Chapter 2
Rotational and Translational Diffusion in Ionic Liquids

Joshua Sangoro, Tyler Cosby and Friedrich Kremer

Abstract Dynamic glass transition and charge transport in a variety of glass-forming aprotic ionic liquids (ILs) are investigated in wide frequency and temperature ranges by means of broadband dielectric spectroscopy (BDS), pulsed-field gradient nuclear magnetic resonance (PFG NMR), differential scanning calorimetry and dynamic mechanical spectroscopy. On the low-frequency side, the dielectric spectra exhibit electrode polarization effects, while hopping conduction in a disordered matrix dominates the spectra of ionic liquids at higher frequencies. Upon systematic variation of the molecular structure of the ionic liquids, it is observed that the absolute values of dc conductivity and viscosity span more than 11 orders of magnitude with temperature. However, quantitative agreement is found between the characteristic charge transport and the structural α-relaxation rates. These results are discussed in the context of dynamic glass transition-assisted hopping as the underlying mechanism of charge transport in the ionic liquids investigated. In addition, a novel approach to determine diffusion coefficients from dielectric spectra in quantitative agreement with PFG NMR is proposed. This makes it possible to separately determine the effective number densities and mobilities of the charge carriers and the type of their temperature dependence. The observed Vogel–Fulcher–Tammann (VFT) dependence of the dc conductivity is shown to be due to a similar temperature dependence of the mobility while Arrhenius type of thermal activation is found for the number density.
Keywords  Ionic liquids · Diffusion · Charge transport rate · DC conductivity · Einstein–Smoluchowski relations · Green–Kubo relations · Effective number density · Random barrier model

2.1 Introduction

Fruitful and exciting periods of scientific and technological research often ensue the discovery of a novel material. As succinctly stated by Yves Chauvin in his 2005 Nobel address: “If you want to find something new, look for something new!” [1]. New breakthroughs offer possibilities to critically re-examine old problems as well as to pose new ones. This is the case with ionic liquids, liquids consisting entirely of cations and anions with melting points below 100 °C. Ionic liquids are interesting for both fundamental as well as technological applications. Depending on the composition and chemical characteristics of the constituent molecular moieties comprising the ionic liquids, they may be classified into two main categories, namely aprotic and protic ionic liquids. This chapter focuses on studies of aprotic ionic liquids. Although no single ionic liquid possesses all these characteristics, aprotic ionic liquids in general show a rich mix of outstanding properties such as low melting temperatures, high ionic conductivity, negligible vapour pressures, wide liquidus ranges, high thermal and electrochemical stability and tunability. Despite reports dating back to Paul Walden’s work in 1914 [2, 3], there has been a heightened interest in ionic liquids during the last two decades due to their unique properties which make them especially attractive for use in reaction media, as electrolytes in electrochemical energy technologies, among many others. Some of the significant areas of applications of ionic liquids are illustrated in Fig. 2.1.

From a fundamental point of view, the fact that ionic liquids can be easily supercooled makes them interesting materials to use as platforms for investigating the interplay between the dynamic glass transition and charge transport in amorphous liquids. In a sense, this involves re-examination of basic relations put forward by Einstein [4], Smoluchowski [5, 6], Maxwell [7, 8], Langevin [9] and Debye [10] concerning rotational and translational diffusion in (conducting) liquids. Although the different terminologies are employed today, certain aspects of the topics addressed by these scientists still remain unsolved. For instance, there is no general quantitative theory of dynamic glass transition (treated by Debye as rotational Brownian motion based on Einstein’s ideas) which is able to reproduce all the observed experimental results to date, notwithstanding the significant advances achieved so far from experimental and theoretical studies. Another outstanding example is Einstein’s work on Brownian motion in which he derived the link between translational diffusion (or charge transport) and viscosity (related to rotational diffusion). One of the objectives of the current chapter is to verify how well these classical relations hold in glass-forming ionic liquids.
Since it measures the complex dielectric function (and consequently, the complex conductivity) over many orders of magnitude in frequency and in a wide temperature interval, broadband dielectric spectroscopy (BDS) has proved to be an ideal experimental tool for addressing basic questions regarding the correlation between ion conduction (translational diffusion) and the dynamic glass transition (rotational diffusion) in broad length- and timescales as well as localized molecular fluctuations (secondary relaxations) \[11–32\]. Detailed knowledge of diffusion in ionic liquids, provided by this technique, is instructive for their optimal utilization in a wide range of scientific and technological applications.

It is estimated that it may be possible to synthesize approximately $10^{18}$ different ionic liquids based on the combinations of cations and anions available \[20, 33\]. This high degree of tunability has its challenges as well. Use of a trial-and-error approach in the synthesis of ionic liquids in search of one exhibiting particular physical and chemical properties is therefore not viable. Thus, it is imperative that more general relationships between the desirable properties such as high conductivities and the nature as well as structure of anions and cations be established. Molecular dynamics simulations are being conducted to make quantitative predictions of the physical properties of ionic liquids. In this chapter, it is shown that characteristic hopping lengths (determined from a combination of broadband dielectric spectroscopy and pulsed-field gradient nuclear magnetic resonance) in a selected series of ionic liquids increase with the molecular volume obtained from quantum chemical simulations.

**Fig. 2.1** Some possible applications of ionic liquids
2.2 Experimental Details

The ionic liquids investigated in this study (1-hexyl-3-methylimidazolium chloride—[HMIM] [Cl], 1-hexyl-3-methylimidazolium bromide—[HMIM] [Br], 1-hexyl-3-methylimidazolium iodide—[HMIM] [I], 1-hexyl-3-methylimidazolium tetrafluoroborate—[HMIM] [BF4], 1-hexyl-3-methylimidazolium hexafluorophosphate—[HMIM] [PF6], 1-(2-hydroxyethyl)-3-methylimidazolium tetrafluoroborate—[HEMIM] [BF4], 1-methyl-3-octylimidazolium tetrafluoroborate—[MOIM] [BF4], 1-butyl-3-methylimidazolium tetrafluoroborate—[BMIM] [BF4], 1,3-dimethylimidazolium dimethylphosphate—[MMIM] [Me2PO4], 1-ethyl-3-methyl-pyridinium ethylsulfate—[3-MEP] [EtSO4], Trioctylmethylammonium bis(trifluoromethylsulfonyl)imide—[OMA] [BTA] and Tetrabutylphosphonium bromide—[TBP] [Br]) were purchased from Solvent Innovation GmbH and Iolitec GmbH. Tributyloctylphosphonium trifluoromethanesulfonate—[TBOP] [OTf] was provided by Prof. Dr. Katsuhiko Tsunashima, National Institute of Technology, Wakayama College. (1-propyl-3-methylimidazolium bis(trifluoromethylsulfonyl) imide—[PrMIM] [NTf2], 1-butyl-3-methylimidazolium bis(trifluoromethylsulfonyl)imide—[BMIM] [NTf2], 1-hexyl-3-methylimidazolium bis(trifluoromethylsulfonyl)imide—[HMIM] [NTf2], 1-octyl-3-methylimidazolium bis(trifluoromethylsulfonyl)imide—[OMIM] [NTf2], 1-decyl-3-methylimidazolium bis(trifluoromethylsulfonyl)imide—[DMIM] [NTf2], 1-pentyl-3-vinylimidazolium bis(trifluoromethylsulfonyl)—[PVIM] [NTf2], 1-octyl-3-vinylimidazolium bis(trifluoromethylsulfonyl)—[OVIM] [NTf2]) were prepared by Prof. Dr. Veronika Strehmel, Hochschule Niederrhein University of Applied Sciences. The chemical structures of the cations and anions investigated are shown in Tables 2.1 and 2.2, respectively. The dielectric measurements were performed between 0.01 Hz and 1.8 GHz using a novocontrol high-resolution alpha analyzer (0.01 Hz–10 MHz) and an HP impedance analyzer (1 MHz–1.8 GHz). The analyzers were assisted by Quatro Temperature Controllers using pure nitrogen as heating agent and assuring a temperature stability better than 0.2 K. An ARES (Advanced Rheometric Expansion System) rheometer from TA Instruments was employed for Dynamic Mechanical Spectroscopy (DMS) measurements. A 400 MHz NMR spectrometer with a home-built gradient device was used to obtain the self-diffusivities at different temperatures.

2.3 Results and Discussion

In this chapter, the dielectric properties of ionic liquids are investigated in broad frequency and temperature ranges. Emphasis is placed on quantitative understanding of the underlying mechanisms as well as the interplay between charge transport and glassy dynamics in these materials. Quantum chemical calculations of molecular volumes of ionic liquids are performed using MOPAC2009 package in
order to re-examine the predictions of Einstein concerning the dependence of the diffusion coefficient on molecular sizes. The following techniques are employed: broadband dielectric spectroscopy, pulsed-field gradient nuclear magnetic resonance, differential scanning calorimetry and dynamic mechanical spectroscopy.

### 2.3.1 Charge Transport and Dynamic Glass Transition in Ionic Liquids

Broadband dielectric spectroscopy (BDS) measures the complex dielectric function, $\varepsilon^*$, which is equivalent to the complex conductivity function, $\sigma^*$. This is expressed as

<table>
<thead>
<tr>
<th>Chemical structures of typical cations in ionic liquids</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1 - alkyl - 3-methylimidazolium</strong></td>
</tr>
<tr>
<td><img src="image1" alt="Structure" /></td>
</tr>
<tr>
<td><strong>Tetra-alkyl-ammonium</strong></td>
</tr>
<tr>
<td><img src="image2" alt="Structure" /></td>
</tr>
<tr>
<td><strong>Tetra-alkyl-phosphonium</strong></td>
</tr>
<tr>
<td><img src="image3" alt="Structure" /></td>
</tr>
<tr>
<td><strong>N-alkyl-pyridinium</strong></td>
</tr>
<tr>
<td><img src="image4" alt="Structure" /></td>
</tr>
<tr>
<td><strong>N-alkyl-N-methylpiperidinium</strong></td>
</tr>
<tr>
<td><img src="image5" alt="Structure" /></td>
</tr>
<tr>
<td>Symbol key</td>
</tr>
<tr>
<td>$R_{1,2,3,4} = \text{CH}_3(\text{CH}_2)_n \cdot (n = 1,3,5,7,9)$, aryl, etc...</td>
</tr>
</tbody>
</table>
\[ \sigma' (\omega, T) = i\varepsilon_0 \omega \varepsilon'' (\omega, T), \] implied that \( \sigma' = \varepsilon_0 \omega \varepsilon'' \) and \( \varepsilon'' = \varepsilon_0 \omega \varepsilon' \) (\( \varepsilon_0 \) being the vacuum permittivity and \( \omega \) the radial frequency) [32]. The dielectric spectra of the ionic liquid [HMIM] [Cl] are presented in Fig. 2.2. The real part of the complex conductivity \( \sigma' \) is characterized on the low-frequency side by a plateau (the value of which directly yields the long range ionic conductivity, \( \sigma_0 \)), and the characteristic rate, \( \omega_c \), at which dispersion sets in and turns into a power law at higher frequencies. On the other hand, the real part of the complex dielectric function \( \varepsilon' \) turns from the high frequency limit to the static value \( \varepsilon_s \) at the characteristic rate \( \omega_c \). At lower frequencies, it is observed that \( \sigma' \) decreases from \( \sigma_0 \) value and this is due to electrode polarization that results from slowing down of charge carriers at the electrodes.

Rescaled with respect to \( \omega_c \) and \( \sigma_0 \), the dielectric spectra—as measured over wide temperature ranges—coincide (see Fig. 2.3), thus proving the uniqueness of

### Table 2.2 Chemical structures of some of the anions comprising the ionic liquids investigated

<table>
<thead>
<tr>
<th>Anion</th>
<th>Structure</th>
</tr>
</thead>
<tbody>
<tr>
<td>[BF(_4)](^-)</td>
<td>![BF(_4)](^-)</td>
</tr>
<tr>
<td>[Me(_2)PO(_4)](^-)</td>
<td>![Me(_2)PO(_4)](^-)</td>
</tr>
<tr>
<td>[NTf(_2)](^-)</td>
<td>![NTf(_2)](^-)</td>
</tr>
<tr>
<td>[ethyl sulfate](^-)</td>
<td>![ethyl sulfate](^-)</td>
</tr>
<tr>
<td>[OTf](^-)</td>
<td>![OTf](^-)</td>
</tr>
<tr>
<td>[PF(_6)](^-)</td>
<td>![PF(_6)](^-)</td>
</tr>
<tr>
<td>[NO(_3)](^-)</td>
<td>![NO(_3)](^-)</td>
</tr>
<tr>
<td>[halides](^-)</td>
<td>![halides](^-)</td>
</tr>
<tr>
<td>Cl(^-), Br(^-), I(^-)</td>
<td>![Cl(^-), Br(^-), I(^-)]</td>
</tr>
</tbody>
</table>
Fig. 2.2 Complex dielectric function ($\varepsilon^* = \varepsilon' - i\varepsilon''$) and complex conductivity function ($\sigma^* = \sigma' + i\sigma''$) of the ionic liquid 1-hexyl-3-methylimidazolium chloride—[HMIM] [Cl] at different temperatures as indicated. The error bars are comparable to the size of the symbols, if not explicitly indicated otherwise. The logarithm is to base 10.

Fig. 2.3 Scaling of the complex dielectric function ($\varepsilon^* = \varepsilon' - i\varepsilon''$) and complex conductivity function ($\sigma^* = \sigma' + i\sigma''$) with respect to $\omega_c$ and $\sigma_0$ for the ionic liquid 1-hexyl-3-methylimidazolium chloride—[HMIM] [Cl] at different temperatures.
the mechanism of charge transport. It can be concluded that the underlying pro-
cesses in the spectral and temperature ranges probed exhibit identical thermal
activation [22, 23, 34–36].

Charge transport in many amorphous ion-conducting systems is reasonably well
described by random barrier model developed by Dyre [37]. In the context of this
model, ions hop in a random spatially distributed potential landscape. The ion
transport is determined by the ability of the charge carriers to hop over the random
energy barriers. The success of the ions in surmounting the highest barrier deter-
mines long range or dc conductivity \( \sigma_0 \) [38]. The time corresponding to the attempt
rate to overcome the highest barrier is denoted by \( \tau_e \). The analytical solution for the
complex dielectric function, obtained within the continuous-time-random walk
approximation, is expressed as

\[
\varepsilon^*(\omega) = \frac{\sigma_0 \tau_e}{\varepsilon_0 \ln(1 + i\omega \tau_e)}. \tag{2.1}
\]

It has been demonstrated that the approximate form of the random barrier model
given in Eq. 2.1 describes the dielectric spectra of many different ion-conducting
systems qualitatively well. However, a close examination of the fits of the dielectric
spectra of ionic liquids reveals the existence of additional ‘relaxation-like’ process.
This contribution is particularly dominant in the real part of the complex dielectric
function. Apparently, this additional process was neglected in many earlier
dielectric studies of ionic liquids but its possible origin is fairly obvious. From a
physical point of view, a successful ionic jump must be accompanied by structural
reorganization of the ionic atmosphere, leading to a relaxation-like contribution not
only to the imaginary part, but also to the real part of the dielectric function. This
reorganization accompanying successful ion jumps causes additional fluctuations in
the polarization, resulting in a relaxation process with characteristic timescales
similar to that of ionic motion.

From the definition of polarization \( \mathbf{P} \) in terms of the complex dielectric function
\( \varepsilon^*(\omega) \), Debye employed the Lorentz field and substituted the static permittivity with
the dynamic permittivity obtaining

\[
\mathbf{P} = \varepsilon_0 (\varepsilon^*(\omega) - 1) \mathbf{E} = N_1 \left[ \varepsilon_\infty + \frac{\mu^2}{3kT(1 + i\omega \tau_D)} \right] \frac{\varepsilon^*(\omega) + 2}{3} \mathbf{E} \tag{2.2}
\]

where \( \varepsilon_\infty \) and \( N_1 \) denote the polarizability and number of dipoles per unit volume,
respectively, and \( \tau_D \) is the Debye macroscopic relaxation time. Defining \( \varepsilon_s \) and \( \varepsilon_\infty \)
as the unrelaxed and relaxed values of the dielectric permittivity, respectively,
Eq. 2.2 can be rearranged as

\[
\frac{\varepsilon^*(\omega) - \varepsilon_\infty}{\varepsilon_s - \varepsilon_\infty} = \frac{1}{1 + i\omega \tau_D} \tag{2.3}
\]
The Debye equations of dielectric permittivity can also be derived by considering the first-order kinetics of rise or decay of the dipolar polarization. Further details can be found in [32, 39].

It should be recalled that although the Debye theory of dielectric relaxation resulted from rigorous treatment of rotational Brownian motion, Debye-like relaxations are only observed in rare cases in glass-forming systems. The usual dielectric spectra are much broader than predicted by Debye’s approach. Thus, a number of phenomenological descriptions aimed at obtaining better fits to experimental data like the Cole-Cole, Cole–Davidson and Havriliak–Negami functions have been proposed. The empirical Havriliak–Negami function is the most commonly used form to fit the spectra of many materials exhibiting dielectric relaxations. Within this approach, the complex dielectric function is given by

$$\varepsilon_{\text{HN}}^* = \varepsilon_\infty + \frac{\Delta \varepsilon}{(1 + (i\omega\tau_{\text{HN}})^\beta)^\gamma}$$  \hspace{1cm} (2.4)

where $\Delta \varepsilon = \varepsilon_s - \varepsilon_\infty$ is the dielectric relaxation strength or intensity with $\varepsilon_s = \lim_{\omega \tau_{\text{HN}} \geq 1} \varepsilon'(\omega)$ and $\varepsilon_\infty = \lim_{\omega \tau_{\text{HN}} \leq 1} \varepsilon'(\omega)$. The shape parameters $\beta$ and $\gamma$ describe symmetric and asymmetric broadening of the complex dielectric function. The position of maximal loss $\omega_p$ depends on the characteristic time obtained from Eq. 2.4 as well as the shape parameters according to $\omega_p = \frac{1}{\tau_{\text{HN}}} \left[ \sin \frac{\beta \pi}{2 + 2\beta} \right]^{\frac{1}{\beta}} \left[ \sin \frac{\beta \gamma \pi}{2 + 2\beta} \right]^{-\frac{1}{\beta}}$. It should be noted that the real and imaginary parts of the complex dielectric function are related by the Kramers–Kronig relations [32, 40].

Thus, to account for ionic motion and the accompanying structural reorganization, the complex dielectric function should be described by Eq. 2.5:

$$\varepsilon^*(\omega) = \frac{\sigma_0 \tau_e}{\varepsilon_0 \ln(1 + i\omega\tau_e)} + \frac{\Delta \varepsilon}{(1 + (i\omega\tau_{\text{HN}})^\beta)^\gamma} + \varepsilon_\infty$$  \hspace{1cm} (2.5)

where $\varepsilon_\infty$ is the (high frequency) relaxed value of $\varepsilon'$. As shown in Fig. 2.4 for [TBOP] [OTf], Eq. 2.5 yields a quantitative fit to the part of the experimental spectra dominated by charge transport.

The dielectric spectra of ionic disordered materials can also be presented in terms of the electrical modulus $M^*$, where $M^* = 1/\varepsilon^*$. Although the three formalisms (complex dielectric, conductivity and modulus functions) are equivalent, they emphasize different aspects of the underlying mechanisms of charge transport and molecular dynamics. The first two forms—the complex dielectric and conductivity functions—have traditionally been used in studies of (dipolar) relaxations and charge transport, respectively. Maxwell’s equations, which describe the interaction of electromagnetic waves with matter, make direct reference to and indicate equivalence of these approaches. At higher frequencies, secondary relaxations...
(denoted by rate $\omega_\beta$) are observed. We recently showed that these dipolar processes are due to librations of the cations [23, 41].

In the intermediate frequency ranges, the dielectric properties are governed by motion of the charge carriers in the bulk. This contribution represents the (translational) diffusion of the ions. This regime of the dielectric spectra can be used to obtain important molecular parameters characterizing translational diffusion of the charge carriers. Electrode polarization dominates the spectra at lower frequencies. Empirically, $\omega_c \approx \omega_M \approx 1/\tau_c$ as shown in Fig. 2.5, where $\omega_M$ is the radial frequency corresponding to the peak in the imaginary part of the electrical modulus and $\omega_c = (1/\tau_c)$ is a characteristic time that defines the attempt rate of the charge carriers to overcome the highest energy barrier (limiting the $\sigma_0$), thereby enabling the physical interpretation of $\omega_c$ within the random barrier model [32, 37].

To find out the impact of molecular structure and composition on charge transport and dynamics of these systems, ionic liquids based on the HMIM cation are systematically investigated upon variation of the anions. The thermal activation of $\sigma_0$ and $\omega_c$—the central quantities describing ion transport—are studied. It is clear that systematic changes of the anions while keeping the same [HMIM] cation result in substantial differences in the charge transport parameters. These differences become more significant as the temperature is lowered towards the calorimetric glass transition temperature of the ionic liquid under study. In order to find out the
impact of the dynamic glass transition on charge transport in ionic liquids, dc conductivity and viscosity measurements were performed for a systematic series of ionic liquids, upon variation of the anions. Figure 2.6 presents the temperature dependence of $\sigma_0$ for the selected ionic liquids.

Systematic variation of the chemical structure leads to remarkable differences exceeding six orders of magnitude in $\sigma_0$ (between the tetrafluoroborate and chloride anions) at lower temperatures. However, upon scaling with the calorimetric glass transition temperature, all the data coincide for the anions examined. This experimental finding highlights the important role played by dynamic glass transition in charge transport in ionic liquids. In addition, viscosity, $\eta$,—well known to be directly related to the dynamic glass transition in liquids—also exhibits a similar thermal activation. The significant changes in the viscosity and $\sigma_0$ upon systematic variation of the anions is also worth pointing out as presented in Fig. 2.7.
Extrapolating $\omega_c$ to the timescale corresponding to 100 s, agreement with the measured calorimetric glass transition temperature $T_g$ within margins of ±2 K is found. Scaling of $\sigma_0(T)$, $\omega_c(T)$, and $1/\eta(T)$ with respect to $T_g$ using the approach proposed by Angell [42–45], coinciding plots for all the investigated ionic liquids are found as shown in Figs. 2.6 and 2.7. On the other hand, upon varying the cation and maintaining the same anion, deviations can be observed in the scaled plot of $\sigma_0(T)$. This is illustrated for the bis(trifluoromethylsulfonyl)imide-based ionic liquids in Fig. 2.8. Our conjecture is that since the size of the cation in ionic liquids is larger than that of the anion, the cations play a more pronounced role in determining the viscosity and structural relaxation (and consequently charge transport). It should be
recalled that electrostatic interactions, which control the dynamic glass transition in these materials, are strongly influenced by both the charge as well as the distance of separation between the interacting ions.

Using the Einstein and Einstein–Smoluchowski equations together with the definition of dc conductivity, one obtains [23, 34]:

\[
\sigma_0(T) = q\mu(T)n(T) = n(T)\frac{q^2D(T)}{kT} = n(T)\frac{q^2\lambda^2\omega_c(T)}{6kT}
\]

where \(n\) denotes the effective number density of charge carriers contributing to ion transport at the timescale of \(\omega_c\), \(\lambda\) refers to the characteristic diffusion length in this timescale, also taken characterizing the cross-over from non-random diffusion to random diffusion [46], \(D\) is the diffusion coefficient, \(\mu\) is the mobility, \(q\) is the elementary charge and \(k\) denotes the Boltzmann constant. Therefore, \(\sigma_0 \propto \omega_c\) as implied by the empirical BNN relation and in full agreement with the results presented in Fig. 2.9. It is noteworthy that the data coincide for all the ionic liquids investigated despite variations of the absolute values of both the dc conductivity and the characteristic rates spanning more than 11 decades upon changes in the temperature, pressure as well as composition.

In his paper on Brownian motion, Einstein [4] derived the first form of what is currently known as the fluctuation—dissipation theorem, linking position fluctuations to a dissipation (viscosity). This relation forms the basis of the linear response theory emphasized by Kubo [47] and is of direct relevance in the current studies. Einstein showed that

\[
\langle x^2 \rangle_t = 2Dt = \frac{RT}{N} \frac{t}{3\pi \eta a} \tag{2.7}
\]

where \(\langle x^2 \rangle_t\) denote the position fluctuations (completely identical to \(\lambda^2\) employed in Eq. 2.6), \(t\) is a measure of the timescale (equivalent to \(1/\omega_c\) in the current work), \(R\) the gas constant, \(N\) the Avogadro number and \(a\) the Stoke’s radius. Upon
rearranging Eq. 2.7, it is clear that the observed universality of charge transport in ionic liquids as displayed in Fig. 2.9 is due to identical thermal activation of the charge transport rate and the viscosity, i.e., the product $\eta(T)\omega_c(T)/T$ exhibits negligible temperature and pressure dependence. This embodies the link between charge transport and structural relaxation in the ionic liquids studied.

Further insight into the correlation between charge transport and dynamic glass transition can be gained by comparison of viscosity obtained from dynamic mechanical spectroscopy (DMS) and the charge transport rate $\omega_c$. Due to technical reasons, DMS only covers a spectral range spanning about 3–4 decades. This is about nine orders of magnitude less than the range accessible by broadband dielectric spectroscopy. To circumvent this difficulty and compare dielectric and DMS results over many decades, the structural relaxation rate $\omega_\eta$ can be approximated using Maxwell relation, $\omega_\eta = 3G_\infty a^2/kT$, assuming a temperature independent instantaneous shear modulus (typically 0.1 GPa) and position fluctuations (typically 0.2 nm). Based on Einstein–Smoluchowski, Stokes–Einstein and Maxwell relations one can easily show that $\omega_c = Po_\eta$, where $P = kT/(3\pi G_\infty a^2)$. Figure 2.10 demonstrates that $\omega_c \approx \omega_\eta$ over 6 decades and $P$ is a constant of order one within experimental accuracy. These two rates can be compared more directly by making dielectric and DMS measurements at temperatures just above the calorimetric glass transition as shown Fig. 2.10. The rate $\omega_\eta$ obtained from the peak in $G''$ is observed to be in quantitative agreement with the charge transport rate $\omega_c$ determined from the dielectric spectra. It is therefore experimentally established that $\omega_c$ is $\omega_\eta$ in

![Fig. 2.10](image_url)
glass-forming ionic liquids. The observed universality is thus understood based on Einstein’s predictions as already discussed.

According to Eq. 2.6, the measured \( \sigma_0 \) is a product of the mobility and the effective number density of charge carriers contributing to ionic transport. It is essential to identify which of the two quantities gives the dc conductivity its characteristic VFT-type temperature dependence. In addition, it is helpful to find out whether one of the two quantities has greater influence on the ionic conductivity. For electronic conductors, Hall Effect measurements provide a means of disentangling the contributions of \( n \) and \( \mu \) to the measured electrical conductivity, but the Hall voltages associated with ionic transport are at the nano-volt levels and are therefore too small to be determined with sufficient accuracy using state-of-the-art equipment. In addition, the validity of Hall measurements on ionic conductors is still in doubt [48].

Estimative determinations of the diffusion coefficients can be made by applying Eq. 2.6 and taking for the hopping length, values in the order of the Pauling diameter [49], i.e. \( \lambda = 0.17 \) nm and \( \lambda = 0.19 \) nm for [BMIM] [BF\(_4\)] and [MMIM] [Me\(_2\)PO\(_4\)], respectively. Independent results from PFG NMR together with the diffusion coefficients obtained from dielectric measurements are shown in Fig. 2.11. This simple method yields diffusion coefficients in agreement with those obtained by PFG NMR [22, 23]. Additionally, these experimentally determined diffusivities have been shown to coincide with those found from computations involving the Green–Kubo and Einstein relations as shown for [BMIM] [Br] in Fig. 2.12. These estimates make it possible to disentangle (using Eq. 2.6) the influence of \( n \), the

![Fig. 2.11](image)

**Fig. 2.11** Diffusion coefficient determined by the novel approach involving application of the Einstein–Smolukowski equation (using \( \omega_s \) as hopping rate and with \( \lambda \) equal to the Pauling diameter of the ions as hopping length [49]), compared with the diffusion coefficient measured by PFG NMR (blue colour) for two ionic liquids: BMIM BF\(_4\) and MMIM Me\(_2\)PO\(_4\) [22, 23]. Inset Effective number of charge carriers as a function of inverse temperature (the respective activation energies are as indicated). The error bars are comparable to the size of the symbols, if not specified otherwise. Log is used to refer to logarithm to base 10.
effective number density, from that of the electrical mobility, μ. It is found that the VFT-type of thermal activation of the ionic conductivity originates exclusively from a similar dependence of the diffusion coefficient. In contrast, the effective number density of the ions exhibits Arrhenius type of temperature dependence (inset of Fig. 2.11). By extrapolating the effective number density to room temperature, one obtains $2.9 \times 10^{27} \text{ m}^{-3}$ for [BMIM] [BF₄], close to $3.4 \times 10^{27} \text{ m}^{-3}$, which is the stoichiometric number of ions in the system. Therefore, around 85% of the available charge carriers participate in ionic transport at room temperature. This result agrees with recent reports suggesting that only a fraction of the available charge carriers actually participate in the conduction process [12–14]. We have demonstrated that the approach to determine diffusion coefficient from dielectric spectra holds for other glass-forming systems as well [35]. Thus, this approach enables the determination of diffusion coefficient spanning many orders of magnitude by broadband dielectric spectroscopy.

2.3.2 Elucidating the Correlation Between Characteristic Hopping Lengths and Molecular Volumes of Ionic Liquids

There is a concerted effort aimed at understanding the physico-chemical properties of ionic liquids (ILs). In general, they exhibit a unique mix of interesting properties such as high conductivities, wide electrochemical windows, thermal stability, negligible vapour pressures, wide liquid ranges and low melting points, which makes them promising for use in power sources for electric vehicles, hybrid cars,
electronic and power storage devices \cite{22, 23, 34, 50, 51}. It is estimated that it is possible to synthesize up to about $10^{18}$ different ILs based on the combinations of cations and anions available \cite{20, 33}. Thus, it is imperative that more general relationships between the desirable properties such as high conductivities and the nature of anions and cations be established. Previous attempts to make quantitative predictions of the physical properties of ILs using quantitative structure–property relationships and molecular mechanics simulations have had some success \cite{20}. The major drawbacks of these approaches include the need for large experimental datasets to derive correlations, time consuming computational methods or at least some experimental data from the IL under study. In this section, the hopping lengths characterizing charge transport are investigated with respect to ionic volumes determined by quantum chemical calculations.

Quantum chemical calculations of molecular volumes of the ionic liquids were carried out using MOPAC2009 \cite{52}. Semi-empirical quantum chemical calculations have been performed using MOPAC2009 and the PM6 \cite{53} Hamiltonian for geometry optimization. The COSMO technique was applied to account for solvent effects \cite{54}. The typical value of the static dielectric permittivity for many imidazolium-based ILs is $\varepsilon_s = 15$, so this value was assumed in the calculations \cite{15, 25}. Because the molecular diameter is strongly dependent on the conformation of the molecule various conformers were considered (e.g. 56 conformers for $[\text{DMIM}]^+$). Molecular volumes, $V$, were obtained by modelling the molecule as a series of intersecting spheres, whose radii are determined by the atom type (as implemented in MOPAC2009 \cite{52} by the COSMO \cite{54} solvation model). By taking the longest distance between two atoms and adding the van der Waals radii of the atoms, the maximum molecular diameters, $D$, were determined. The van der Waals radii reported by Bondi \cite{55} were assumed for the calculation of the volume $V$ as well as the diameter $D$. The ranges of molecular dimensions obtained are presented in Table 2.3. The technique described has been shown to reproduce the well-established ionic volumes of many common ions \cite{56}. The values obtained in our studies are systematically lower than volumes determined from crystal structures but they exhibit the same trends \cite{20}. The volumes corresponding to those of the crystal structures could only be reproduced by choosing different van der Waals radii.

**Table 2.3** Range of molecular volumes, $V$, and maximum diameters, $D$, for different conformations of the ions constituting the ionic liquids

<table>
<thead>
<tr>
<th>Ion</th>
<th>$V$/nm$^3$</th>
<th>$D$/pm</th>
</tr>
</thead>
<tbody>
<tr>
<td>$[\text{MVIM}]$</td>
<td>0.1205–0.1208</td>
<td>966–1018</td>
</tr>
<tr>
<td>$[\text{PropMIM}]$</td>
<td>0.1492–0.1524</td>
<td>989–1131</td>
</tr>
<tr>
<td>$[\text{PVIM}]$</td>
<td>0.2601–0.2722</td>
<td>1084–1522</td>
</tr>
<tr>
<td>$[\text{OVIM}]$</td>
<td>0.1694–0.1738</td>
<td>1118–1905</td>
</tr>
<tr>
<td>$[\text{OMIM}]$</td>
<td>0.2483–0.2619</td>
<td>1030–1768</td>
</tr>
<tr>
<td>$[\text{DMIM}]$</td>
<td>0.2899–0.3139</td>
<td>1053–2021</td>
</tr>
<tr>
<td>($[\text{CF}_3\text{SO}_2\text{N}]$ (or $[\text{NTf}_2]$)</td>
<td>0.1693–0.1748</td>
<td>1000–1027</td>
</tr>
</tbody>
</table>
Using a combination of PFG NMR and dielectric measurements, it is possible to calculate the characteristic hopping length $k_h$ from diffusion coefficients and rates in the temperature range where the two techniques coincide. Based on Eq. 2.7, one can expect only a weak temperature dependence (if any) of $k_h$ given the universality of charge transport in ionic liquids as already discussed. The values of $k_h$ obtained are then used to determine diffusion coefficients from dielectric spectra of a series of ionic liquids as presented in Fig. 2.13. By that, it becomes possible to extend the range of diffusion coefficients measured by PFG NMR from about 4 to over 11 decades by employing dielectric spectroscopy. Consequently, electrical mobilities and effective number densities as well as their type of temperature dependence can be determined. The latter shows a weak Arrhenius-type dependence and is practically independent of the cation whereas the former exhibits a VFT—type behaviour and shows a pronounced dependence on the nature of the cation (especially at lower temperatures).

Central to the concept of diffusion within the linear response regime is the magnitude of the position fluctuations (also referred to as hopping lengths in the current chapter) $\lambda$ of molecules in thermal equilibrium. Based on Einstein’s [4] theory, the direct link to the diffusion coefficient was established. According to Eq. 2.7, molecular size has a direct influence on $\lambda$. The molecular volume of an IL...
is a measure of its size and is more physically meaningful than the radius (or diameter). Based on quantum chemical calculations, it is possible to compute the molecular volumes of ionic liquids. Table 2.3 presents the results obtained for the series of ILs investigated. These are then compared with the values of $\lambda$ obtained from diffusion measurements. It is observed that $\lambda$ increases with the volume of the cation (see inset of Fig. 2.13). This can be interpreted in terms of the increase in the average distance between the ions for larger cations. One can expect that larger position fluctuations are only possible if adequate space is available. It has been shown that the knowledge of volumes of ILs can be used to successfully predict their physical properties [20].

Diffusion in dilute electrolytes comprised of multiple ionic species is often described by the Nernst–Hartley approach which neglects the contributions arising from ion–ion cross-correlation effects [57]. However, a general analytical expression of the diffusion coefficient for concentrated systems incorporating the contributions from the ion-correlation is yet to be found. It is therefore instructive to investigate the deviation from the predictions of the Nernst–Hartley equation and employ it as a means of quantifying the interaction of ionic species in ionic liquids. In two-component monovalent electrolytes, the effective diffusion coefficient within this framework is given by the Nernst-Hartley diffusion coefficient, $D_{NH}$, expressed as [57]:

$$D_{NH} = \frac{2D_AD_C}{D_A + D_C}$$

(2.8)

where $D_A$ and $D_C$ denote the diffusion coefficients of the anion and cation, respectively. Based on Einstein–Smoluchowski relation, the effective characteristic diffusion (hopping) length, $\lambda_h$ [in the timescale of the characteristic diffusion (charge transport) rate, $\omega_c$, (described in Sect. 2.3.1)] can be defined with respect to the corresponding hopping lengths of the anions and cations given by $\lambda_A$ and $\lambda_C$, respectively. If a single diffusion rate is further assumed, then the effective hopping length $\lambda_h$ can be expressed as

$$\lambda_h = \sqrt{\frac{\lambda_A^2 \lambda_C^2}{\lambda_A^2 + \lambda_C^2}}$$

(2.9)

As experimentally established in Sect. 2.3.1 of the current chapter, the diffusion rate, $\omega_c$ and the structural $\alpha$-relaxation rate are identical. Given that the molecules experience mean displacements comparable to their diameters [58], it is worthwhile to estimate the average hopping lengths using Eq. 2.9 and diameters of anions as well as cations obtained from quantum chemical calculations. Based on the data in Table 2.3 as well as Eq. 2.9, this consideration yields effective hopping lengths of the order of 5 Å in contrast to the typical 2–3 Å determined experimentally for the
ionic liquids investigated. This disparity is indicative of the role of the cation–anion correlation in ionic liquids. Thus, the cross-correlation terms in ionic liquids are shown to be non-negligible. An adequate theory capable of describing diffusion coefficients in highly concentrated electrolytes such as ionic liquids is yet to be obtained. The Nernst–Hartley approach overestimates the diffusion coefficients because it does not consider the interaction between the anions and cations.

2.4 Conclusions

Because of the ease with which they can be supercooled, ionic liquids offer new opportunities to investigate long-standing problems regarding the nature of dynamic glass transition as well as its impact on charge transport. Despite the significant progress achieved so far from experimental and theoretical studies, no generally accepted quantitative theory of dynamic glass transition capable of reproducing all the experimentally observed features exists to date. In this chapter, we discuss recent studies on the interplay between charge transport and glassy dynamics in ionic liquids as investigated by a combination of several experimental techniques. Using Einstein–Smoluchowski relations, we suggest a simple approach to determine diffusion coefficients in a broad range spanning more than ten orders of magnitude from dielectric spectra of ionic liquids in quantitative agreement with independent pulsed-field gradient nuclear magnetic resonance measurements. This provides a new possibility to separately determine the electrical mobility and effective number density of charge carriers from the measured dc conductivity. The origin of the remarkable universality of charge transport in different classes of glass-forming ionic liquids is also unravelled.

Acknowledgments J. S. and T. C. acknowledge the National Science Foundation for financial support through the award number DMR-1508394. The authors gratefully acknowledge financial support from the Deutsche Forschungsgemeinschaft under the DFG SPP 1191 Priority Program on Ionic Liquids.

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