was discovered in 1974 [27, 28] and required the introduction of a fourth quark, the quark model became more popular. It was further strengthened by the observation of additional states including the new quark and the subsequent discovery of the particles of the third family which was completed 1995 with the discovery of the top quark [29, 30]. Deep inelastic scattering showed that there are electrically neutral constituents inside the proton that are identified with the mediators of the strong force, the gluons. Further evidence for gluons was found for example in the jet structure characteristics of inelastic scattering at high energies [31]. Based on these observations the current picture of a proton is as follows: Its ‘macroscopic’ properties like charge and spin are defined by its valence quarks content. The valence quarks of a proton are two $u$- and one $d$-quark. They are held together by the strong
force, i.e. by the exchange of gluons. These gluons can again fluctuate into quark-antiquark pairs, which form the **quark sea**, or split into gluons. The gluons, valence- and sea-quarks are commonly referred to as **partons**.

The 8 generators of the $SU(3)_C$ symmetry of the strong interaction can be represented via the Gell-Mann matrices $\lambda_\rho$, $\rho = 1, 2, \ldots, 8$. The commutators of these matrices define the totally antisymmetric structure functions $f^{abc}$ of the $SU(3)$:

$$[\lambda^a, \lambda^b] = 2i f^{abc} \lambda^c. \quad (2.29)$$

The covariant derivative is given as

$$D_\mu = \partial_\mu - ig_s \frac{\lambda_\rho}{2} G^\rho_\mu, \quad (2.30)$$

with the $\rho$-component $G^\rho_\mu$ of the gluon field. Using this, the Lagrangian of quantum chromodynamics (QCD) can be formulated as

$$L_{QCD} = \sum_q \bar{q} (i D - m_q) q - \frac{1}{4} G^\rho_\mu G^{\rho \mu}. \quad (2.31)$$

The $\rho$th gluon field tensor is written as

$$G^\rho_{\mu\nu} = \partial_{\mu} G^\rho_{\nu} - \partial_{\nu} G^\rho_{\mu} + g_s f^{\rho\beta\gamma} G^\beta_{\mu} G^\gamma_{\nu}. \quad (2.32)$$

The strong coupling constant $\alpha_s$ is related to the coupling $g_s$ above as

$$\alpha_s = \frac{g^2_s}{4\pi}. \quad (2.33)$$

The last term in Eq. (2.32) describes the self-coupling of gluons with each other due to the fact that they carry colour charge as well (more precisely, one colour and one anticolour charge). This leads to special features of the strong interaction. At small distances, the self-coupling of gluons leads to “anti-screening” effects, resulting in a weakening of the coupling constant $\alpha_s$. This is referred to as **asymptotic freedom**, as the quarks are quasi-free and can be treated perturbatively. On the other hand, the coupling constant becomes large for large distances, which leads to the so-called **confinement** of quarks in hadrons: When trying to separate two quarks, the energy needed becomes so large, that it exceeds the threshold for the creation of new quark-antiquark pairs, which then again form colourless states with the original quarks. This process is also referred to as **hadronisation**.

The dependence of the strong coupling constant on the energy—parameterised as momentum transfer $Q$—can in leading order be expressed as

$$\alpha_s(Q) = \frac{12\pi}{(33 - 2n_f) \log \frac{Q^2}{\Lambda^2}}, \quad (2.34)$$
2.3 Quantum Chromodynamics

Fig. 2.1 Summary of measurements of $\alpha_s$, illustrating the running of the coupling constant as a function of the energy scale $Q$ [4]

with some arbitrary scale $\Lambda$ for which $\alpha_s$ is assumed to be known. The number $n_f$ is the number of quark flavours accessible at the chosen energy scale, i.e. for which $Q^2 > m_q^2$. Due to this energy dependence $\alpha_s$ is called a running coupling constant.

From Eq. (2.34), it can be seen that, in case $n_f < 17$, for $Q \rightarrow \infty$ the coupling strength approaches 0—the quarks are asymptotically free. On the other hand, $\alpha_s$ grows for small values of $Q$ and becomes greater than 1 for values of $Q$ below a few hundred MeV. In this regime, no perturbation expansion is possible any more, confinement sets in. A typical scale is the mass of the $Z$-boson: $\alpha_s(m_Z) = 0.1185(6)$ [4]. The running of the coupling constant is also apparent from Fig. 2.1 which shows measurements of its value at various scales.

2.4 Open Questions and Extensions

The Standard Model of particle physics is surely one of the most successful theories in physics. So far, it withstands all tests and has been experimentally verified with tremendous precision. One of its latest triumphs is the discovery of a Higgs-boson candidate particle which to date appears to have the properties predicted by the SM. But even if it does, there remain several phenomena that cannot be explained within the SM and hence require the existence of some kind of yet undiscovered physics—commonly referred to as new physics or physics beyond the SM (BSM).

Within the SM, neutrinos are treated as massless, but observation of neutrino oscillation demands that neutrinos in fact do have a non-vanishing mass, albeit a very small one.

Another challenge for the SM is the so-called hierarchy problem: The standard model gives no explanation for the enormous difference between the electroweak scale ($\mathcal{O}(100 \text{ GeV})$), the scale at which electroweak and strong forces become equally strong (due to the running coupling constants) which is of the order of $10^{16} \text{ GeV}$ and
the Planck scale of \( \sim 10^{19} \text{ GeV} \), at which also the gravitational interaction becomes as strong as the other forces. Similarly, while the masses of the fundamental particles can be generated via the Higgs-mechanism in electroweak symmetry breaking, the theory gives no explanation for the large range of the masses. Moreover, additional particles are needed in order to cancel diverging loop-corrections to the Higgs mass. There is also no explanation within the SM as to why there are three generations of fundamental fermions.

The origin of the matter-antimatter asymmetry in the universe is another open question in particle physics: If at the big bang particles and antiparticles were created in the same amount, they should all have annihilated again. However, the annihilation appears to be asymmetric as there is today only matter observed in the universe while the antimatter has disappeared. This requires CP violation by an amount that cannot be accommodated in the SM.

Finally, cosmological and astrophysical observations lead to the conclusion, that radiation and matter made of SM particles only account for about 5% of the mass and energy content in the universe. Roughly 27% are attributed to non-luminous dark matter and the remaining roughly 68% are so-called dark energy. Neither of these last two components finds any explanation within the SM. Dark matter will be discussed in more detail in Chap. 3.

One proposed explanation for several of the phenomena listed above provides the theory of super-symmetry (SUSY), in which the particle content is doubled by assigning a super-partner to each SM particle. The partners of fermions—sfermions—are bosons and the partners of gauge bosons—gauginos—are fermions. For example, the SUSY-partner of a neutrino would be called a sneutrino, that of a W-boson a Wino. Electrically neutral mixtures of gauginos are referred to as neutralinos.

Another class of extensions to the Standard Model are theories of extra spacial dimensions. In most of these models, the usual \((3 + 1)\)-dimensional spacetime—referred to as a brane—is embedded in the bulk, a \((3 + \delta + 1)\)-dimensional spacetime, i.e. adding \( \delta \) extra spacial dimensions. Such scenarios are often proposed as solutions to the hierarchy problem, for example in the Arkani-Hamed, Dimopoulos and Dvali (ADD) model [32], where all of the large extra dimensions (LED) are compactified on some topology with size \( R \), which leads to the fundamental Planck scale being lowered to approximately the electroweak scale. Another possibility to achieve this are warped extra dimensions, i.e. extra dimensions with large curvature, as in the so-called Randall–Sundrum model [33]. In the aforementioned models, it is assumed that the SM fields propagate in the brane only, and only gravity is allowed to propagate in the bulk. In addition, there are universal extra dimensions models (UED) with flat extra dimension that are much smaller than the ones in the ADD model, for example. In these UED models all particles can propagate in the extra dimensions.

In Chap. 3, Dark Matter candidates that these models provide will be discussed.
References

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