

Contents

Part I Introduction to Forward Sensitivity Method

1	Introduction	3
1.1	Predictability Limits	3
1.2	Sources of Error that Limit Predictability	6
1.3	Data Assimilation in Service to Prediction	7
1.4	Overview and Goals	9
1.5	A Classification of Forecast Dynamics and Errors	9
1.5.1	Forecast Dynamics	9
1.5.2	Forecast Errors	10
1.6	Organization of the Monograph	11
1.7	Exercises	12
1.7.1	Demonstrations and Problems	12
2	Forward Sensitivity Method: Scalar Case	19
2.1	Evolution of First-Order Sensitivities	20
2.2	First-Order Sensitivities Used in FSM	24
2.2.1	First-Order Analysis	26
2.3	Evolution of Second-Order Sensitivities	31
2.3.1	Evolution of $\partial^2 x(t)/\partial x^2(0)$	32
2.3.2	Evolution of $\partial^2 x(t)/\partial \alpha^2$	33
2.3.3	Evolution of $\partial^2 x(t)/\partial \alpha \partial x(0)$	34
2.4	Data Assimilation Using FSM: A Second-Order Method	35
2.5	FSM: Discrete Time Formulation	37
2.5.1	Discrete Evolution of First-Order Forward Sensitivities	37
2.5.2	Discrete Evolution of Second-Order Forward Sensitivity	41
2.6	Sensitivity to Initial Conditions and Lyapunov Index	42
2.6.1	Continuous Time Model	42
2.6.2	Discrete Time Model	43

2.7	Exercises	44
2.7.1	Demonstrations	47
2.8	Notes and References	56
3	On the Relation Between Adjoint and Forward Sensitivity	57
3.1	On the Structure of Adjoint Sensitivity	57
3.1.1	Adjoint Method	58
3.1.2	Computing $\nabla_{x(0)}\mathbf{J}$ in (3.1.20)	61
3.1.3	Computation of $\nabla_{\alpha}J$ in (3.1.22)	63
3.1.4	4D-VAR method	64
3.2	On the Relation Between FSM and 4D-VAR	65
3.3	Investigation of the Impact of Observations Using 4D-VAR and FSM	66
3.3.1	Experiments	67
3.4	Exercises	72
3.4.1	Demonstrations	72
3.5	Notes and References	91
4	Forward Sensitivity Method: General Case	93
4.1	Dynamics of Evolution of First Order Forward Sensitivities	93
4.1.1	Dynamics of Evolution of $\mathbf{u}_1(k)$	94
4.1.2	Dynamics of Evolution of $\mathbf{v}_1(k)$	95
4.1.3	Propagation of Perturbation and Forward Sensitivity	96
4.2	On the Relation Between Adjoint and Forward Sensitivities	97
4.3	Data Assimilation Using FSM	100
4.3.1	Case 1 Single Observation	100
4.3.2	Case 2: Multiple Observations	102
4.4	Exercises	103
4.4.1	Demonstrations	104
4.5	Notes and References	106
5	Forecast Error Correction Using Optimal Tracking	107
5.1	Pontryagin's Minimum Principle (PMP) in Discrete Time	108
5.1.1	Condition 1: Model Dynamics	111
5.1.2	Condition 2: Co-State or Adjoint Dynamics	111
5.1.3	Condition 3: Stationarity Condition	112
5.1.4	Condition 4: Boundary Conditions	113
5.2	Connection to 4D-VAR	114
5.2.1	Condition 1: Model Dynamics	115
5.2.2	Condition 2: Co-State or Adjoint Dynamics	116
5.2.3	Condition 3: Boundary Conditions	116
5.3	Optimal Tracking: Linear Case	117
5.3.1	Structure of the Optimal Control	118
5.3.2	The TPBVP	118
5.3.3	Affine Relation Between λ_k and \mathbf{x}_k	118
5.3.4	Conversion of TPBVP to Two Initial Value Problems	119

- 5.3.5 Structure of the Off-Line Optimal Control..... 120
- 5.3.6 Optimal Trajectory 121
- 5.3.7 Identification of Model Error 121
- 5.4 Computation of Model Errors..... 122
 - 5.4.1 Gradient of $\alpha(\mathbf{S}, \mathbf{x})$ 123
 - 5.4.2 Gradient of $\beta(\mathbf{S}, \mathbf{x}, \mathbf{y})$ 124
 - 5.4.3 Gradient of $\mathbf{Q}(\mathbf{S})$ 124
- 5.5 An Application: Linear Advection Equation Model and Its Solution 124
 - 5.5.1 The Low-Order Model (LOM) 126
 - 5.5.2 Solution of LOM(n) in (5.5.8)..... 129
 - 5.5.3 Identification of Model Error 134
- 5.6 Exercises 137
 - 5.6.1 Demonstrations..... 137
- 5.7 Notes and References 140
- Appendix 142

Part II Applications of Forward Sensitivity Method

- 6 The Gulf of Mexico Problem: Return Flow Analysis 149**
 - 6.1 Observations in Support of Mixed Layer Modeling 152
 - 6.2 The Five Variable Model 152
 - 6.3 Evolution of Forward Sensitivities 160
 - 6.3.1 Sensitivity with Respect to the Parameters 160
 - 6.3.2 Sensitivity with Respect to Initial Conditions 163
 - 6.3.3 Sensitivity at the End of the Simulation..... 190
 - 6.4 Data Assimilation Using Forward Sensitivity Method:
 - Numerical Experiments 191
 - 6.4.1 Observations at Multiple Time 192
 - 6.5 Problems 192
 - 6.6 Notes and References 202
 - Appendix 203
- 7 Lagrangian Tracer Dynamics 207**
 - 7.1 Shallow Water Model and Tracer Dynamics 208
 - 7.1.1 Low-Order Model (LOM) 208
 - 7.1.2 Solution of LOM 210
 - 7.1.3 Tracer Dynamics 211
 - 7.2 Analysis of Equilibria of Tracer Dynamics 212
 - 7.2.1 Case 1: Equilibria in Geostrophic Mode 212
 - 7.2.2 Case 2: Equilibria in Inertia-Gravity Mode 214
 - 7.2.3 Case 3: Equilibria in General Case 217
 - 7.2.4 Conditions for the Sign Definiteness of $\Delta_i(t)$ 221
 - 7.3 Analysis of Bifurcation..... 221
 - 7.3.1 Case 1 224
 - 7.3.2 Case 2..... 225
 - 7.4 Dynamics of Evolution of Forward Sensitivities..... 227

- 7.5 Sensitivity Plots 236
 - 7.5.1 Sensitivity to Initial Conditions 236
 - 7.5.2 Sensitivity to Parameters..... 236
- 7.6 Data Assimilation Using FSM 236
 - 7.6.1 Methodology 243
 - 7.6.2 Data Assimilation Experiment..... 244
- 7.7 Notes and References 248
- Appendix 249
- Epilogue 251

- A Basic Notation 255**
 - A.1 Basic Notation 255
 - A.2 Inner Product 255
 - A.3 Gradient, Jacobian, Hessian..... 255
 - A.4 Taylor Expansion 257
 - A.5 First Variations..... 257
 - A.6 Conditions for Minima 258

- References 259**

- Index 265**



<http://www.springer.com/978-3-319-39995-9>

Forecast Error Correction using Dynamic Data
Assimilation

Lakshmivarahan, S.; Lewis, J.M.; Jabrzemski, R.
2017, XVI, 270 p. 125 illus., 104 illus. in color.,
Hardcover

ISBN: 978-3-319-39995-9