In 2015, we celebrated the 100th anniversary of the development of general relativity theory (GRT). Einstein presented his theory at the Prussian Academy of Science in Berlin on November 25th, 1915. In GRT, he replaced the absolute space and time of Newton in favor of a changing arena called “spacetime,” in which gravity appeared as curvature. The equivalence principle linked every acceleration locally with gravitation. In principle, GRT poses the possibility of understanding all forces in the world using geometry. Galileo Galilei expressed this thought nearly 400 years ago when he pronounced: *He who understands geometry, understands anything in the world.* Therefore it was logical that Einstein continued this program even after completing his GRT, with the development of proposals for a unified field theory.

Carl H. Brans chose to investigate such theories for his undergraduate thesis at Loyola University in New Orleans. It was the beginning of a lifelong engagement with GRT. Even the mathematical beauty of GRT and the unified field theory attracted him. As a 10-year-old boy, he taught himself differential and integral calculus, and difficult books on mathematics and mathematical physics held a great appeal for him. His preference for GRT was a little bit unusual at the time. Since the 1920s, the GRT had lost its prominent role in theoretical physics. The advent of quantum mechanics and elementary particle physics, together with new work on quantum electrodynamics, inspired more interest among physicists in those days.

In the 1950s, the situation began to change. John A. Wheeler, at Princeton University, started to develop geometrodynamics as a new representation of GRT. At the same time, Wheeler established his by-now famous working group, which focused on problems in GRT and on the foundations of quantum mechanics. In parallel, Robert Dicke, Wheeler’s colleague at Princeton, began to work on the experimental problems in GRT during his sabbatical year of 1954. He also became interested in Mach’s principle, which Einstein had used as a guide during the development of GRT. Dicke considered Mach’s principle to imply: *The gravitational constant, \( \kappa \), should be a function of the mass distribution in the universe.* Paul A.M. Dirac had earlier conjectured that there is a relation between the coupling
constant of gravity and the mass and radius of the universe (now known as Dirac’s “large number conjecture”). For an expanding universe, one thereby obtains a variable gravitational coupling constant!

In 1957, Carl Brans arrived in Princeton to undertake his graduate study and Ph.D. thesis. In his contribution to this book, he writes extensively about that time. He heard some lectures and visited the seminar of Wheeler, who had established his famous group. Charles Misner, who had recently completed his Ph.D. thesis, introduced Carl to fiber bundle theory. Hence, Carl planned to write his Ph.D. thesis about the application of fiber bundles in physics. At that time, he also began to be interested in the mathematical structure of spacetime. But the time was not yet ripe for these ideas; fiber bundles would only become commonly used in theoretical physics in the 1970s. Instead, Misner recommended that Carl should contact Dicke, who was searching for a theoretical physicist. It was the beginning of a lifelong and fruitful collaboration.

Mach’s principle and Dirac’s large-numbers hypothesis formed the basis for the discussions between Dicke and Brans. They wondered if they could create a version of GRT with a variable gravitational coupling. Brans pursued the idea and developed it in his Ph.D. thesis in 1961. Today this renowned theory is known as the Brans-Dicke theory. They introduced a scalar field to represent the variable coupling. Pascual Jordan had described a similar theory in his 1955 book, *Schwerkraft und Weltall*, though Jordan’s work was not well known at the time. Brans and Dicke’s work quickly received much more attention within the physics community, helping to establish the importance of “scalar-tensor” theories of gravitation, as Carl describes further in his contribution to this volume.

In the Brans-Dicke theory, one arbitrary parameter (usually denoted by $\omega$) quantifies the coupling between the scalar field and spacetime curvature. Dicke proposed to express $\omega$ in terms of other physical constants; failing that, most experimental tests of the theory concentrated on possible restrictions on $\omega$. An outstanding experimenter, Dicke was strongly interested in the experimental verification of the Brans-Dicke theory. As an important side-effect of these efforts, many effects of GRT were tested with unprecedented precision. Among them included classic experiments like the Eötvös experiment to confirm the weak equivalence principle, as well as various NASA missions. Martin McHugh’s contribution in this volume presents an overview of these experiments, as well as Dicke’s endeavor to confirm the Brans-Dicke theory.

In 1961, Brans and Dicke’s paper appeared in the *Physical Review*. Following its publication, the Brans-Dicke theory had wide repercussions. The meaning and importance of scalar fields in physics increased significantly, from their role in spontaneous symmetry breaking, as in the Higgs mechanism, to the dynamics of the very early universe, as in models of cosmic inflation. Other theories, which incorporated a scalar field to model variable cosmological effects, such as quintessence, used Brans-Dicke theory as a prototype. Interest in Brans-Dicke theory increased further during the 1980s and 1990s in the context of string theory. Finally, the discovery of the Higgs boson in 2012 marked the first experimental detection of a fundamental scalar field in nature.
The present volume includes a collection of invited papers by renowned colleagues. The contributions range over various aspects of scalar fields to Mach’s principle, Bell’s inequality, and spacetime structure. Together, the chapters illustrate how Carl’s ideas have been developed even further over the years. The volume is organized into three parts, reflecting the scientific foci of Carl’s career.

The first part concerns the scalar-tensor theory. In the decades since the development of Brans-Dicke theory, scalar fields have come to play a diverse set of roles in physics, from the inflaton that drove cosmic inflation, to the axion that breaks chiral symmetry in QCD, to the Higgs boson that generates mass for elementary particles and the dilaton field that breaks global scale invariance (Weyl symmetry). Chapters in this part focus on this diversity of scalar fields in the context of GRT.

David Kaiser (MIT, USA) describes the role of Brans-Dicke (or non-minimal) couplings between scalar fields and spacetime curvature in the context of inflationary model-building. As he discusses, recent observational data, such as collected by the Planck satellite, place strong constraints on models of early-universe inflation. Models with Brans-Dicke couplings provide a natural way of realizing inflation while matching all the latest observations. Yasunori Fujii (Waseda University, Japan) focuses on a possible relation between microscopic physics and the cosmological model of Brans and Dicke. According to Brans-Dicke theory, the mass of an electron would not be constant in an expanding universe. However, Fujii demonstrates, one may introduce a massive scalar field (akin to a dilaton) to address this feature, and further estimate the dilaton mass. Roman Jackiw (MIT, USA) and So-Young Pi (Boston University, USA) focus on a special version of Brans-Dicke theory which is independent of the underlying scale (Weyl symmetry), which should affect short-scale behavior.

The appearance of different scalar fields naturally leads to the question of how those fields might relate or interact with each other. Friedrich Hehl (University of Cologne, Germany, and University of Missouri-Columbia, USA) addresses such questions. First he shows that the dilaton and axion fields appear naturally in the context of Einstein–Cartan theory. Next he constructs the metric as well as the axion and dilaton fields directly from an electromagnetic model of the universe (“premetric electrodynamics”).

Many researchers have implicitly assumed that Brans-Dicke theory would yield small deviations from the usual predictions of GRT. But what about more radical departures, such as contributions that are quadratic in the curvature? This question is discussed by Tirthabir Biswas (Loyola University New Orleans, USA) in collaboration with Alexey Koshelev (Universidade da Beira Interior, Portugal) and Anupam Mazumdar (Lancaster University, UK). They demonstrate the appearance of the Brans-Dicke model as a stable solution to physically well-motivated consistency conditions.

What is the influence of the scalar field on objects in the universe and on the universe as a whole? These fascinating questions are investigated by Eckehard W. Mielke (Universidad Autónoma Metropolitana Iztapalapa, Mexico) and Israel Quiros (Universidad de Guanajuato, Mexico). As shown by Mielke, the gravitational collapse of a boson cloud of scalar fields would lead to a boson star as a
new type of a compact object. Moreover, as a coherent state (like the vortices of Bose–Einstein condensates), such collapse would allow for rotating solutions with quantized angular momentum. Quiros focuses on the cosmological impact of Brans-Dicke theory. Is the standard model of cosmology (the so-called $\Lambda$CDM model) a stable solution of Brans-Dicke theory? Assuming a Friedmann-Robertson-Walker metric in the Brans-Dicke theory, he demonstrates that the de Sitter solution of GRT is an attractor of the Jordan frame (dilatonic) Brans-Dicke theory only for special values of the coupling constant $\omega$ and for special scalar-field potentials. Only for these values does one obtain the $\Lambda$CDM model from Brans-Dicke theory.

The first part of the volume closes with the contribution by Martin McHugh (Loyola University New Orleans, USA) about the history of the Brans-Dicke theory and its experimental tests. Dicke became famous for this experimental work and was a popular contact to discuss unexplainable experimental results. At the end of 1965, he received a call from Arno Penzias and Robert W. Wilson at nearby Bell Laboratory, who had found a mysterious microwave signal. They had spent nearly a year searching for the cause of the signal in their antenna. Dicke immediately identified the signal as the long-sought cosmic microwave background (CMB), which he had dubbed the “ash of the Big Bang.” In 1978, Penzias and Wilson received the Nobel Prize for their discovery.

The Brans-Dicke theory occupied Carl Brans for twenty years after its initial publication in 1961, and he continued to return to the topic after that. But Brans made contributions to several other topics as well. (Indeed, even beyond the research topics covered in this volume, Carl made additional, important contributions to the Petrov classification, numerical GRT, and complex GRT.) The second part of this volume includes contributions reflecting on Carl’s work during the 1980s.

The original motivation for Brans-Dicke theory concerned Mach’s principle, and the notion that the gravitational constant, $\kappa$, should be a function of the mass distribution of the universe. In his contribution for this volume, Bahram Mashoon (University of Missouri-Columbia, USA) describes the application of Mach’s principle to particles’ inertial property of spin. The inertia of intrinsic spin is studied via the coupling of intrinsic spin with rotation, a coupling which has recently been measured in neutron polarimetry. The implications of the inertia of intrinsic spin are critically examined in the light of the hypothesis that an electromagnetic wave cannot stand completely still with respect to an accelerated observer.

The second chapter in this part, by Michael J.W. Hall (Griffith University Brisbane, Australia), concerns Bell’s inequality. Carl’s colleague A.R. Marlow (Loyola University New Orleans, USA) notes that Carl developed an interest in quantum logic and interpretational problems in quantum mechanics. In particular, Carl became interested in Bell’s theorem and the effort to decide whether any hidden variables determine the outcomes of measurements, or if the probabilistic framework of quantum mechanics is complete. In 1988, Carl published an article in which he noticed a circular argument in the derivation of Bell’s theorem. Bell had to assume that an experimenter’s selection of detector settings in an experimental
test of quantum entanglement was entirely uncorrelated with any possible hidden variables that could affect the outcomes of those measurements—even though the events that determined the detector settings presumably shared an enormous causal past with any events that could have influenced the outcome of the measurements. Put another way, whatever hidden variables could have classically determined the outcomes of measurements could also have determined the experimenter’s selection of detector settings. Hence, in order to derive strong no-go results like Bell’s inequality, one must assume “measurement independence.” Hall discusses the importance of such an assumption as well as means to relax it within the context of Bell’s inequality. He further generalizes Brans’s 1988 model to demonstrate that no more than $2 \log d$ bits of prior correlation between the hidden variables and the detector settings are required for a local deterministic model to reproduce the quantum-mechanical predictions for any $d$-dimensional system.

More recently, Carl’s research has focused on the structure of spacetime, and in particular on exotic smoothness. These topics occupy the third part of the volume. As noted above, Charles Misner introduced Carl to such questions with his lecture on fiber bundle theory in 1957, and Norman Steenrod’s book on *The Topology of Fiber Bundles* (1951) provided further inspiration. Exploiting similar methods, including cobordism theory, John Milnor made an unexpected discovery in 1956: there exist exotic 7-spheres.

To appreciate the importance of this result, one must dig deeply into manifold theory. The weak equivalence principle in GRT implies the usage of the manifold concept: every neighborhood of a point in spacetime must be locally flat, that is, it must be a subset of $\mathbb{R}^n$. Then spacetime is a smooth manifold, i.e. it is covered by smooth charts with smooth transition functions forming an atlas. A smooth atlas is a smoothness structure. Conventional wisdom had long held that every topological manifold could be smoothed (by smoothing the corners), so that there would only be one smoothness structure (given by the smoothness structure of the $\mathbb{R}^n$). But Milnor found seven 7-dimensional spheres $S^7$ which agreed topologically but differed in their smoothness structure, thereby providing the first counterexample to the higher-dimensional Poincaré conjecture. Milnor thus founded the new topic of differential topology and received the highest mathematical honor, the Fields medal, in 1962.

As Carl noticed, this revolution occurred only “some doors away from him” at Princeton university. From the physics point of view, the 7-sphere is not particularly interesting, except perhaps in string theory (in which Edward Witten used it to cancel the global gravitational anomalies in 1985). Moreover, exotic smoothness is difficult to visualize, because no exotic smoothness structure exists in dimension smaller than four. For dimension 5 and higher, there are only finitely many exotic smoothness structures, as shown by Kervaire and Milnor in 1963. But what about 4-manifolds as models of our spacetime?

The riddle was solved in the 1980s with the work of many mathematicians, including Michael Freedman, Simon Donaldson, Robert Gompf, and Clifford Taubes. Most compact 4-manifolds admit (countable) infinitely many different
smoothness structures, whereas most non-compact 4-manifolds—including $\mathbb{R}^4$—admit (uncountable) infinitely different ones. Therefore, the physical dimension 4 is mathematically distinguished from any other dimension!

Carl attended a lecture by Ron Fintushel at Tulane University to hear about these results. It is typical for Carl that he immediately asked about their relevance for physics. In his first article in collaboration with the mathematician Duane Randall, Brans published the first deep results. It was the start of a long and fruitful collaboration between mathematicians and physicists on this topic. Indeed, many of Carl’s questions remain open to this day. His questions helped to shape the direction for current research.

A driving force was the Brans conjecture from 1994. In an article from that year, Carl constructed an exotic $\mathbb{R}^4$ in which the exoticness is localized (now known as small exotic $\mathbb{R}^4$). The Brans conjecture is that this localized exoticness can act as a source for some externally regular field, just as matter or a wormhole can. This conjecture was partly proven by Jan Sładkowski and Torsten Asselmeyer-Maluga. In a 2002 paper by Brans and Asselmeyer-Maluga, this conjecture was extended:

"... In summary, what we want to emphasize is that without changing the Einstein equations or introducing exotic, yet undiscovered forms of matter, or even without changing topology, there is a vast resource of possible explanations for recently observed surprising astrophysical data at the cosmological scale provided by differential topology. ..."

Results in this area of research up through 2007 may be found in Brans and Asselmeyer-Maluga’s book, Exotic Smoothness and Physics (World Scientific, 2007), which has become a standard reference for the topic. An introduction to the topic may also be found in Carl’s contribution to the present volume. The third part of this book describes more recent developments.

Jan Sładkowski (University of Silesia Katowice, Poland) aims to describe spacetime structure from the physics point of view. He considers the algebra of all real functions over a manifold containing the information about the topology of the manifold. A generalization of these functions leads to Alain Connes’s model of noncommutative geometry as a possible description of the standard model in elementary particle physics.

Jerzy Król (University of Silesia Katowice, Poland) studies model-theoretic aspects of exotic smoothness, uncovering unexpected relations to noncommutative spaces and quantum theory. Forcing, as a special extension of the axioms in set theory, is used to obtain the deformation of the algebra of usual complex functions to the noncommutative algebra of operators on a Hilbert space. The results in the context of the Epstein-Glaser renormalization in QFT are also discussed.

In the contribution by Duane Randall (Loyola University New Orleans, USA), a question of Milnor is answered: is there always an exotic $n$–sphere for $n > 6$ and $n \neq 12, 61$? In the next chapter, Torsten Asselmeyer-Maluga (German Aerospace Center Berlin, Germany) extensively discusses the following questions: Is it possible to construct a quantum gravity theory by using exotic smoothness? Is it possible to construct quantum gravity directly, i.e. without any quantization of a
classical theory? In his chapter, the richness of exotic smoothness in dimension 4 is used to construct a quantum gravity theory directly. The use of this geometrical approach implies one problem: one has to construct a geometrical expression for a quantum state (the ψ-ontic interpretation as implied by current experiments). This construction, using wild embeddings (like Alexander’s horned sphere), gives a fractal space. Moreover, quantum fluctuations arise from an unpredictable chaotic dynamics. The consequences for decoherence, the measurement problem, and cosmology are discussed.

The contributions in this volume are dedicated to Carl Brans on the occasion of his 80th birthday, and were written exclusively for this volume. The chapters were contributed by renowned colleagues who collaborated directly with Carl or who were inspired by his ideas. Though Carl never founded a formal school or group, his influence has been felt by many young scientists, across many countries and communities.

Throughout his career, colleagues and students have appreciated Carl’s critical questions and his ambition to understand problems at a very deep level. Always approachable, Carl has inspired generations with his deep questions and important insights. Israel Quiros expressed it best in his dedication: “He is one of the greatest minds of the twentieth century.” It is a great pleasure to honor Carl Brans with this collection. Happy Birthday, Carl!

Berlin Torsten Asselmeyer-Maluga
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