Contents

1 Some Introductory Material 1
1.1 Johann Bernoulli's Challenge 3
1.2 Before Johann Bernoulli 4
1.2.1 Fermat's Principle of Least Time 5
1.2.2 Isoperimetric Problems. 9
1.2.3 Solving the New Minimum Problems 10
1.2.4 Newton's Problem. 11
1.2.5 Galilei: The Quickest Descent Curve 12
1.3 The Infinitesimal Calculus in the Continent: Leibniz and Euler 16
1.3.1 Leibniz's View 17
1.3.2 Euler's View 22
1.4 The Cycloid 27

2 The Brachistochrone Problem: Johann and Jakob Bernoulli 39
2.1 Johann Bernoulli's 1697 Paper 39
2.2 Jakob Bernoulli's Solution 45
2.3 Leibniz's Solution 49
2.4 Newton's Solutions 51
2.5 The *Addendum* in Johann Bernoulli's 1718 Paper 53

3 Isoperimetrical Problems: Jakob and Johann Bernoulli 59
3.1 Jakob Bernoulli's 1701 Paper 65
3.2 Johann Bernoulli's 1706 Paper 81
3.3 Johann Bernoulli's 1718 Paper 85

4 Shortest Lines and Geodesics 101
4.1 Bernoulli's Theorem 102
4.2 Geodesics on a Surface 106
4.2.1 Euler's 1732 Paper 106
4.2.2 Clairaut's 1733 Paper 117

5 Euler's Memoirs of 1738 and 1741 123
5.1 Euler's Classification: the First Class 127
5.2 Isoperimetric Problems: Euler's Multiplier Rule 131
5.3 Integrands Depending on the Path Length. 135
5.4 Euler's 1741 Paper 144
5.5 A Few Remarks 151

6 Euler's *Methodus Inveniendi* 155
6.1 The Simplest Problem 156
6.2 Integrands Depending on Integrals. 162
6.3 Isoperimetric Problems. 169
6.4 The Two *Additamenta* 172
6.4.1 Elastic Curves. 173
6.4.2 The Principle of Least Action 181

7 Lagrange's δ–Calculus 193
7.1 The Lagrange–Euler Correspondence on the δ–Calculus 194
7.2 Lagrange's First Published Papers on $\delta-$Calculus 212
7.2.1 The *Essai*. 212
7.2.2 The *Application* 227
7.3 Euler's Reading of Lagrange's Method 236
7.4 More on Lagrange's δ–Calculus 239
7.4.1 Lagrange's *Méthode des Variations* 239
7.4.2 Lagrange's *Late Treatises*. 244
7.4.3 The Method of Variation of Constants 253
7.5 Euler's Paper of 1771 256
7.6 A Glimpse at the More Recent Vision 262
7.6.1 Euler–Lagrange Equations 262
7.6.2 On the Existence and Regularity of Minimizers 267
7.6.3 Constrained Variational Problems 272

Bibliography 279

Index of Names 289

Subject Index 291

http://www.springer.com/978-3-319-38944-8

The Early Period of the Calculus of Variations

Freguglia, P.; Giaquinta, M.

2016, XII, 293 p. 59 illus., Hardcover
ISBN: 978-3-319-38944-8
A product of Birkhäuser Basel