Chapter 2
Evolving a Plan: Design and Planning with Complexity

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Abstract Producing physical plans that manipulate urban form and function to generate optimal designs with respect to an affected community is a many-stage process of resolving inherent conflicts between those who represent the interests of the community. Here we introduce a class of decision models that involve resolving conflicts between a series of opinions that differ from one another and are associated with a set of agents who act as designers. These opinions are expressed as differing interest and control in factors that influence the design and these are articulated as spatial plans based on the suitability or desirability of different map locations for physical development. We define a set of agents who motivate the process and whose interactions which involve resolving their conflicting opinions, are used to pool opinions where, at each stage, some degree of resolution takes place. Ultimately because every opinion relates to every other through the network of relations that bind agents together, a consensus is reached that can be interpreted as a process of weighted averaging whose formal properties mirror the operation of a first-order Markov chain. The elaboration of this process that we invoke here is based on a process of exchange due to Coleman (Foundations of Social Theory. Belknap Press, Cambridge, MA, 1994) in which we characterise the problem as one of resolving conflicts between agents which we call the primal or differences between factors in terms of opinions which we call the dual. We define several variants of this process and then demonstrate this for a semi-real 'toy' problem of land development in the heart of London where a small set of stakeholder agents have different degrees of interest and control in a small set of land and building sites (parcels). In terms of the model, we show how the problem is already in equilibrium if interest and control are the same and this provides a benchmark for differences between interest and control which characterise the actual problem. We conclude with proposals for making the model more realistic and extending it to deal with problems where conflicts are only partially resolved or not resolved at all.

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2.1 The Complexity of Design: Evolving Plans Through Opinion Pooling

Design which we consider here as the arrangement of physical forms that satisfy some functions in an optimal or best way, is seldom accomplished immediately in one step. Some argue that good design can be traced to moments of instant inspiration and insight but these are rare and in any case when we are dealing with forms whose function is to enable others to meet their needs, design is usually accomplished in some iterative manner, whether the designer be a lone individual or as is more likely, several individuals, a collectivity, a team, any group working together. In our context where we are focussed on physical forms that range from buildings to landscapes, urban morphologies, and regions, design is very much a communal activity which involves resolving different viewpoints about what constitutes a best solution. In this sense, design is sequential, perhaps even based on a process of argumentation, and this is particularly so where the forms being designed have wide and powerful interests and control involving many stakeholders whose context is often political. In fact, in urban planning, the language of design is often based on defining solutions as plans or policies and although they usually have physical and spatial implications, there are many non- or a-spatial features that bring the process directly into the political and social arena. In this sense, design is inherently complex and the processes that characterise complex systems provide useful analogies.

In this chapter, we will introduce a model of design as a process of evolution, where the aim is to generate design solutions that resolve conflict between stakeholders. Models for this kind of conflict resolution go back 60 years or more (French 1956; Harary 1959) and involve the notion that different opinions about the best design associated with the stakeholders, need to be resolved in some way, leading to a compromise or consensus. The process we will present has often been compared to Markov averaging which produces a solution as a weighted average of the initial opinions as to the best design. This process has quite tractable and rather simple statistical properties (de Groot 1974; Kelly 1981) but more recently, a new wave of interest in this model has characterised it as 'opinion pooling' where the focus is much more on generating equilibria using various forms of computation such as flocking (Blondel et al. 2005; Motsch and Tadmor 2014). There are also social network interpretations which link these ideas to network science (Jackson 2011) and the model is also being explored in the study of opinion dynamics (Jia et al. 2013; d'Errico et al. 2014).

We will develop these ideas of opinion pooling on the basis of predicting the form of the networks that tie together various elements of the problem that different opinions are associated with. These networks can be generated from correlations between the opinions of the actors, agents or stakeholders—terms we will use interchangeably—with higher correlations in general being associated with stronger ties between actors. In an equivalent way, we can consider correlations between the elements making up an opinion across the agents and in this way consider that networks can be formed by concatenating these bipartite relations between agents and the elements forming their opinions. In this way, we will draw on ideas from exchange theory specifically introducing Coleman's (1994) model of collective action which represents an unpacking of the more aggregate French-Harary opinion pooling models noted above.

We will thus articulate the problem of choosing a plan in terms of how a group of ***agents*** relate to a series of ***factors*** in which the agents have varying opinions reflecting the degrees of interest in and control they have over the factors they consider important to the best plan. The factors imply something about the plan which is defined in terms of spatial locations. It might, for example, be defined in terms of the relative weights that agents ascribe to factors relevant to the plan or to different plans themselves that agents consider define their interest and control. In our subsequent exposition, we will present different conceptions of the problem defined in terms of different agents and different factors. In this sense, our problem is conceived in terms of the relationships between the social system defined through the agents and the spatial system defined through the factors. We might even consider the rules that define how agents behave socially as the 'genotype' of the problem and the spatial factors as the 'phenotype' but there the analogy ends. It only serves to show that the system can be thought of as a social collective based on relations between agents defined through spatial factors or as a system of relations defined across spatial factors with respect to social agents.

To summarise, agents relate to plans with respect to factors that affect the plan. If we define these consistently, then we can measure the relationships between agents —as a kind of social network—through their varying interest and control over factors. This defines the ***primal*** problem for which there is a natural ***dual*** based on the relations between spatial factors—a kind of spatial network but not in locational terms—through the relative coincidence of interest and control by agents over factors. In the sequel, we will elaborate this conception in several different ways first by introducing a generic framework based on social exchange, namely Coleman's (1994) theory of collective action. We use this framework to define many different variants of the plan-design problem and once we have elaborated its implications, we will produce a key simplification of the structure which pertains to thinking of factors as partial solutions to the planning problem—different plans— which in turn are defined as physical maps. In this way, the framework connects up to plan-design problems which lie at the basis of geo-design. These in turn build on older ideas about map overlay analysis which is a cornerstone of GIS (McHarg 1969; Steinitz et al. 1976; Steinitz 2012) and we will trace these links to spatial averaging in the sequel.

2.2 Social Exchange: A Theory of Collective Action

We first define a set of n agents who each have a degree of interest $\mathbf{X}$ as well as a degree of control $\mathbf{C}$ in a set of m factors. An agent may have a very low degree of interest in a factor but a high degree of control over it and vice versa, and these differences between interest and control define the relative interaction between agents with respect to factors as well as the relative interaction between factors with respect to agents. The first conception we refer to as the 'primal problem' and the second the 'dual'. This is equivalent to thinking of the primal as being soluble through interactions between agents over factors and the dual as interactions between factors with respect to agents. To give this some formal meaning, we first consider the interest which each agent i has in a factor j as an $n \times m$ matrix $\mathbf{X}$ which we define as

$$\mathbf{X} = \begin{bmatrix} X_{11} & X_{12} & X_{13} & \dots & X_{1m} \\ X_{21} & X_{22} & X_{23} & \dots & X_{2m} \\ X_{31} & X_{32} & X_{33} & \dots & X_{3m} \\ . & . & . & & . \\ X_{n1} & X_{n2} & X_{n3} & \dots & X_{nm} \end{bmatrix}, \quad \sum_{j=1}^{m} X_{ij} = 1, \quad \forall i = 1, 2, 3, \ldots, n. \quad (2.1)$$

The matrix of interests is structured in probability form as a stochastic matrix where each element X_{ij} is the proportion of interest that an agent i has in a particular factor j. An analogous stochastic matrix can be defined for the degree of control which is an $m \times n$ matrix $\mathbf{C}$ which we define as

$$\mathbf{C} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & \dots & C_{1n} \\ C_{21} & C_{22} & C_{23} & \dots & C_{2n} \\ C_{31} & C_{32} & C_{33} & \dots & C_{3n} \\ . & . & . & & . \\ C_{m1} & C_{m2} & C_{m3} & \dots & C_{mn} \end{bmatrix}, \quad \sum_{k=1}^{n} C_{jk} = 1, \quad \forall k = 1, 2, 3, \ldots, m, \quad (2.2)$$

where C_{jk} is the degree of control over a factor which each agent exercises. Note that the two matrices are defined as being the transpose of one another with respect to agents and factors so that we can relate them directly in the manner used below.

We define the primal problem as one where the interaction between agents is formed by correlating the degree of interest with respect to the degree of control between any two agents. We thence define the probability of interaction between any two agents i and k as the $n \times n$ stochastic matrix $\mathbf{P}$ defined as

$$\mathbf{P} = [P_{ik}] = \sum_{j} X_{ij} C_{jk}, \sum_{k} P_{ik} = \sum_{k} \sum_{j} X_{ij} C_{jk} = \sum_{j} X_{ij} \sum_{k} C_{jk} = 1, \quad (2.3)$$

and this is the key set of relative interactions that define the primal problem. The dual problem is defined in analogous terms as the pattern of interactions between the factors formed as the correspondence between the degree of control and interest across the profile of the agents. We define the probability of interaction between any two factors j and ℓ as the $m \times m$ stochastic matrix $\mathbf{Q}$ defined as

$$\mathbf{Q} = [Q_{j\ell}] = \sum_k C_{jk} X_{k\ell}, \sum_\ell Q_{j\ell} = \sum_\ell \sum_k C_{jk} X_{k\ell} = \sum_k C_{jk} \sum_\ell X_{k\ell} = 1. \quad (2.4)$$

These interactions compose a dual problem which is easily seen as being entirely consistent in formal terms with the primal. If we define processes of consistently resolving conflicts between interest and control with respect to the agents or the factors—on the primal or the dual—then we will be able to generate a consistent set of power relations that are reflected in the solutions generated from (2.3) and (2.4).

Essentially the plan-design problem can be thought of as a process of resolving the differences between interests and control with respect to the agents or with respect to the factors. In this sense we can define a process of conflict resolution on the primal or dual problems with one following from the other. Let us illustrate this for the primal problem. Imagine that we begin with an arbitrary distribution of resources across the agents which is the $1 \times n$ vector $\mathbf{r}(0)$ which we can normalise as $\sum_i r_i(0) = 1$. Now if we assume that the agents examine the resources that other agents have and thus rationally pool them first according to their collective interest and then their control, the agents produce new allocations of resources called $\mathbf{r}(1)$. In terms of the interaction matrix between agents which relates to the relative importance of communications based on their interest and control, this can be written as $r_k(1) = \sum_i r_i(0) P_{ik}$ which in matrix terms is $\mathbf{r}(1) = \mathbf{r}(0)\mathbf{P}$.

Now this is a Markov process with very well defined properties. The resources vector will converge to a unique equilibrium where the relative pooling of resources will stabilise as a function of the interaction matrix (which reflects interests and control). In the steady state, it is easy to show that the resources vector $\mathbf{r}(t+1) \to \mathbf{r}$ as $t+1 \to \infty$; formally this equilibrium is

$$\left.\begin{array}{l} \mathbf{r}(t+1) = \mathbf{r}(t)\mathbf{P} = \mathbf{r}(0)\mathbf{P}^t \\ \mathbf{r} = \mathbf{r}\mathbf{P}^\infty \to \mathbf{r} = \mathbf{r}\mathbf{P} \end{array}\right\}. \quad (2.5)$$

An exactly analogous process occurs if we begin the resources pooling with a vector of the values of the factors that will ultimately compose the plan. Start with a normalised arbitrary $1 \times m$ vector $\mathbf{v}(0)$ where $\sum_j v_j(0) = 1$. Then we produce new values for each factor $\mathbf{v}(1)$ which in terms of the interaction matrix between factors is written as $v_\ell(1) = \sum_j v_j(0) Q_{j\ell}$ or in matrix terms $\mathbf{v}(1) = \mathbf{v}(0)\mathbf{Q}$. This has a steady state equivalent to that in (2.5) reflecting the convergence of values of each factor and giving the importance of each in the final solution where $\mathbf{v}(t+1) \to \mathbf{v}$ as $t+1 \to \infty$; formally this is

$$\left.\begin{array}{l} \mathbf{v}(t+1) = \mathbf{v}(t)\mathbf{Q} = \mathbf{v}(0)\mathbf{Q}^t \\ \mathbf{v} = \mathbf{v}\mathbf{Q}^\infty \to \mathbf{v} = \mathbf{v}\mathbf{Q} \end{array}\right\}. \quad (2.6)$$

Equations (2.5) and (2.6) are interconnected in an entirely consistent way but before we sketch the relationships of these interlinked Markov processes, we need to present a much more intuitive way of illustrating the meaning of these processes that make crystal clear what these processes of social exchange and their equilibrium imply.

It is relatively straightforward to directly connect the two Markov processes. If we multiply the steady state equations for agents $\mathbf{r} = \mathbf{rP} = \mathbf{rXC}$ by the interest matrix $\mathbf{X}$, we can write this as $\mathbf{rX} = \mathbf{rPX} = \mathbf{rXCX} = \mathbf{rXQ}$. However the vector $\mathbf{v}$ in the steady state equation $\mathbf{v} = \mathbf{vQ}$ is unique and therefore it is clear that $\mathbf{v} = \mathbf{rX} = \mathbf{vCX} = \mathbf{vQ}$. In an analogous way, we can multiply by the control matrix $\mathbf{C}$ and from this it is clear that $\mathbf{vC} = \mathbf{vQC} = \mathbf{vCXC} = \mathbf{vCP}$, from which the unique steady state vector $\mathbf{r} = \mathbf{vC}$. Collecting these two results, we can state the equilibrium relations as

$$\left.\begin{array}{l}\mathbf{v} = \mathbf{rX} \\ \mathbf{r} = \mathbf{vC}\end{array}\right\}. \tag{2.7}$$

From this it is clear that there is a much more intuitive explanation of the process of social exchange, of social interaction, which leads to the steady state. We can now iterate on (2.7) by first forming an arbitrary distribution of resources $\mathbf{r}(0)$ or, if we are able to define this, an observed distribution and this generates a distribution of values $\mathbf{v}(0)$, in short, $\mathbf{v}(0) = \mathbf{r}(0)\mathbf{X}$. This essentially means that we take the resources of each agent and we distribute them to each factor in proportion to how much interest they have in that factor; that is the distribution of resources $r_i(0)$ is mapped into the interest in a factor X_{ij} and then this component of the resource $r_i(0)X_{ij}$ is summed over all the agents to find the value that is invested in the factor as $v_j(0) = \sum_i r_i(0)X_{ij}$. Now we have the investment in the factor and we have to consider how much control we have over that factor. This involves us in working out the component of the value in that factor which is controlled by an agent, that is $v_j(0)C_{j\ell}$. We then add up the value that is controlled by each agent in each factor to the total value controlled by the agent in all factors and this gives the amount of resource that is now assigned to the agent, that is $r_k(1) = \sum_j v_j(0)C_{jk}$. If $r_k(1) \neq r_k(0)$ which will always be the case in the initial rounds of iteration, we need to repeat the process, each time indulging our interest and modifying it by exercising our control until the system moves to the unique equilibrium defined above in (2.5)–(2.7).

2.3 Complementary Exchange: Averaging Processes from Markov Chains

Another way of thinking about the processes of moving to an equilibrium distribution of agent resources and factor values is to consider an agent or a factor as changing their opinions or values of their attributes successively in proportion to their resources or values in the following way. For an agent, the initial distribution of resources $\mathbf{r}(0)$ (which is a probability vector) can be interpreted as reflecting the probability that an agent takes on a particular opinion about the problem in question. If we assume a set of agents continually hopping from one state to another, we might even think of the distribution of resources as being the probability that an agent is in a particular state, or rather as the problem is based on agents, the probability of each agent in question having a particular resource. This is continually changing as the agents compare their resources but if we think of agents as states of the system, then the probability of the system being in a particular state is the resource vector $\mathbf{r}(t)$ at any time t. As the pooling of resources moves towards equilibrium, then the steady state vector $\mathbf{r}$ reflects the probability of any agent being in the state associated with that agent. The same process can be considered in terms of the factors and their values which reflect the state of the system that any factor find itself in. This is perhaps a little tortuous but it is the conventional form of a Markov probability process which provides a more traditional interpretation.

At the other extreme, however, we have two complementary processes pertaining to the primal and dual which can be interpreted as the averaging of initial differences within the set of agents, and the set of factors, and these follow the same equilibrium relations that we defined around (2.5)–(2.7). Imagine now that each agent holds a certain attitude about a plan defined as a number in the vector $\mathbf{a}(0) = [a_i(0)]$ which is the value that they start with. Each factor also has a value that we can define equivalently as $\mathbf{f}(0) = [f_j(0)]$. Now consider first the primal process where the agents pool their values on the basis of how they interact with one another which is given by the matrix $\mathbf{P}$. A new set of values at the second iteration of the process is defined from $\mathbf{a}(1) = \mathbf{P}\ \mathbf{a}(\mathbf{0})$ and if we iterate in the normal fashion, it is clear that the set of attitudes will converge to $\mathbf{a} = \lim\ t \to \infty\ [\mathbf{a}(t+1) = \mathbf{P}^{t+1}\ \mathbf{a}(\mathbf{0})]$ and $\mathbf{a} = \mathbf{P}\ \mathbf{a}$. Now as $\mathbf{P}$ is a stochastic matrix, and sums to 1 over its rows, then each agent will move to the same numerical value and this represents a weighted average which is reflected in the structure of $\mathbf{P}$. This is a little easier to see if we note that the steady state matrix $\lim\ t \to \infty\ \mathbf{P}^{t+1} = \mathbf{R}$ where each row is the steady state vector $\mathbf{r}$. Thus $\mathbf{a} = \mathbf{R}\ \mathbf{a}(0)$ or $a = a_k = \sum_i R_{ik} a_i(0) = \sum_i r_i a_i(0),\ \forall k$. Each agent thus has the same attitude at equilibrium and it might be said that a consensus has been reached. This is the classic process of Markov averaging first introduced by French (1956), formalised by Harary (1959) and further explored by many others, in particular de Groot (1974) and Kelly (1981) and more recently by Jia et al. (2013) amongst others.

An exactly analogous process pertains to the averaging of factors. If we start with a vector of factor values $\mathbf{f}(0)$, we form $\mathbf{f}(1) = \mathbf{Q}\ \mathbf{f}(0)$ and then through iteration, factor values will converge to $\mathbf{f} = \lim\ t \to \infty\ [\mathbf{f}(t+1) = \mathbf{Q}^{t+1}\ \mathbf{f}(0)]$. As $\lim\ t \to \infty\ \mathbf{Q}^{t+1} = \mathbf{S}$, then $\mathbf{f} = \mathbf{S}\ \mathbf{f}(0)$, and the final factor value $f = f_j = \sum_\ell S_{j\ell} f_\ell(0) = \sum_j v_j f_j(0),\ \forall \ell$. Now let us write the equilibrium relations for both these averages as

$$\left.\begin{aligned} \mathbf{a} = \mathbf{P}\ \mathbf{a} = \mathbf{XCa} \\ \mathbf{f} = \mathbf{Qf} = \mathbf{CXf} \end{aligned}\right\} \tag{2.8}$$

which can be considerably simplified by noting that as $\mathbf{a} = [a_i] = a,\ \forall i$ and $\mathbf{f} = [a_j] = f,\ \forall j$, then the relations $\mathbf{Ca}$ and $\mathbf{Xf}$ are degenerate in that $\sum_k C_{jk} a_k =$

$a \sum_k C_{jk} = a, \ \forall j$ and $\sum_j X_{ij} f_j = f \sum_j X_{ij} = f, \ \forall i$. Relations equivalent to (2.7) do not hold for the averaging processes. We will explore this convergence in more detail when we use it below to examine how social agents can define solutions to spatial problems through the notion of factors being equivalent to map layers. A full statement of the model is presented by the author (Batty 2013) where there are various additional interpretations of the convergence to a steady state. We assume that the matrices **X** and **C** are defined so that **P** and **Q** are strongly connected which is a basic requirement of articulating the problem in the first place. This implies that every agent is linked to every other agent and every factor to every other factor, directly or indirectly though the networks associated with **P** and **Q**.

2.4 Maps as Factors: Using the Model to Simulate the Map Overlay Problem

We will now focus on the averaging problem where we define the agents as social entities that have an interest and control over a planning solution which they articulate as a set of maps. We will define the series of maps in the $n \times m$. matrix $\mathbf{M}(0)$ as

$$\mathbf{M}(0) = \begin{bmatrix} M_{11}(0) & M_{12}(0) & M_{13}(0) & \dots & M_{1m}(0) \\ M_{21}(0) & M_{22}(0) & M_{23}(0) & \dots & M_{2m}(0) \\ M_{31}(0) & M_{32}(0) & M_{33}(0) & \dots & M_{3m}(0) \\ . & . & . & & \\ M_{n1}(0) & M_{n2}(0) & M_{n3}(0) & \dots & M_{nm}(0) \end{bmatrix}, \tag{2.9}$$

where each agent $i, i = 1, 2, 3, \ldots, n$ expresses an initial value that they ascribe to each map cell or location at time $t = 0$ as $M_{ij}(0), j = 1, 2, 3, \ldots, m$. In fact the map associated with agent i is strung out as a $1 \times m$ vector whose values define the relative importance that the agents ascribe to the problem. To fix ideas, we might consider each map at this stage to represent the development potential that an agent i ascribes to the location or cell j of the map, with the differences between each map vector in terms of these values as pertaining to differences between agents which need to be resolved as a solution to the problem. The differences between the column map vectors then describe the differences in potential between the agents with respect to a particular map cell or location. In short the process of averaging or compromising is one that irons out these differences according the balance of interest and control that first, the agents have in factors which are now assumed to be individual maps, and second, locations or map cells associated with agents which are location profiles across all agents. These define the primal and dual respectively.

We can now use all the results we have derived to illustrate what happens if the agents resolve conflicts between their different maps through averaging. We will now consider each set of maps $\mathbf{M}(0)$ defined as a row of cells for each agent and its transpose $\mathbf{M}^T(0)$ as a column of cells for each agent. The averaging across agents is based on the process defined as $\mathbf{M}(t+1) = \mathbf{P}^{t+1}\mathbf{M}(0)$ and this converges to $\mathbf{M} = \mathbf{PM}$ while the averaging across cells is $\mathbf{M}^T(t+1) = \mathbf{Q}^{t+1}\mathbf{M}^T(0)$ and this converges to $\mathbf{M}^T = \mathbf{QM}^T$. We can write these equilibrium averages explicitly as

$$\left.\begin{array}{l} \sum_k P_{ik}M_{kj} = M_j, \forall j \\ \sum_\ell Q_{j\ell}M_{\ell k} = M_\ell, \forall k \end{array}\right\}, \tag{2.10}$$

where M_{ij} is the transpose of M_{ji} and vice versa. When we write these transposes explicitly, we simply interchange the rows and columns which are always defined with respect to agents as i and k and maps (factors) as j and ℓ. In fact we can compute these averages directly from knowledge of the steady state resources and values from (2.5) and (2.6) and these are $M_j = \sum_i r_i M_{ij}$ and $M_i = \sum_j v_j M_{ji}$.

Now there is a dramatic simplification of this process when we define the interest agents have in maps and the control they have over maps in terms of the same values they ascribe to the map. That is, we will define the two matrices $\mathbf{X}$ and $\mathbf{C}$ directly from $\mathbf{M}(0)$ and $\mathbf{M}^T(0)$. Then

$$X_{ij} = \frac{M_{ij}(0)}{\sum_\ell M_{i\ell}(0)} = \frac{M_{ij}(0)}{M_i(0)}, \quad \sum_j X_{ij} = 1, \text{and} \tag{2.11}$$

$$C_{jk} = \frac{M_{jk}(0)}{\sum_i M_{ji}(0)} = \frac{M_{jk}(0)}{M_j(0)}, \quad \sum_k C_{jk} = 1. \tag{2.12}$$

Note that $M_i(0)$ is the sum of the rows or the total interest that an agent has in all the maps, that is in matrix terms $\mathbf{M}(0)\mathbf{1}^T$ where $\mathbf{1}$ is the relevant unit column vector, and $M_j(0)$ is the sum of the columns or the total control that all agents vest in a cell of the map, in matrix terms $\mathbf{M}^T(0)\mathbf{1}$. If we now write these sums in their appropriate diagonal matrix, we can write the matrix equations for interest and control as

$$\left.\begin{array}{l} \mathbf{X} = \mathbf{DM} = \begin{bmatrix} M_1^{-1}(0) & 0 & 0 & \dots & 0 \\ 0 & M_2^{-1}(0) & 0 & \dots & 0 \\ 0 & 0 & M_3^{-1}(0) & \dots & 0 \\ . & . & . & \dots & . \\ 0 & 0 & 0 & \dots & M_n^{-1}(0) \end{bmatrix} \begin{bmatrix} X_{11} & X_{12} & X_{13} & \dots & X_{1m} \\ X_{21} & X_{22} & X_{23} & \dots & X_{2m} \\ X_{31} & X_{32} & X_{33} & \dots & X_{3m} \\ . & . & . & & . \\ X_{n1} & X_{n2} & X_{n3} & \dots & X_{nm} \end{bmatrix} \\ C = \boldsymbol{\delta}\mathbf{M}^T = \begin{bmatrix} M_1^{-1}(0) & 0 & 0 & \dots & 0 \\ 0 & M_2^{-1}(0) & 0 & \dots & 0 \\ 0 & 0 & M_3^{-1}(0) & \dots & 0 \\ . & . & . & \dots & . \\ 0 & 0 & 0 & \dots & M_m^{-1}(0) \end{bmatrix} \begin{bmatrix} C_{11} & C_{12} & C_{13} & \dots & C_{1n} \\ C_{21} & C_{22} & C_{23} & \dots & C_{2n} \\ C_{31} & C_{32} & C_{33} & \dots & C_{3n} \\ . & . & . & & . \\ C_{m1} & C_{m2} & C_{m3} & \dots & C_{mn} \end{bmatrix} \end{array}\right\} \tag{2.13}$$

Note that the inverses M_i^{-1} and M_j^{-1} in $\mathbf{D}$ and $\boldsymbol{\delta}$ respectively are inverses of the row and column sums of the original map matrix $\mathbf{M}(0)$. We can now simplify the equilibrium relations directly by substituting these expanded representations of the interest and control matrices into any of (2.5)–(2.7). In fact using (2.7), we can write these as

$$\left.\begin{array}{l}\mathbf{v} = \mathbf{rX} = \mathbf{rDM}(0) \\ \mathbf{r} = \mathbf{vC} = \mathbf{v}\boldsymbol{\delta}\mathbf{M}^T(0)\end{array}\right\}. \tag{2.14}$$

If we set $r_i = M_i(0)$ and $v_j = M_j(0)$, then $\mathbf{rD} = \mathbf{1}$ and $\mathbf{v}\boldsymbol{\delta} = \mathbf{1}$ and (2.14) become $\mathbf{v} = \mathbf{1M}(0)$ and $\mathbf{r} = \mathbf{1M}^T(0)$. In short the equilibrium vectors—the relative values of each map and the resources for each agent—are the values invested in each cell of the map by all the agents, and the values invested by each agent in all the maps. This is a very interesting result. It means that if the basic map matrix determines both the interest and control, the system is already in equilibrium, that is, interest and control align exactly. In fact this also implies what is obvious from this exposition, that is, that the differences between interest and control determine the need to compromise. In this sense, we might consider the difference between the final resource and value vectors from a situation where interest and control differ and these simplified vectors a measure of how far from equilibrium a system of interaction such as this one is. In short, the differences $\mathbf{v} - \mathbf{1M}(0)$ and $\mathbf{r} - \mathbf{1M}^T(0)$ determine how far from equilibrium the system is.

There is one last variant of this process that does not rely on defining probability relationships between agents and factor maps through interest and control but simply takes the agent-map matrices and forms social networks based on the relative similarity between agents in the primal problems and map factors in the dual. This is much more akin to the traditional averaging networks first introduced by French (1956) and Harary (1959). Comparing the values that agents ascribe to maps with respect to how similar agents are to one another, we can form the interaction matrix $\tilde{\mathbf{P}} = \mathbf{M}(0)\mathbf{M}^T(0)$. This is a symmetric matrix that gives the strength of connections between pairs of agents and this defines our primal problem. This is in one sense an un-normalised version of the map matrix for the normalised interest and control matrices in (2.11) and (2.12) above. The dual symmetric interactions between map factors which is a comparison of the similarity between any two maps over all agents is given in an analogous way as $\tilde{\mathbf{Q}} = \mathbf{M}^T(0)\mathbf{M}(0)$. From these matrices, we can form stochastic matrices $\mathbf{P}$ and $\mathbf{Q}$ which form the essence of the two probability and averaging processes which define the equilibrium relations associated with the primal and the dual. As above, these can be written in diagonal and map matrix form as

$$P_{ik} = \frac{\sum_j M_{ij}M_{jk}}{\sum_j \sum_z M_{ij}M_{jz}} = \mathbf{DM}(0)\mathbf{M}^T(0), \quad \sum_k P_{ik} = 1, \text{and} \tag{2.15}$$

$$Q_{j\ell} = \frac{\sum_k M_{jk}M_{k\ell}}{\sum_k \sum_z M_{jk}M_{kz}} = \boldsymbol{\delta}\mathbf{M}^T(0)\mathbf{M}(0), \quad \sum_\ell Q_{j\ell} = 1, \tag{2.16}$$

with the equilibrium relations now defined as

$$\left.\begin{array}{l}\mathbf{r} = \mathbf{rP} = \mathbf{rDM}(0)\mathbf{M}^T(0) \\ \mathbf{v} = \mathbf{vQ} = \mathbf{v}\boldsymbol{\delta}\,\mathbf{M}^T(0)\mathbf{M}(0)\end{array}\right\}. \qquad (2.17)$$

If we assume that the resource and value equilibrium vectors are defined from the normalisation factors in each of the diagonal matrices as

$$\left.\begin{array}{l} r_i \propto \sum_j \sum_z M_{ij} M_{jz} \\ v_j \propto \sum_k \sum_z M_{jk} M_{kz} \end{array}\right\}. \qquad (2.18)$$

The matrix equations in (2.17) then simplify to

$$\left.\begin{array}{l}\mathbf{r} = \mathbf{1M}(0)\mathbf{M}^T(0) \\ \mathbf{v} = \mathbf{1M}^T(0)\mathbf{M}(0)\end{array}\right\}. \qquad (2.19)$$

As these matrices $\mathbf{M}(0)\mathbf{M}^T(0)$ and $\mathbf{M}^T(0)\mathbf{M}(0)$ are symmetric, it is easy to show that the column sums are the same as the row sums for each relation thus proving (2.18). In fact this is the result that is presented by the author (Batty 2013) for the French-Harary model, the first in the development of this kind of conflict resolution.

At this stage we have presented the essential logic of opinion pooling as it might be dimensioned to a problem where agents have different interests and control in locations specified by a spatial system represented by a map. We have also introduced a set of variants of the collective action model and in the remaining part of the chapter, we will demonstrate how these models might be applied to a semi-real problem of land use allocation in the heart of world city. The model of course can be generalised to any system where there are two sets of characteristics and in fact, most opinion pooling models are non-spatial. But the logic of developing the model in this context is strikingly similar to that used in map overlay analysis in GIS and urban design and this is the focus we will exploit and demonstrate here.

2.5 Applications of the Approach: Competition for Land Use in a World City

To demonstrate how we determine a solution equivalent to evolving a plan from the differing plans of the relevant agents or stakeholders, we have chosen a problem of reconciling different interests in land development in the heart of a world city, London. The area we have chosen is some 5 hectares in size, immediately north of St. Paul's cathedral in an area that for the last 200 years (until quite recently) has been the location of the General Post Office and is now largely occupied by financial services, medical-hospital uses, and private apartments. It is in an area that is undergoing rapid change of use and redevelopment as investors and developers attempt to realise ever more profits from its location in the ‘square mile’, the financial quarter of London and the borough which is called the ‘City’. The pressures on development are huge while the control over what happens is severely constrained by the City Corporation. All this conflicts with the massive amounts of capital that are tied up in the buildings which house some of the world’s most prominent financial services.

This application is a caricature of the development process and in this sense, it is a ‘toy’ example simply to illustrate the nature of the solution process rather than to provide any realistic resolution of conflicts in the area. We will define a limited number of key agents—but only 6 in all—who dominate the scene. In reality, however, there are many more, including groups whose interest originates from issues beyond the area in question. In terms of the way we define the map, we deal simply with the location of 8 land parcels, most containing buildings and we do not consider the streets between the buildings as uses that would change in any way. In fact the land parcels and streets are not assumed to change their configuration in the plan, and this is consistent with the relatively inert structure of land parcels in this part of London, notwithstanding considerable change in the usage of land and in the way it is developed and occupied. We list the agents and the sites that compose the map in Table 2.1 which gives an immediate sense of the nature of the problem.

The area is shown in Fig. 2.1 where the buildings are defined by the numbers in Table 2.1, and some sense of the character of the area is given by the thumbnail pictures showing individual viewsheds in the area which are shown in Fig. 2.2. We will say something about each of the sites or land parcels in the order shown in Table 2.1. Site 1, the Aldersgate Complex, is a large postmodern ziggurat-type building adjacent to the Museum of London at the western end of London Wall (the original Roman wall of the city). The site was redeveloped in the early 1990s and it is unlikely to be changed physically in the next decade although it has recently been reconfigured after the financial crisis and now contains a number of financial services companies in contrast to its previous tenant which was a large law firm. South of this is the building complex that bounds site 5, Postman’s Park. On the southern side of the Park is site 3, the 1880 General Post Office building which is now owned

Table 2.1 Agents and sites-land parcels-buildings

$n = 6$	Agents	$m = 8$	Sites-land parcels-buildings
1	City Corporation	1	Aldersgate Complex
2	Residents	2	St. Botolph’s Church
3	Hospital NHS	3	Nomura House
4	Developers	4	Milton House
5	Property Speculators	5	Postman’s Park
6	Investment Banks	6	Bank of America
		7	Barts New Building
		8	Barts Old Building

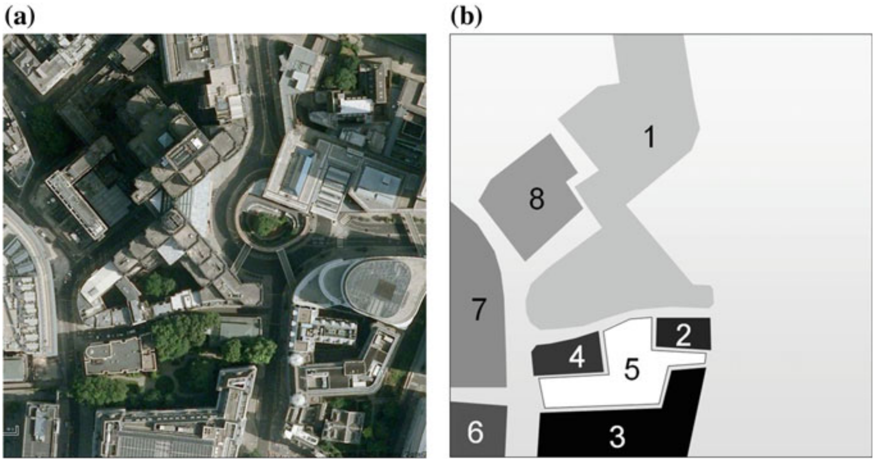

Fig. 2.1 The building complex. **a** The physical form. **b** The sites/land parcels

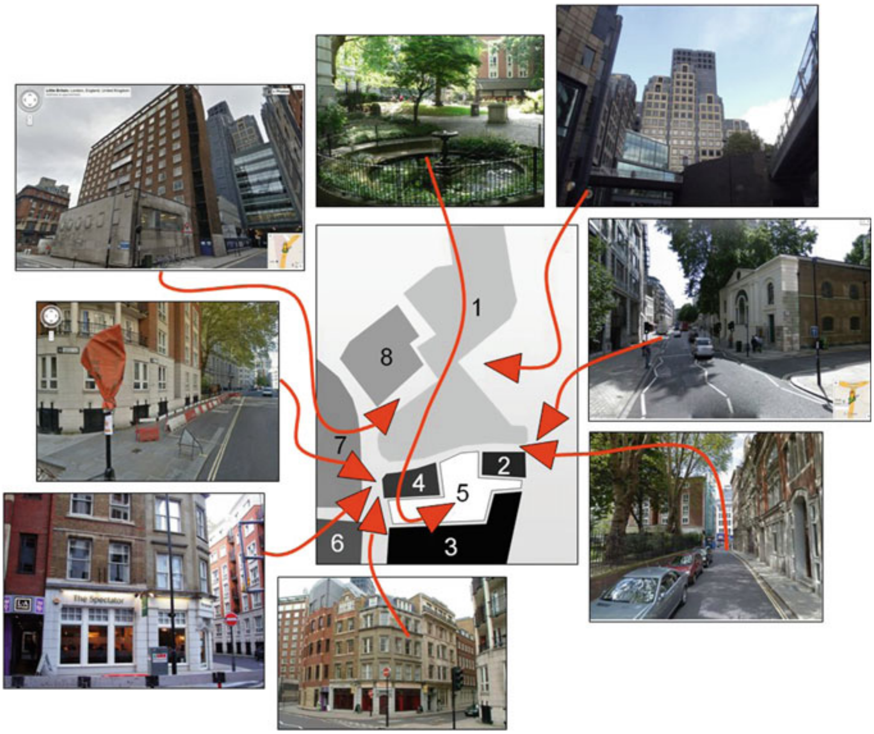

Fig. 2.2 The problem context: The Little Britain—St. Bartholomew's Hospital Site

by Nomura Bank. On the east of the Park is site 2, St. Botolph's Georgian Church, while the street that bounds the Aldersgate Complex running along Postman's Park is Little Britain, the place of John Wesley's conversion in 1738 which established Methodism a year later. At the western end of this street is site 4, residential apartments called Milton House built around the same time as the Aldersgate complex in the early 1990s. King Edward Street divides these buildings in the north-south direction with site 7, the St. Bartholomew's Hospital Old Building on its north (currently being redeveloped as apartments), site 8, the new St. Bartholomew's Hospital building south of the Old Building on the east, and south of this, site 6, the old Post Office Extension. This is now owned by Bank of America (acquired Merrill Lynch in the financial crash in 2009) who in turn had bought it from the Post Office in 2004.

This area does not have any particular symbolic imagery as it is of mixed use and much influenced by the eastward expansion of the city from the Bank of England area to St. Paul's due to the change in the location of the London Stock Exchange which is just south of the Bank of America on High Holborn. As Fig. 2.2 reveals, the area is quite attractive, particularly the complex of buildings around Postman's Park which is a classic New York City style pocket park. What is clear is that the area is subject to continual pressures relating to the fact that financial services now dominate the city and this is an area that is subject to the volatility of the financial economy, continual acquisitions and mergers that define these firms, and the extreme competition for office space that dominates different areas of the city and its extension westwards towards Bloomsbury and eastwards into the London Docklands.

We have defined 6 agents or actors who are the key stakeholders with both interest and control over the various sites. In fact we could define many more than 6 for each of the groups we identify could be broken down into different types but we need to keep the 'toy' problem manageable to illustrate the method. The first group is the City Corporation which is the arbiter of all that happens in the square mile. It controls most of the land which it owns and is leased to the many businesses and residents who make up the economic and social activity in the city. In fact its main control is over development and it tends to operate a pro-business policy but at the same time exercising considerable control over the type and visual appearance of development. The next group 2 are Residents who currently live in the complex centred on site 4 Milton House and the northern side of Little Britain which is attached to the Aldersgate Complex. This group is not particularly well-organised but as there is more apartment building planned for the area and as the streets are being reconfigured for cyclists, the residents' group is potentially a greater power broker than it has been so far. The third group is the St. Bartholomew's Hospital run by the National Health Service (NHS) who are a powerful public agency but who tend not to be interested in property except insofar as they have been selling off parts of the old Hospital buildings to private developers for apartments. The massive redevelopment of their own buildings which is almost complete now has been financed by external public finance initiatives. Group 4 are Developers who have a predatory interest in all buildings except those that they know they can never control such as churches, parks etc. The fifth group are the Property Speculators.

These are not the same as Developers as they tend not to be interested in the eventual usage or building form except insofar as they are interested in capital and finance. Last but not least there are the Investment Banks who have more ad hoc interests in property insofar as they are concerned with their international profile and nearness to other financial institutions.

We will define four different variants of the group decision problem beginning with the last that we specified, the French-Harary model. Here interest and control are not specified separately and the interactions between agents in the primal problem and map sites in the dual problem are determined as in (2.15) and (2.16). We then use the map matrix for both interest and control in the standard problem which is based on Coleman's model as we specified in (2.11) and (2.12) and this constitutes what we called the 'baseline' model that we can use to compare with any other. We then move to two variants of the full model where interest and control are specified quite separately. The first is where we simply assign random values to the **X** and **C** matrices and the second (and last) one is based on a full specification of what we regard as appropriate interest and control matrices that differ in plausible ways from one another. We can then make comparisons between the four different applications in terms of their equilibrium resource and value vectors and draw conclusions as to the sensitivity of the model, and the size of actual applications that are needed to implement this way of thinking in contrast to our 'toy' application. We then speculate on ways in which the model might be taken further to explore how planning problems of this type do not lead to consensus or solution, which some would argue is the recurrent condition in such contexts.

As a starting point we will define the map matrix which we use to define aggregate interactions and also use in the baseline model. We can write this matrix as

$$\mathbf{M}(0) = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}, \tag{2.20}$$

where Fig. 2.3 annotates this with respect to the agents and the sites. It is worth going through the elements of this matrix to focus on the rationale for associating an agent with a site for this is the essence of both their interest and control. To an extent, we might think of this as simply the interest that the agent has in change of use for the site which determines the agent interactions while the dual is simply the overall common interest that all agents have with respect to any two sites. Starting with the City Corporation, it has a strong interest in the two banks which have changed use frequently during the boom which preceded the financial crisis and its aftermath and although Bank of America now looks stable, Nomura are seeking new tenants. The Corporation also have an interest in the Barts Old Building, the new apartment complex as do the Residents whose interest is in the enhancement of the residential quality of the area. When we say that an agent or group has no

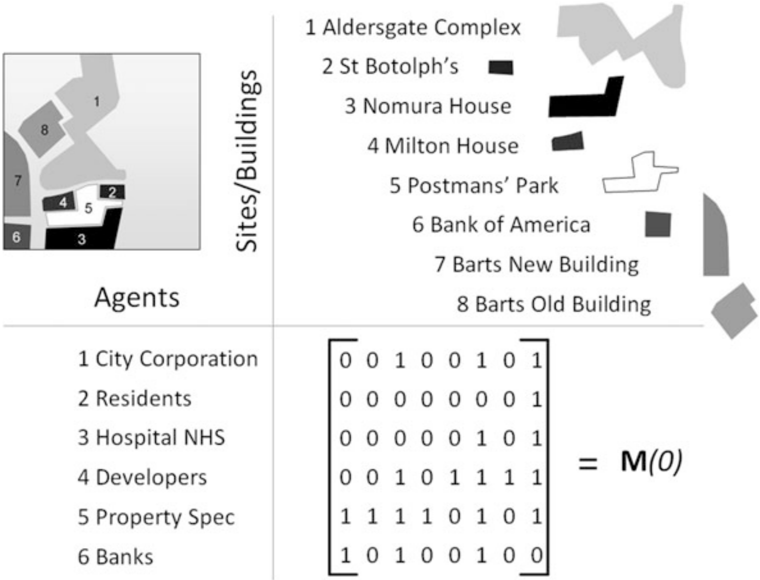

Fig. 2.3 The n × m map matrix based on relating agents to sites and buildings

interest, this is as much likely to mean that the agent has an interest in keeping a site in the same use and in this sense, all the agents apart from Developers and Property Speculators have a strong conservative outlook on what happens in this area.

The Hospital NHS Trust (St. Bartholomew's) is a state-of-the-art cancer hospital with a very old foundation which has recently been extensively redeveloped, hence the selling off of its Old Building for residential development. They have little interest in anything other than adjacent buildings but in this sense, do have a mild interest in Bank of America. Were the funds to be available for purchase of surrounding buildings, the NHS Trust would probably have a stronger interest but it is unlikely that this would ever be possible for the organisation of the hospital is based on much wider considerations that pertain to the Trust that runs it, and the somewhat parlous state of the NHS in Britain. Developers are much more predatory and have an interest in everything in the area with the exception of the residential development, the church and the Aldersgate Complex which are all protected or unlikely to be changed in any form in the immediate future. Arguably the Park is protected and unlikely to change but it could be developed more actively. Property Speculators have the widest interest but with little interest in Postman's Park or the Barts New Building. There is a mild interest in the church but only for its use value. Finally the Banks have an interest in their own use of their two sites as well as in the Aldersgate Complex which is still has space for let and which contains several financial companies that service banks in the wider city.

2.5.1 Solutions and Comparisons

Our first model is now very easy to construct as we form $\hat{\mathbf{P}}$ by multiplying $\mathbf{M}(0)$ by its transpose $\mathbf{M}^T(0)$ and its dual $\hat{\mathbf{Q}}$ by multiplying the transpose $\mathbf{M}^T(0)$ by the basic map matrix $\mathbf{M}^T(0)$. We can write these out explicitly as

$$\left.\begin{aligned}
\hat{\mathbf{P}} = \begin{bmatrix} 312332 \\ 111110 \\ 212221 \\ 312532 \\ 312363 \\ 201233 \end{bmatrix} = \begin{bmatrix} 00100101 \\ 00000001 \\ 00000101 \\ 00101111 \\ 11110101 \\ 10100100 \end{bmatrix} \begin{bmatrix} 000011 \\ 000010 \\ 100111 \\ 000010 \\ 000100 \\ 101111 \\ 000100 \\ 111110 \end{bmatrix} = \mathbf{M}(0)\mathbf{M}^T(0) \\
\hat{\mathbf{Q}} = \begin{bmatrix} 21210201 \\ 11110101 \\ 21411413 \\ 11110101 \\ 00101111 \\ 21411514 \\ 00101111 \\ 11311415 \end{bmatrix} = \begin{bmatrix} 000011 \\ 000010 \\ 100111 \\ 000010 \\ 000100 \\ 101111 \\ 000100 \\ 111110 \end{bmatrix} \begin{bmatrix} 00100101 \\ 00000001 \\ 00000101 \\ 00101111 \\ 11110101 \\ 10100100 \end{bmatrix} = \mathbf{M}^T(0)\mathbf{M}(0)
\end{aligned}\right\}. \tag{2.21}$$

We form the probability matrices in the usual fashion as $\mathbf{P}$ and $\mathbf{Q}$ and we can picture the stochastic interactions in these two social networks as in Fig. 2.4. Note that we will do this for each of the models in this section but our graph program does not produce directional interactions and thus what we see is the maximum interaction between agents and between sites. In fact in this first model, the ultimate steady state weights can be read off directly from the interaction matrices $\hat{\mathbf{P}}$ and $\hat{\mathbf{Q}}$. Equations (2.18) and (2.19) above can be written explicitly as

$$\left.\begin{aligned}
r_i \propto \sum_k \hat{P}_{ik} = \sum_j M_{ij} M_{jk} \\
v_j \propto \sum_\ell \hat{Q}_{j\ell} = \sum_k M_{jk} M_{k\ell}
\end{aligned}\right\}, \tag{2.22}$$

which from (2.21) can be written out directly as

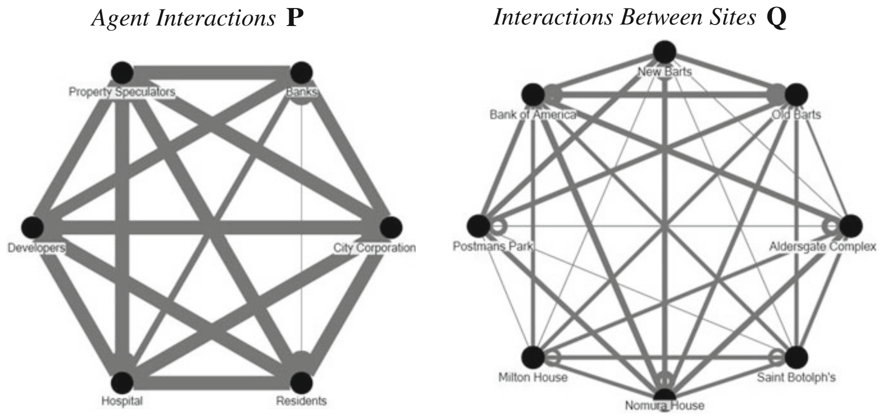

Fig. 2.4 The French-Harary interaction networks

$$\left.\begin{array}{l} \mathbf{r} = [14 \quad 5 \quad 10 \quad 16 \quad 18 \quad 11] \\ \mathbf{v} = [9 \quad 6 \quad 17 \quad 6 \quad 5 \quad 19 \quad 5 \quad 17] \end{array}\right\}. \tag{2.23}$$

The French-Harary model is in fact another kind of baseline, and we show these values in (2.23) scaled to sum to 100 in Table 2.2, where, henceforth, for the three other variants of the model we will use the same scaling. What this implies for the

Table 2.2 The steady state weighting vectors for the four models

Models	1	2	3	4	Differences		
Agents	French-Harary	Coleman Baseline	Coleman Random	Coleman Real	(2)–(1)	(2)–(3)	(2)–(4)
1	19 (3)	15 (3=)	16 (5)	34 (1)	−4	−1	−19
2	7 (6)	5 (6)	18 (1)	12 (5)	−2	−13	−7
3	13 (5)	10 (5)	15 (6)	9 (6)	−3	−5	1
4	22 (2)	25 (2)	17 (2=)	15 (2=)	3	8	10
5	24 (1)	30 (1)	17 (2=)	15 (2=)	6	13	15
6	15 (4)	15 (3=)	17 (2=)	15 (2=)	0	−2	0
Sites							
1	11 (4)	10 (4)	11 (6=)	16 (4=)	−1	−1	−6
2	7 (5=)	5 (5=)	13 (4)	3 (8)	−2	−8	2
3	20 (2=)	20 (3)	14 (3)	16 (4=)	0	6	4
4	7 (5=)	5 (5=)	9 (8)	5 (7)	−2	−4	0
5	6 (7=)	5 (5=)	15 (1=)	17 (3)	−1	−10	−12
6	23 (1)	25 (1=)	11 (6=)	19 (1)	2	14	6
7	6 (7=)	5 (5=)	12 (5)	6 (6)	−1	−7	−1
8	20 (2=)	25 (1=)	15 (1=)	18 (2)	5	10	7

The weights sum to 100 and can thus be interpreted as percentages; the numbers in brackets are their rank agents is that the most influential (for these values pertain to their weight in any ultimate consensus) are the Developers and the Speculators. The Residents have very little influence while the City Corporation is almost as powerful as the land and property interests, and although the Corporation wields great power, it is a little less interested in the sites in this area than Developers and Speculators. This makes good sense. The NHS in fact has modest power but only in relation to its adjacent sites. When we examine the sites, the most important are the banks—Nomura and the Bank of America with the new redevelopment site of Barts Old Building also having some significance. The existing residential sites, the church and the park are less important as these are shielded from an interest in further development.

Our second model uses the same data as the first but this time we have articulated it using specific interest and control as defined from the maps in (2.11) and (2.12). This is the simplification of Coleman's model where the final equilibrium vectors are simply the sum of the rows and columns of the map matrix, that is $\mathbf{v} \propto \mathbf{1M}(0)$ and $\mathbf{r} \propto \mathbf{1M}^T(0)$ which we can write out explicitly as

$$\left.\begin{array}{l} \mathbf{r} = [3 \quad 1 \quad 2 \quad 5 \quad 6 \quad 3] \\ \mathbf{v} = [2 \quad 1 \quad 4 \quad 1 \quad 1 \quad 5 \quad 1 \quad 5] \end{array}\right\}. \tag{2.24}$$

In fact these values in (2.24) are quite similar but much less sharpened versions of those generated by the French-Harary model in (2.23). This is clear from Table 2.2 where we scale them to sum to 100 and also use (2.24) as the baseline to make comparisons for all the other three models that we test. The only significant difference from the previous model is that the Barts Old Building is of top importance while the city corporation is of equal importance to the banks. It is not worth speculating on these differences for it is clear that this kind of analysis only comes into its own when the problem is scaled up with many more agents and sites and this is for future applications.

Our third model is yet another kind of baseline where we simply set each element of interest and control to random values, that is $X_{ij} \sim random(1)$ and $C_{jk} \sim random(1)$ where these values are then scaled to sum to 1 so that the two matrices are stochastic. We show the equilibrium values for a run of this model in Table 2.2 where the predictions of resources **r** and value **v** are truly random and have no meaning in terms of our set of agents and sites. Our fourth and last model is where we define interest and control in much more realistic ways. For example, the Residents may have a lot of interest in Postman's Park but no real control over doing anything about their interest. In fact this is not quite the case for as residents they can petition their local aldermen to act on their behalf but in general it is those who own sites and those who have capital to acquire them and change their use that have greater control. We have defined two matrices of interest and control that differ in these terms. The City Corporation have interest in the use of the larger buildings but no interest in the residential apartments or church or new hospital buildings because these are not going to change in the future. The Residents have an interest in the other residential building and planned apartments and the park while the hospital trust simply has an interest in its own buildings.

The Developers and Speculators have pretty similar interests in the usage of the larger buildings and banks while the Banks themselves have an interest in the buildings they themselves own. In terms of control, the Corporation has pretty much control over the entire area while the residents and the Hospital Trust's control matches their interest. Developers and Speculators control the large buildings and the banks are controlled by those who own them. The correlation between interest and control is low but positive with the percent of variance explained between **X** and $\mathbf{C}^T$ as 35 and thus there is a sufficient measure of difference between interest and control to ensure that some compromise is required.

In terms of the equilibrium distribution of resources and value, then it is the City Corporation that is by far the most powerful with the Developers, Speculators and Banks having a more or less equal control of resources. The sites that are most valuable are the largest bank buildings but the new apartments and the Aldersgate Complex are also highly valued in terms of change of use. The park is highly valued but the church is not because there is no likelihood of it being developed. The configurations of resources and values are given in Table 2.2 where it is clear that there are substantial differences between these measures and the baseline and where the biggest office building sites are the most valued. The real strength of this model is of course in sensitivity testing—to pose the question 'how can the resources of a particular agent be increased or the value of a site increased by manipulating the networks of interest and control?' And 'how easy or difficult this would be to accomplish?'

To conclude this analysis, it is worth exploring a little further the data in terms of the social networks that are implied by this application. We have already shown the

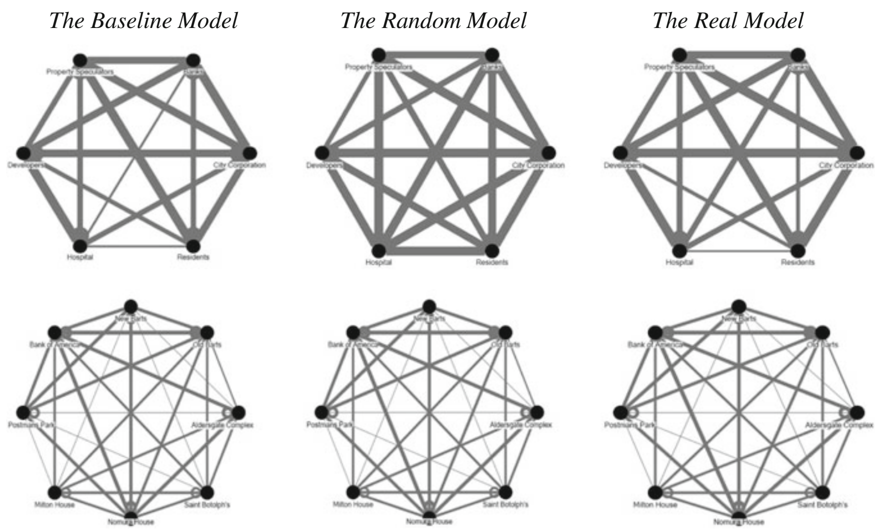

Fig. 2.5 The three variants of Coleman's model

network associated with the French-Harary model in Fig. 2.1 but in terms of Coleman's model, we will show the primal and dual networks for **P** and for **Q** for each of the three problems – the baseline where the map matrix determines both interest and control from $\mathbf{M}(0)$ associated with model 2 in Table 2.2, the **X** and **C** random matrices associated with $\mathbf{P} = \mathbf{XC}$ and $\mathbf{Q} = \mathbf{CX}$ in model 3, and the realistic matrices of interest and control associated with model 4. We show these networks in Fig. 2.5, where the key differences between interactions between agents and between sites are clearly illustrated by the amounts of interaction associated with the primal and dual probability matrices.

2.6 Conclusions and Next Steps

The models introduced here are all variants of an averaging process that assumes that networks of agents and their interest and control over development/building sites are sufficiently connected to ensure that their communication with respect to resolving differences between themselves is functional. Moreover we have assumed that rational compromise takes place, but in reality we know that this is invariably not the case. In fact, it is quite likely that in many problems of this kind, networks are not strongly connected and agents do not communicate leading to all kinds of log-jams and conflicts that often can only be resolved at a much higher level. In this sense, conflict resolution may ultimately take place but outside of the limits of the kind of problem posed here. The City of London it is likely that conflict would be resolved because the City Corporation is so powerful that it can bring massive resources in that it is the prime land owner in all development transactions.

It is quite easy to modify the model to illustrate how conflict can be resolved in such a way that the equilibrium weightings implied by these variants are distorted or modified by additional factors. For example, it is possible to build in inexorable and constant pressures where exogenous resources are continually introduced to pressure the conflict resolution towards certain directions. As the models have a linear structure, it is possible to add additional inputs in the manner sketched by Friedkin (1998) and illustrate how different exogenous weights can influence the ultimate balance of resources and values. It is also possible to add generic trends to the outcomes in the manner introduced by Blondel et al. (2005) where flocking and following are used to converge solutions that have their own dynamic. To explore these however, we need to move to much bigger problems and to problems where observational data pertaining to the actual processes of conflict resolution is to the fore. This involves grappling with group dynamics and engaging with the long stream of work on how decisions are actually reached in empirical contexts, while at the same time ensuring that the notion of design as optimisation remains to the fore.

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