Chapter 2
Non-negative Fuzzy Optimal Solution of Fully Fuzzy Linear Programming Problems with Equality Constraints

Lotfi et al. [1] pointed out that there is no method in the literature for solving fully fuzzy linear programming problems with equality constraints and proposed a method for the same. In this chapter, the limitations and shortcoming of the existing method [1] are pointed out and to overcome the limitations as well as to resolve the shortcoming, Kumar et al.'s method [2] is presented to find the non-negative fuzzy optimal solution of fully fuzzy linear programming problems with equality constraints.

2.1 Preliminaries

In this section, some basic definitions and arithmetic operations for trapezoidal fuzzy numbers are presented [3].

2.1.1 Basic Definitions

In this section, some basic definitions are presented.

Definition 2.1 Let X be a classical set of objects. Then, the set of ordered pairs $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\}$, where $\mu_{\tilde{A}} : X \to [0, 1]$, is called a fuzzy set in X. The evaluation function $\mu_{\tilde{A}}(x)$ is called the membership function.

Definition 2.2 Let $\tilde{A}$ be a fuzzy set in X and $\lambda \in [0, 1]$ be a real number. Then, a classical set $A^{\lambda} = \{x \in X : \mu_{\tilde{A}}(x) \geq \lambda\}$ is called an λ-level set or λ-cut or parametric form of $\tilde{A}$.

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Definition 2.3 A fuzzy set $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\}$ is called a normalized fuzzy set if and only if $\underset{x \in X}{\text{Supremum}}\{\mu_{\tilde{A}}(x)\} = 1$.

Definition 2.4 A fuzzy set $\tilde{A}$ is called a convex fuzzy set if and only if $\mu_{\tilde{A}}(\alpha x_1 + (1-\alpha)x_2) \geq \min\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\}, \ \forall\, x_1, x_2 \in X, \ \alpha \in [0, 1]$.

Definition 2.5 A convex normalized fuzzy set $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\}$ is called a fuzzy number if and only if $\mu_{\tilde{A}}(x)$ is piecewise continuous in X.

Definition 2.6 A fuzzy number $\tilde{A}$ is said to be a non-negative fuzzy number if and only if $\mu_{\tilde{A}}(x) = 0, \ \forall\, x < 0$.

Definition 2.7 A fuzzy number $\tilde{A}$ defined on the universal set of real numbers $\mathbb{R}$, denoted as $\tilde{A} = (a, b, c, d)$, is said to be a trapezoidal fuzzy number if its membership function, $\mu_{\tilde{A}}(x)$, is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{(x-a)}{(b-a)} & a \leq x < b \\ 1 & b \leq x \leq c \\ \frac{(x-d)}{(c-d)} & c < x \leq d \\ 0 & \text{otherwise} \end{cases}$$

Definition 2.8 Let $\tilde{A} = (a, b, c, d)$ be a trapezoidal fuzzy number. Then, its λ-cut A^{λ} is defined as follows:

$$A^{\lambda} = [a + (b-a)\lambda, d - (d-c)\lambda], \ 0 \leq \lambda \leq 1$$

Definition 2.9 A trapezoidal fuzzy number $\tilde{A} = (a, b, c, d)$ is said to be symmetric trapezoidal fuzzy number if and only if $b - a = d - c$, otherwise $\tilde{A}$ is said to be an asymmetric trapezoidal fuzzy number.

A symmetric trapezoidal fuzzy number $\tilde{A} = (a, b, c, d)$ can be denoted as (b, c, σ), where $[b, c]$ is the core and $\sigma = b - a = d - c$ is the spread of the symmetric trapezoidal fuzzy number $\tilde{A}$.

Definition 2.10 A trapezoidal fuzzy number $\tilde{A} = (a, b, c, d)$ is said to be non-negative trapezoidal fuzzy number if and only if $a \geq 0$ and is said to be non-positive trapezoidal fuzzy number if and only if $a \leq 0$.

Definition 2.11 A trapezoidal fuzzy number $\tilde{A} = (a, b, c, d)$ is said to be unrestricted trapezoidal fuzzy number if and only if a is a real number.

Definition 2.12 Two trapezoidal fuzzy numbers $\tilde{A}_1 = (a_1, b_1, c_1, d_1)$ and $\tilde{A}_2 = (a_2, b_2, c_2, d_2)$ are said to be equal i.e., $\tilde{A}_1 = \tilde{A}_2$ if and only if $a_1 = a_2, b_1 = b_2, c_1 = c_2$ and $d_1 = d_2$.

2.1.2 Arithmetic Operations

In this section, the arithmetic operations for trapezoidal fuzzy numbers and intervals are presented.

2.1.2.1 Arithmetic Operations for Trapezoidal Fuzzy Numbers

In this section, some arithmetic operations for two trapezoidal fuzzy numbers, defined on universal set of real numbers $\mathbb{R}$, are presented.

(i) Let $\tilde{A}_1 = (a_1, b_1, c_1, d_1)$ and $\tilde{A}_2 = (a_2, b_2, c_2, d_2)$ be two trapezoidal fuzzy numbers. Then, $\tilde{A}_1 \oplus \tilde{A}_2 = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2)$.

(ii) Let $\tilde{A}_1 = (a_1, b_1, c_1, d_1)$ and $\tilde{A}_2 = (a_2, b_2, c_2, d_2)$ be two trapezoidal fuzzy numbers. Then, $\tilde{A}_1 \ominus \tilde{A}_2 = (a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2)$.

(iii) Let $\tilde{A}_1 = (a_1, b_1, c_1, d_1)$ and $\tilde{A}_2 = (a_2, b_2, c_2, d_2)$ be two non-negative trapezoidal fuzzy numbers. Then, $\tilde{A}_1 \otimes \tilde{A}_2 = (a_1a_2, b_1b_2, c_1c_2, d_1d_2)$.

(iv) Let $\tilde{A} = (a, b, c, d)$ be any trapezoidal fuzzy number. Then,

$$\gamma\tilde{A} = \begin{cases} (\gamma a, \gamma b, \gamma c, \gamma d) & \gamma \geq 0 \\ (\gamma d, \gamma c, \gamma b, \gamma a) & \gamma \leq 0 \end{cases}$$

2.1.2.2 Arithmetic Operations for Intervals

In this section, some arithmetic operations for two intervals are presented.

(i) Let $A_1 = [a_1, b_1]$ and $A_2 = [a_2, b_2]$ be two intervals. Then, $A_1 + A_2 = [a_1 + a_2, b_1 + b_2]$

(ii) Let $A_1 = [a_1, b_1]$ and $A_2 = [a_2, b_2]$ be two non-negative intervals. Then, $A_1A_2 = [a_1a_2, b_1b_2]$

Remark 2.1 An interval $A = [a, b]$ is said to be non-negative interval if and only if $a \geq 0$.

Remark 2.2 If $b = c$ then a trapezoidal fuzzy number (a, b, c, d) is said to be triangular fuzzy number and is denoted as (a, b, b, d) or (a, c, c, d) or (a, b, d) or (a, c, d).

Remark 2.3 [1] Let $\tilde{a}^\lambda = [\underline{a}(\lambda), \overline{a}(\lambda)]$ be a parametric form of an asymmetric triangular fuzzy number $\tilde{a}$ then its nearest symmetric triangular fuzzy number is (a_0, σ), where the core 'a_0' and the spread 'σ' of the symmetric triangular fuzzy number can be obtained as $a_0 = \frac{3}{2}\int_0^1 (\overline{a}(\lambda) - \underline{a}(\lambda))(1 - \lambda)d\lambda$, $\sigma = \frac{1}{2}\int_0^1 (\overline{a}(\lambda) + \underline{a}(\lambda))d\lambda$.

Remark 2.4 In the entire thesis 'minimum' and 'maximum' are represented by 'min' and 'max' respectively.

2.2 Existing Method for Solving Fully Fuzzy Linear Programming Problems with Equality Constraints

Lotfi et al. [1] pointed out that there is no method in literature for solving fully fuzzy linear programming problems with equality constraints and proposed the following method to find the fuzzy optimal solution of fully fuzzy linear programming problems with equality constraints (2.1):

$$\begin{aligned} &\text{Maximize } \sum_{j=1}^{n} \tilde{c}_j \otimes \tilde{x}_j \\ &\text{subject to} \\ &\quad \sum_{j=1}^{n} \tilde{a}_{ij} \otimes \tilde{x}_j = \tilde{b}_i \ \ \forall\, i = 1, 2, \ldots, m \end{aligned} \tag{2.1}$$

where $\tilde{c}_j$, $\tilde{x}_j$, $\tilde{a}_{ij}$ and $\tilde{b}_i$ are non-negative triangular fuzzy numbers.

Step 1 Assuming $\tilde{c}_j = (p_j, q_j, r_j), \tilde{a}_{ij} = (a_{ij}, b_{ij}, c_{ij}), \tilde{x}_j = (x_j, y_j, z_j)$ and $\tilde{b}_i = (b_i, g_i, h_i)$ the fully fuzzy linear programming problem (2.1) can be written as:

$$\begin{aligned} &\text{Maximize } \sum_{j=1}^{n} (p_j, q_j, r_j) \otimes (x_j, y_j, z_j) \\ &\text{subject to} \\ &\quad \sum_{j=1}^{n} (a_{ij}, b_{ij}, c_{ij}) \otimes (x_j, y_j, z_j) = (b_i, g_i, h_i) \ \ \forall\, i = 1, 2, \ldots, m \end{aligned} \tag{2.2}$$

where (x_j, y_j, z_j) is a non-negative triangular fuzzy number.

Step 2 Using Definitions 2.8 and 2.10, the fully fuzzy linear programming problem (2.2) can be converted into problem (2.3):

$$\begin{aligned} &\text{Maximize } \sum_{j=1}^{n} [p_j + (q_j - p_j)\lambda, r_j - (r_j - q_j)\lambda][x_j + (y_j - x_j)\lambda, z_j - (z_j - y_j)\lambda] \\ &\text{subject to} \\ &\sum_{j=1}^{n} [a_{ij} + (b_{ij} - a_{ij})\lambda, c_{ij} - (c_{ij} - b_{ij})\lambda][x_j + (y_j - x_j)\lambda, z_j - (z_j - y_j)\lambda] \\ &= [b_i + (g_i - b_i)\lambda, h_i - (h_i - g_i)\lambda] \ \ \forall\, i = 1, 2, \ldots, m \\ &x_j \geq 0, y_j - x_j \geq 0, z_j - y_j \geq 0 \ \ \forall\, j = 1, 2, \ldots, n \end{aligned} \tag{2.3}$$

Step 3 Using the arithmetic operations of intervals, defined in Sect. 2.1.2.2, the problem (2.3) can be converted into the problem (2.4):

$$\begin{aligned}
&\text{Maximize } [\sum_{j=1}^{n}(p_j x_j + \lambda(p_j y_j + q_j x_j - 2p_j x_j) + \lambda^2(q_j - p_j)(y_j - x_j)), \sum_{j=1}^{n}(r_j z_j - \lambda(2r_j z_j \\
&- r_j y_j - q_j z_j) + \lambda^2(r_j - q_j)(z_j - y_j))] \\
&\text{subject to} \\
&\sum_{j=1}^{n}[a_{ij}x_j + \lambda(a_{ij}y_j + b_{ij}x_j - 2a_{ij}x_j) + \lambda^2(b_{ij} - a_{ij})(y_j - x_j), c_{ij}z_j - \lambda(2c_{ij}z_j - c_{ij}y_j \\
&- b_{ij}z_j) + \lambda^2(c_{ij} - b_{ij})(z_j - y_j)] = [b_i + \lambda(g_i - b_i), h_i - \lambda(h_i - g_i)] \ \forall\, i = 1, 2, \ldots, m \\
&x_j \geq 0, y_j - x_j \geq 0, z_j - y_j \geq 0 \ \ \forall\, j = 1, 2, \ldots, n
\end{aligned} \tag{2.4}$$

Step 4 Using Remark 2.3, the problem (2.4) can be converted into the problem (2.5):

$$\begin{aligned}
&\text{Maximize } (\sum_{j=1}^{n}(\frac{1}{3}q_j y_j + \frac{1}{12}q_j z_j + \frac{1}{12}r_j y_j + \frac{1}{6}r_j z_j + \frac{1}{12}q_j x_j + \frac{1}{12}p_j y_j + \frac{1}{6}p_j x_j), \sum_{j=1}^{n}(\frac{1}{8}q_j z_j \\
&+ \frac{1}{8}r_j y_j + \frac{3}{8}r_j z_j - \frac{1}{8}q_j x_j - \frac{1}{8}p_j y_j - \frac{3}{8}p_j x_j)) \\
&\text{subject to} \\
&\sum_{j=1}^{n}(\frac{1}{3}b_{ij}y_j + \frac{1}{12}b_{ij}z_j + \frac{1}{12}c_{ij}y_j + \frac{1}{6}c_{ij}z_j + \frac{1}{12}b_{ij}x_j + \frac{1}{12}a_{ij}y_j + \frac{1}{6}a_{ij}x_j, \frac{1}{8}b_{ij}z_j + \frac{1}{8}c_{ij}y_j \\
&+ \frac{3}{8}c_{ij}z_j - \frac{1}{8}b_{ij}x_j - \frac{1}{8}a_{ij}y_j - \frac{3}{8}a_{ij}x_j) = (\frac{1}{4}b_i + \frac{1}{2}g_i + \frac{1}{4}h_i, \frac{1}{2}h_i - \frac{1}{2}b_i) \ \forall\, i = 1, 2, \ldots, m \\
&x_j \geq 0, y_j - x_j \geq 0, z_j - y_j \geq 0 \ \forall\, j = 1, 2, \ldots, n
\end{aligned} \tag{2.5}$$

Step 5 Using Definition 2.12, convert the obtained fully fuzzy linear programming (2.5) into the crisp linear programming problem (2.6) to maximize the core:

$$\begin{aligned}
&\text{Maximize } \sum_{j=1}^{n}(\frac{1}{3}q_j y_j + \frac{1}{12}q_j z_j + \frac{1}{12}r_j y_j + \frac{1}{6}r_j z_j + \frac{1}{12}q_j x_j + \frac{1}{12}p_j y_j + \frac{1}{6}p_j x_j) \\
&\text{subject to} \\
&\quad \sum_{j=1}^{n}(\frac{1}{3}b_{ij}y_j + \frac{1}{12}b_{ij}z_j + \frac{1}{12}c_{ij}y_j + \frac{1}{6}c_{ij}z_j + \frac{1}{12}b_{ij}x_j + \frac{1}{12}a_{ij}y_j + \frac{1}{6}a_{ij}x_j) = \frac{1}{4}b_i + \frac{1}{2}g_i + \frac{1}{4}h_i \\
&\quad \forall\, i = 1, 2, \ldots, m \\
&\quad \sum_{j=1}^{n}(\frac{1}{8}b_{ij}z_j + \frac{1}{8}c_{ij}y_j + \frac{3}{8}c_{ij}z_j - \frac{1}{8}b_{ij}x_j - \frac{1}{8}a_{ij}y_j - \frac{3}{8}a_{ij}x_j) = \frac{1}{2}h_i - \frac{1}{2}b_i \ \ \forall\, i = 1, 2, \ldots, m \\
&\quad x_j \geq 0, y_j - x_j \geq 0, z_j - y_j \geq 0 \ \ \forall\, j = 1, 2, \ldots, n
\end{aligned} \tag{2.6}$$

Step 6 If the crisp linear programming problem (2.6) has a unique optimal solution x_j^*, y_j^* and z_j^* then the fuzzy optimal solution of (2.1) will be (x_j^*, y_j^*, z_j^*). If it has alternative optimal solutions then solve the crisp linear programming problem (2.7) to minimize spread:

$$\text{Minimize} \sum_{j=1}^{n}(\frac{1}{8}q_jz_j + \frac{1}{8}r_jy_j + \frac{3}{8}r_jz_j - \frac{1}{8}q_jx_j - \frac{1}{8}p_jy_j - \frac{3}{8}p_jx_j)$$

subject to

$$\sum_{j=1}^{n}(\frac{1}{3}b_{ij}y_j + \frac{1}{12}b_{ij}z_j + \frac{1}{12}c_{ij}y_j + \frac{1}{6}c_{ij}z_j + \frac{1}{12}b_{ij}x_j + \frac{1}{12}a_{ij}y_j + \frac{1}{6}a_{ij}x_j) = \frac{1}{4}b_i + \frac{1}{2}g_i + \frac{1}{4}h_i$$

$$\forall\, i = 1, 2, \ldots, m \tag{2.7}$$

$$\sum_{j=1}^{n}(\frac{1}{8}b_{ij}z_j + \frac{1}{8}c_{ij}y_j + \frac{3}{8}c_{ij}z_j - \frac{1}{8}b_{ij}x_j - \frac{1}{8}a_{ij}y_j - \frac{3}{8}a_{ij}x_j) = \frac{1}{2}h_i - \frac{1}{2}b_i \;\; \forall\, i = 1, 2, \ldots, m$$

$$\sum_{j=1}^{n}(\frac{1}{3}q_jy_j + \frac{1}{12}q_jz_j + \frac{1}{12}r_jy_j + \frac{1}{6}r_jz_j + \frac{1}{12}q_jx_j + \frac{1}{12}p_jy_j + \frac{1}{6}p_jx_j) = a^*$$

$$x_j \geq 0, y_j - x_j \geq 0, z_j - y_j \geq 0 \;\; \forall\, j = 1, 2, \ldots, n$$

where a^* is the optimal value of the crisp linear programming problem (2.6).

Step 7 Let x_j^*, y_j^* and z_j^* be the optimal solution of crisp linear programming problem (2.7). Then, the fuzzy optimal solution of fully fuzzy linear programming problem (2.1) is (x_j^*, y_j^*, z_j^*).

2.3 Limitations and Shortcoming of the Existing Method

In this section, the limitations and shortcoming of the existing method [1] are pointed out.

2.3.1 Limitations of the Existing Method

In this section, the limitations of the existing method [1] are pointed out.

The existing method [1] can be used to find the non-negative fuzzy optimal solution of such fully fuzzy linear programming problems with equality constraints in which all the parameters are represented by non-negative triangular fuzzy numbers. However, the existing method [1] cannot be used to find the non-negative fuzzy optimal solution of the following problems:

(i) Fully fuzzy linear programming problems with equality constraints in which all the parameters are represented by non-negative trapezoidal fuzzy numbers:

$$\begin{aligned} &\text{Maximize/Minimize} \sum_{j=1}^{n} \tilde{c}_j \otimes \tilde{x}_j \\ &\text{subject to} \\ &\sum_{j=1}^{n} \tilde{a}_{ij} \otimes \tilde{x}_j = \tilde{b}_i \ \ \forall\, i = 1, 2, \ldots, m \end{aligned} \tag{2.8}$$

where $\tilde{c}_j, \tilde{a}_{ij}, \tilde{b}_i$ and $\tilde{x}_j$ are non-negative trapezoidal fuzzy numbers.

Example 2.1

$$\begin{aligned} &\text{Maximize } ((1, 2, 3, 4) \otimes \tilde{x}_1 \oplus (2, 3, 4, 5) \otimes \tilde{x}_2) \\ &\text{subject to} \\ &(0, 1, 2, 3) \otimes \tilde{x}_1 \oplus (1, 2, 3, 4) \otimes \tilde{x}_2 = (2, 10, 24, 44) \\ &(1, 2, 3, 4) \otimes \tilde{x}_1 \oplus (0, 1, 2, 3) \otimes \tilde{x}_2 = (1, 8, 21, 40) \end{aligned}$$

where $\tilde{x}_1$ and $\tilde{x}_2$ are non-negative trapezoidal fuzzy numbers.

(ii) Fully fuzzy linear programming problems with equality constraints in which some or all the coefficients are either represented by unrestricted triangular or trapezoidal fuzzy numbers:

$$\begin{aligned} &\text{Maximize/Minimize} \sum_{j=1}^{n} \tilde{c}_j \otimes \tilde{x}_j \\ &\text{subject to} \\ &\sum_{j=1}^{n} \tilde{a}_{ij} \otimes \tilde{x}_j = \tilde{b}_i \ \ \forall\, i = 1, 2, \ldots, m \end{aligned} \tag{2.9}$$

where $\tilde{c}_j, \tilde{a}_{ij}, \tilde{b}_i$ are unrestricted triangular or trapezoidal fuzzy numbers and $\tilde{x}_j$ is a non-negative triangular or trapezoidal fuzzy number.

Example 2.2

$$\begin{aligned} &\text{Maximize } ((1, 6, 9, 12) \otimes \tilde{x}_1 \oplus (2, 3, 8, 9) \otimes \tilde{x}_2) \\ &\text{subject to} \\ &(2, 3, 4, 5) \otimes \tilde{x}_1 \oplus (1, 2, 3, 4) \otimes \tilde{x}_2 = (6, 16, 30, 48) \\ &(-1, 1, 2, 3) \otimes \tilde{x}_1 \oplus (1, 3, 4, 6) \otimes \tilde{x}_2 = (0, 17, 30, 54) \end{aligned}$$

where $\tilde{x}_1$ and $\tilde{x}_2$ are non-negative trapezoidal fuzzy numbers.

2.3.2 *Shortcoming of the Existing Method*

In this section, the shortcoming of the existing method [1] is pointed out.

For solving the fully fuzzy linear programming problems by using the existing method [1] there is a need to approximate all the coefficients into its nearest symmetric fuzzy numbers. Due to this conversion, the obtained solutions are approximate and do not satisfy the constraints exactly e.g., on solving the fully fuzzy linear programming problem, chosen in Example 2.3, by using the existing method [1], the obtained fuzzy optimal solution is $\tilde{x}_1 = (1.45, 1.45, 3.31)$ and $\tilde{x}_2 = (0.88, 4.32, 4.32)$ which does not satisfy the constraints exactly.

Example 2.3

$$\begin{aligned}
&\text{Maximize } ((0,1,4) \otimes \tilde{x}_1 \oplus (2,4,5) \otimes \tilde{x}_2)\\
&\text{subject to}\\
&(2,3,7) \otimes \tilde{x}_1 \oplus (2,4,5) \otimes \tilde{x}_2 = (6,18,46)\\
&(0,2,4) \otimes \tilde{x}_1 \oplus (3,5,8) \otimes \tilde{x}_2 = (6,19,52)
\end{aligned}$$

where $\tilde{x}_1$ and $\tilde{x}_2$ are non-negative triangular fuzzy numbers.

2.4 Product of a Non-negative Trapezoidal Fuzzy Number with Unrestricted Trapezoidal Fuzzy Number

For solving the fully fuzzy linear programming problems, there is a need to find the product of fuzzy coefficients and fuzzy variables. Since, in the fully fuzzy linear programming problems, the fuzzy coefficients are known and the product of fuzzy numbers depends upon the nature of fuzzy numbers. So, in this section, on the basis of nature of fuzzy coefficients product, proposed by Kumar et al. [2], with the help of existing product of two fuzzy numbers [3], is presented.

Let $\tilde{A} = (a, b, c, d)$ be an unrestricted trapezoidal fuzzy number and $\tilde{X} = (x, y, z, w)$ be a non-negative trapezoidal fuzzy number. Then

$$\tilde{A} \otimes \tilde{X} = \begin{cases} (ax, by, cz, dw) & a \geq 0 \\ (aw, by, cz, dw) & a < 0 \text{ and } b \geq 0 \\ (aw, bz, cz, dw) & b < 0 \text{ and } c \geq 0 \\ (aw, bz, cy, dw) & c < 0 \text{ and } d \geq 0 \\ (aw, bz, cy, dx) & \text{otherwise.} \end{cases}$$

2.5 Kumar et al.'s Method to Find the Non-negative Fuzzy Optimal Solution of Fully Fuzzy Linear Programming Problems with Equality Constraints

In this section, to overcome the limitations as well as to resolve the shortcoming of the existing method [1], discussed in Sect. 2.3, Kumar et al.'s method [2] is presented to find the non-negative fuzzy optimal solution of fully fuzzy linear programming problems with equality constraints (2.9).

The steps of the method are as follows:

Step 1 Assuming $\tilde{c}_j = (p_j, q_j, r_j, s_j)$, $\tilde{x}_j = (x_j, y_j, z_j, w_j)$, $\tilde{a} = (a_{ij}, b_{ij}, c_{ij}, d_{ij})$ and $\tilde{b} = (b_i, g_i, h_i, k_i)$ the fully fuzzy linear programming problem (2.9) can be converted into (2.10):

$$\begin{aligned}
&\text{Maximize/Minimize} \sum_{j=1}^{n} (p_j, q_j, r_j, s_j) \otimes (x_j, y_j, z_j, w_j) \\
&\text{subject to} \\
&\quad \sum_{j=1}^{n} (a_{ij}, b_{ij}, c_{ij}, d_{ij}) \otimes (x_j, y_j, z_j, w_j) = (b_i, g_i, h_i, k_i) \;\; \forall\, i = 1, 2, \ldots, m
\end{aligned} \tag{2.10}$$

where (x_j, y_j, z_j, w_j) is a non-negative trapezoidal fuzzy number.

Step 2 Using the product of trapezoidal fuzzy numbers, presented in Sect. 2.4 and assuming $(a_{ij}, b_{ij}, c_{ij}, d_{ij}) \otimes (x_j, y_j, z_j, w_j) = (a'_{ij}, b'_{ij}, c'_{ij}, d'_{ij})$, the fully fuzzy linear programming problem (2.10) can be converted into (2.11):

$$\begin{aligned}
&\text{Maximize/Minimize} \sum_{j=1}^{n} (p_j, q_j, r_j, s_j) \otimes (x_j, y_j, z_j, w_j) \\
&\text{subject to} \\
&\quad \sum_{j=1}^{n} (a'_{ij}, b'_{ij}, c'_{ij}, d'_{ij}) = (b_i, g_i, h_i, k_i) \;\; \forall\, i = 1, 2, \ldots, m
\end{aligned} \tag{2.11}$$

where (x_j, y_j, z_j, w_j) is a non-negative trapezoidal fuzzy number.

Step 3 Using arithmetic operations, defined in Sect. 2.1.2.1 and Definition 2.12, the fully fuzzy linear programming problem (2.11) can be converted into (2.12):

$$\begin{aligned}
&\text{Maximize/Minimize} \sum_{j=1}^{n} (p_j, q_j, r_j, s_j) \otimes (x_j, y_j, z_j, w_j) \\
&\text{subject to} \\
&\qquad \sum_{j=1}^{n} a'_{ij} = b_i \;\; \forall\, i = 1, 2, \ldots, m \\
&\qquad \sum_{j=1}^{n} b'_{ij} = g_i \;\; \forall\, i = 1, 2, \ldots, m \\
&\qquad \sum_{j=1}^{n} c'_{ij} = h_i \;\; \forall\, i = 1, 2, \ldots, m \\
&\qquad \sum_{j=1}^{n} d'_{ij} = k_i \;\; \forall\, i = 1, 2, \ldots, m \\
&x_j \geq 0, y_j - x_j \geq 0, z_j - y_j \geq 0, w_j - z_j \geq 0 \;\; \forall\, j = 1, 2, \ldots, n
\end{aligned} \tag{2.12}$$

Step 4 Suppose the fuzzy linear programming problem (2.12) have 'l' basic feasible solutions and $\{x_j^t, y_j^t, z_j^t, w_j^t\}$ is the tth basic feasible solution then our aim is to find that basic feasible solution out of all 'l' basic feasible solutions corresponding to which the value of objective function is maximum (or minimum) i.e., our aim is to find $\max\limits_{1 \leq t \leq l} (\text{or min})\{\sum_{j=1}^{n} (p_j, q_j, r_j, s_j) \otimes (x_j^t, y_j^t, z_j^t, w_j^t)\}$. Liou and Wang [4] proposed the concept that if $\max\limits_{1 \leq t \leq l} (\text{or min})\{\Re(\sum_{j=1}^{n} (p_j, q_j, r_j, s_j) \otimes (x_j^t, y_j^t, z_j^t, w_j^t))\}$ is $\Re(\sum_{j=1}^{n} (p_j, q_j, r_j, s_j) \otimes (x_j^\theta, y_j^\theta, z_j^\theta, w_j^\theta))$ then $\max\limits_{1 \leq t \leq l} (\text{or min})\{\sum_{j=1}^{n} (p_j, q_j, r_j, s_j) \otimes (x_j^t, y_j^t, z_j^t, w_j^t)\}$ will also be $\sum_{j=1}^{n} (p_j, q_j, r_j, s_j) \otimes (x_j^\theta, y_j^\theta, z_j^\theta, w_j^\theta)$, where $\Re(a, b, c, d) = \frac{1}{4}(a + b + c + d)$, i.e., according to the existing method [4], the fuzzy optimal solution of (2.12) can be obtained by solving the crisp linear programming problem (2.13):

$$\text{Maximize/Minimize}\, \Re\left(\sum_{j=1}^{n}(p_j, q_j, r_j, s_j) \otimes (x_j, y_j, z_j, w_j)\right)$$

subject to

$$\begin{aligned}
&\sum_{j=1}^{n} a'_{ij} = b_i \;\; \forall\, i = 1, 2, \ldots, m \\
&\sum_{j=1}^{n} b'_{ij} = g_i \;\; \forall\, i = 1, 2, \ldots, m \\
&\sum_{j=1}^{n} c'_{ij} = h_i \;\; \forall\, i = 1, 2, \ldots, m \\
&\sum_{j=1}^{n} d'_{ij} = k_i \;\; \forall\, i = 1, 2, \ldots, m \\
&x_j \geq 0, y_j - x_j \geq 0, z_j - y_j \geq 0, w_j - z_j \geq 0 \;\; \forall\, j = 1, 2, \ldots, n
\end{aligned} \tag{2.13}$$

Step 5 Assuming $(p_j, q_j, r_j, s_j) \otimes (x_j, y_j, z_j, w_j) = (p'_j, q'_j, r'_j, s'_j)$ the crisp linear programming problem (2.13) can be written as (2.14):

$$\text{Maximize/Minimize}\, \Re\left(\sum_{j=1}^{n}(p'_j, q'_j, r'_j, s'_j)\right)$$

subject to

$$\begin{aligned}
&\sum_{j=1}^{n} a'_{ij} = b_i \;\; \forall\, i = 1, 2, \ldots, m \\
&\sum_{j=1}^{n} b'_{ij} = g_i \;\; \forall\, i = 1, 2, \ldots, m \\
&\sum_{j=1}^{n} c'_{ij} = h_i \;\; \forall\, i = 1, 2, \ldots, m \\
&\sum_{j=1}^{n} d'_{ij} = k_i \;\; \forall\, i = 1, 2, \ldots, m \\
&x_j \geq 0, y_j - x_j \geq 0, z_j - y_j \geq 0, w_j - z_j \geq 0 \;\; \forall\, j = 1, 2, \ldots, n
\end{aligned} \tag{2.14}$$

Step 6 Using the linearity property $\Re(\sum_{j=1}^{n} \tilde{A}_i) = \sum_{j=1}^{n} \Re(\tilde{A}_i)$, where $\tilde{A}_i$ is a fuzzy number, the crisp linear programming problem (2.14) can be converted into (2.15):

$$\begin{aligned}
&\text{Maximize/Minimize} \sum_{j=1}^{n} \Re(p'_j, q'_j, r'_j, s'_j) \\
&\text{subject to} \\
&\qquad \sum_{j=1}^{n} a'_{ij} = b_i \ \ \forall\, i = 1, 2, \ldots, m \\
&\qquad \sum_{j=1}^{n} b'_{ij} = g_i \ \ \forall\, i = 1, 2, \ldots, m \\
&\qquad \sum_{j=1}^{n} c'_{ij} = h_i \ \ \forall\, i = 1, 2, \ldots, m \\
&\qquad \sum_{j=1}^{n} d'_{ij} = k_i \ \ \forall\, i = 1, 2, \ldots, m \\
&x_j \geq 0, y_j - x_j \geq 0, z_j - y_j \geq 0, w_j - z_j \geq 0 \ \ \forall\, j = 1, 2, \ldots, n
\end{aligned} \tag{2.15}$$

Step 7 Using $\Re(a, b, c, d) = \frac{1}{4}(a+b+c+d)$ the crisp linear programming problem (2.15) can be converted into (2.16):

$$\begin{aligned}
&\text{Maximize/Minimize} \sum_{j=1}^{n} \frac{1}{4}(p'_j + q'_j + r'_j + s'_j) \\
&\text{subject to} \\
&\qquad \sum_{j=1}^{n} a'_{ij} = b_i \ \ \forall\, i = 1, 2, \ldots, m \\
&\qquad \sum_{j=1}^{n} b'_{ij} = g_i \ \ \forall\, i = 1, 2, \ldots, m \\
&\qquad \sum_{j=1}^{n} c'_{ij} = h_i \ \ \forall\, i = 1, 2, \ldots, m \\
&\qquad \sum_{j=1}^{n} d'_{ij} = k_i \ \ \forall\, i = 1, 2, \ldots, m \\
&x_j \geq 0, y_j - x_j \geq 0, z_j - y_j \geq 0, w_j - z_j \geq 0 \ \ \forall\, j = 1, 2, ..., n
\end{aligned} \tag{2.16}$$

Step 8 Solve the crisp linear programming problem (2.16) by using an appropriate existing method [5] to find the optimal solution $\{x_j^*, y_j^*, z_j^*, w_j^*\}$.

Step 9 Find the fuzzy optimal solution $\{\tilde{x}_j^*\}$ of the fully fuzzy linear programming problem (2.9) by putting the values of x_j^*, y_j^*, z_j^* and w_j^* in $\tilde{x}_j^* = (x_j^*, y_j^*, z_j^*, w_j^*)$.

Step 10 Find the fuzzy optimal value by putting the values of $\tilde{x}_j^*$, obtained from Step 9, in $\sum_{j=1}^{n} \tilde{c}_j \otimes \tilde{x}_j^*$.

2.6 Illustrative Examples

In this section, Kumar et al.'s method [2], presented in Sect. 2.5, is illustrated with the help of fully fuzzy linear programming problems, chosen in Examples 2.1 and 2.2, which cannot be solved by using the existing method [1]. Moreover, a fully fuzzy linear programming problem, which can be solved by using the existing method [1], is also solved by using the Kumar et al.'s method [2].

2.6.1 Fuzzy Optimal Solution of the Chosen Fully Fuzzy Linear Programming Problems

In this section, fully fuzzy linear programming problems, chosen in Examples 2.1, 2.2 and 2.3, are solved by using the method, presented is Sect. 2.5.

2.6.1.1 Fuzzy Optimal Solution of the Fully Fuzzy Linear Programming Problem Chosen in Example 2.1

The fuzzy optimal solution of the fully fuzzy linear programming problem, chosen in Example 2.1, can be obtained by using the following steps:

Step 1 Assuming $\tilde{x}_1 = (x_1, y_1, z_1, w_1)$ and $\tilde{x}_2 = (x_2, y_2, z_2, w_2)$ the fully fuzzy linear programming problem, chosen in Example 2.1, can be written as:

$$\begin{aligned}
&\text{Maximize } ((1,2,3,4) \otimes (x_1, y_1, z_1, w_1) \oplus (2,3,4,5) \otimes (x_2, y_2, z_2, w_2))\\
&\text{subject to}\\
&(0,1,2,3) \otimes (x_1, y_1, z_1, w_1) \oplus (1,2,3,4) \otimes (x_2, y_2, z_2, w_2) = (2,10,24,44)\\
&\quad (1,2,3,4) \otimes (x_1, y_1, z_1, w_1) \oplus (0,1,2,3) \otimes (x_2, y_2, z_2, w_2) = (1,8,21,40)
\end{aligned}$$

where (x_1, y_1, z_1, w_1) and (x_2, y_2, z_2, w_2) are non-negative trapezoidal fuzzy numbers.

Step 2 Using the product, presented in Sect. 2.4, the fully fuzzy linear programming problem, obtained in Step 1, can be written as:

$$\begin{aligned}
&\text{Maximize } ((x_1, 2y_1, 3z_1, 4w_1) \oplus (2x_2, 3y_2, 4z_2, 5w_2))\\
&\text{subject to}\\
&(0x_1, y_1, 2z_1, 3w_1) \oplus (x_2, 2y_2, 3z_2, 4w_2) = (2, 10, 24, 44)\\
&(x_1, 2y_1, 3z_1, 4w_1) \oplus (0x_2, y_2, 2z_2, 3w_2) = (1, 8, 21, 40)
\end{aligned}$$

where (x_1, y_1, z_1, w_1) and (x_2, y_2, z_2, w_2) are non-negative trapezoidal fuzzy numbers.

Step 3 Using the arithmetic operations defined in Sect. 2.1.2.1 and Definition 2.12, the fully fuzzy linear programming problem, obtained in Step 2, can be written as:

$$\begin{aligned}
&\text{Maximize } (x_1 + 2x_2, 2y_1 + 3y_2, 3z_1 + 4z_2, 4w_1 + 5w_2)\\
&\text{subject to}\\
&\qquad 0x_1 + x_2 = 2\\
&\qquad x_1 + 0x_2 = 1\\
&\qquad y_1 + 2y_2 = 10\\
&\qquad 2y_1 + y_2 = 8\\
&\qquad 2z_1 + 3z_2 = 24\\
&\qquad 3z_1 + 2z_2 = 21\\
&\qquad 3w_1 + 4w_2 = 44\\
&\qquad 4w_1 + 3w_2 = 40\\
&x_1 \geq 0, y_1 - x_1 \geq 0, z_1 - y_1 \geq 0, w_1 - z_1 \geq 0\\
&x_2 \geq 0, y_2 - x_2 \geq 0, z_2 - y_2 \geq 0, w_2 - z_2 \geq 0
\end{aligned}$$

Step 4 Using Step 4 of the method, presented in Sect. 2.5, the fuzzy linear programming problem, obtained in Step 3, can be written as:

$$\begin{aligned}
&\text{Maximize } \frac{1}{4}(x_1 + 2x_2 + 2y_1 + 3y_2 + 3z_1 + 4z_2 + 4w_1 + 5w_2)\\
&\text{subject to}\\
&\qquad 0x_1 + x_2 = 2\\
&\qquad x_1 + 0x_2 = 1\\
&\qquad y_1 + 2y_2 = 10\\
&\qquad 2y_1 + y_2 = 8\\
&\qquad 2z_1 + 3z_2 = 24\\
&\qquad 3z_1 + 2z_2 = 21\\
&\qquad 3w_1 + 4w_2 = 44\\
&\qquad 4w_1 + 3w_2 = 40\\
&x_1 \geq 0, y_1 - x_1 \geq 0, z_1 - y_1 \geq 0, w_1 - z_1 \geq 0\\
&x_2 \geq 0, y_2 - x_2 \geq 0, z_2 - y_2 \geq 0, w_2 - z_2 \geq 0
\end{aligned}$$

Step 5 The optimal solution of the crisp linear programming problem, obtained in Step 4, is $x_1 = 1, y_1 = 2, z_1 = 3, w_1 = 4, x_2 = 2, y_2 = 4, z_2 = 6$ and $w_2 = 8$.

Step 6 Putting the values of $x_1, y_1, z_1, w_1, x_2, y_2, z_2$ and w_2 in $\tilde{x}_1 = (x_1, y_1, z_1, w_1)$ and $\tilde{x}_2 = (x_2, y_2, z_2, w_2)$, the exact fuzzy optimal solution is $\tilde{x}_1 = (1, 2, 3, 4), \tilde{x}_2 = (2, 4, 6, 8)$.

Step 7 Putting the values of $\tilde{x}_1$ and $\tilde{x}_2$, obtained from Step 6, in the objective function the fuzzy optimal value of the fully fuzzy linear programming problem is (5, 16, 33, 56).

2.6.1.2 Fuzzy Optimal Solution of the Fully Fuzzy Linear Programming Problem Chosen in Example 2.2

The fuzzy optimal solution of the fully fuzzy linear programming problem, chosen in Example 2.2, can be obtained by using the following steps:

Step 1 Assuming $\tilde{x}_1 = (x_1, y_1, z_1, w_1)$ and $\tilde{x}_2 = (x_2, y_2, z_2, w_2)$ the fully fuzzy linear programming problem, chosen in Example 2.2, can be written as:

$$\begin{aligned}
&\text{Maximize } ((1, 6, 9, 12) \otimes (x_1, y_1, z_1, w_1) \oplus (2, 3, 8, 9) \otimes (x_2, y_2, z_2, w_2))\\
&\text{subject to}\\
&(2, 3, 4, 5) \otimes (x_1, y_1, z_1, w_1) \oplus (1, 2, 3, 4) \otimes (x_2, y_2, z_2, w_2) = (6, 16, 30, 48)\\
&(-1, 1, 2, 3) \otimes (x_1, y_1, z_1, w_1) \oplus (1, 3, 4, 6) \otimes (x_2, y_2, z_2, w_2) = (0, 17, 30, 54)
\end{aligned}$$

where (x_1, y_1, z_1, w_1) and (x_2, y_2, z_2, w_2) are non-negative trapezoidal fuzzy numbers.

Step 2 Using the product, presented in Sect. 2.4, the fully fuzzy linear programming problem, obtained in Step 1, can be written as:

$$\begin{aligned}
&\text{Maximize } ((x_1, 6y_1, 9z_1, 12w_1) \oplus (2x_2, 3y_2, 8z_2, 9w_2))\\
&\text{subject to}\\
&(2x_1, 3y_1, 4z_1, 5w_1) \oplus (x_2, 2y_2, 3z_2, 4w_2) = (6, 16, 30, 48)\\
&(-w_1, y_1, 2z_1, 3w_1) \oplus (x_2, 3y_2, 4z_2, 6w_2) = (0, 17, 30, 54)
\end{aligned}$$

where (x_1, y_1, z_1, w_1) and (x_2, y_2, z_2, w_2) are non-negative trapezoidal fuzzy numbers.

Step 3 Using the arithmetic operations, defined in Sect. 2.1.2.1 and Definition 2.12, the fuzzy linear programming problem, obtained in Step 2, can be written as:

$$
\begin{aligned}
&\text{Maximize } (x_1 + 2x_2, 6y_1 + 3y_2, 9z_1 + 8z_2, 12w_1 + 9w_2)\\
&\text{subject to}\\
&\qquad 2x_1 + x_2 = 6\\
&\qquad -w_1 + x_2 = 0\\
&\qquad 3y_1 + 2y_2 = 16\\
&\qquad y_1 + 3y_2 = 17\\
&\qquad 4z_1 + 3z_2 = 30\\
&\qquad 2z_1 + 4z_2 = 30\\
&\qquad 5w_1 + 4w_2 = 48\\
&\qquad 3z_1 + 6z_2 = 54\\
&x_1 \geq 0, y_1 - x_1 \geq 0, z_1 - y_1 \geq 0, w_1 - z_1 \geq 0\\
&x_2 \geq 0, y_2 - x_2 \geq 0, z_2 - y_2 \geq 0, w_2 - z_2 \geq 0
\end{aligned}
$$

Step 4 Using Step 4 of the method, presented in Sect. 2.5, the fuzzy linear programming problem, obtained in Step 3, can be written as:

$$
\begin{aligned}
&\text{Maximize } \frac{1}{4}(x_1 + 2x_2 + 6y_1 + 3y_2 + 9z_1 + 8z_2 + 12w_1 + 9w_2)\\
&\text{subject to}\\
&\qquad 2x_1 + x_2 = 6\\
&\qquad -w_1 + x_2 = 0\\
&\qquad 3y_1 + 2y_2 = 16\\
&\qquad y_1 + 3y_2 = 17\\
&\qquad 4z_1 + 3z_2 = 30\\
&\qquad 2z_1 + 4z_2 = 30\\
&\qquad 5w_1 + 4w_2 = 48\\
&\qquad 3z_1 + 6z_2 = 54\\
&x_1 \geq 0, y_1 - x_1 \geq 0, z_1 - y_1 \geq 0, w_1 - z_1 \geq 0\\
&x_2 \geq 0, y_2 - x_2 \geq 0, z_2 - y_2 \geq 0, w_2 - z_2 \geq 0
\end{aligned}
$$

Step 5 The optimal solution of the crisp linear programming problem, obtained in Step 4, is $x_1 = 1, y_1 = 2, z_1 = 3, w_1 = 4, x_2 = 4, y_2 = 5, z_2 = 6$ and $w_2 = 7$.

Step 6 Putting the values of $x_1, y_1, z_1, w_1, x_2, y_2, z_2$ and w_2 in $\tilde{x}_1 = (x_1, y_1, z_1, w_1)$ and $\tilde{x}_2 = (x_2, y_2, z_2, w_2)$, the exact fuzzy optimal solution is $\tilde{x}_1 = (1, 2, 3, 4), \tilde{x}_2 = (4, 5, 6, 7)$.

Step 7 Putting the values of $\tilde{x}_1$ and $\tilde{x}_2$, obtained from Step 6, in the objective function the fuzzy optimal value of the fully fuzzy linear programming problem is $(9, 27, 75, 111)$.

2.6.1.3 Fuzzy Optimal Solution of the Fully Fuzzy Linear Programming Problem Chosen in Example 2.3

The fuzzy optimal solution of the fully fuzzy linear programming problem, chosen in Example 2.3, can be obtained by using the following steps:

Step 1 Assuming $\tilde{x}_1 = (x_1, y_1, z_1)$ and $\tilde{x}_2 = (x_2, y_2, z_2)$ the fully fuzzy linear programming problem, chosen in Example 2.3, can be written as:

$$\begin{aligned}
&\text{Maximize } ((0, 1, 4) \otimes (x_1, y_1, z_1) \oplus (2, 4, 5) \otimes (x_2, y_2, z_2))\\
&\text{subject to}\\
&(2, 3, 7) \otimes (x_1, y_1, z_1) \oplus (2, 4, 5) \otimes (x_2, y_2, z_2) = (6, 18, 46)\\
&(0, 2, 4) \otimes (x_1, y_1, z_1) \oplus (3, 5, 8) \otimes (x_2, y_2, z_2) = (6, 19, 52)
\end{aligned}$$

where (x_1, y_1, z_1) and (x_2, y_2, z_2) are non-negative triangular fuzzy numbers.

Step 2 Using the product, presented in Sect. 2.4, the fully fuzzy linear programming problem, obtained in Step 1, can be written as:

$$\begin{aligned}
&\text{Maximize } ((0x_1, y_1, 4z_1) \oplus (2x_2, 4y_2, 5z_2))\\
&\text{subject to}\\
&(2x_1, 3y_1, 7z_1) \oplus (2x_2, 4y_2, 5z_2) = (6, 18, 46)\\
&(0x_1, 2y_1, 4z_1) \oplus (3x_2, 5y_2, 8z_2) = (6, 19, 52)
\end{aligned}$$

where (x_1, y_1, z_1) and (x_2, y_2, z_2) are non-negative triangular fuzzy numbers.

Step 3 Using the arithmetic operations, defined in Sect. 2.1.2.1 and Definition 2.12, the fuzzy linear programming problem, obtained in Step 2, can be written as:

$$\begin{aligned}
&\text{Maximize } (0x_1 + 2x_2, y_1 + 4y_2, 4z_1 + 5z_2)\\
&\text{subject to}\\
&\qquad 2x_1 + 2x_2 = 6\\
&\qquad 0x_1 + 3x_2 = 6\\
&\qquad 3y_1 + 4y_2 = 18\\
&\qquad 2y_1 + 5y_2 = 19\\
&\qquad 7z_1 + 5z_2 = 46\\
&\qquad 4z_1 + 8z_2 = 52\\
&x_1 \geq 0, y_1 - x_1 \geq 0, z_1 - y_1 \geq 0\\
&x_2 \geq 0, y_2 - x_2 \geq 0, z_2 - y_2 \geq 0
\end{aligned}$$

Step 4 Using Step 4 of the method, presented in Sect. 2.5, the fuzzy linear programming problem, obtained in Step 3, can be written as:

$$\text{Maximize } \frac{1}{4}(0x_1 + 2x_2 + 2y_1 + 8y_2 + 4z_1 + 5z_2)$$

subject to

$$2x_1 + 2x_2 = 6$$
$$0x_1 + 3x_2 = 6$$
$$3y_1 + 4y_2 = 18$$
$$2y_1 + 5y_2 = 19$$
$$7z_1 + 5z_2 = 46$$
$$4z_1 + 8z_2 = 52$$
$$x_1 \geq 0, y_1 - x_1 \geq 0, z_1 - y_1 \geq 0$$
$$x_2 \geq 0, y_2 - x_2 \geq 0, z_2 - y_2 \geq 0$$

Step 5 The optimal solution of the crisp linear programming problem, obtained in Step 4, is $x_1 = 1, y_1 = 2, z_1 = 3, x_2 = 2, y_2 = 3$ and $z_2 = 5$.

Step 6 Putting the values of x_1, y_1, z_1, x_2, y_2 and z_2 in $\tilde{x}_1 = (x_1, y_1, z_1)$ and $\tilde{x}_2 = (x_2, y_2, z_2)$, the exact fuzzy optimal solution is $\tilde{x}_1 = (1, 2, 3), \tilde{x}_2 = (2, 3, 5)$.

Step 7 Putting the values of $\tilde{x}_1$ and $\tilde{x}_2$, obtained from Step 6, in the objective function the fuzzy optimal value of the fully fuzzy linear programming problem is (4, 14, 37).

2.7 Advantages of the Kumar et al.'s Method

In this section, the advantages of Kumar et al.'s method, presented in Sect. 2.5, over the existing method [1] are discussed.

(i) It is easy to apply the Kumar et al.'s method, presented in Sect. 2.5, as compared to the existing method [1].

(ii) The fuzzy optimal solution, obtained by using the existing method [1], does not exactly satisfy the constraints of the fully fuzzy linear programming problems while the fuzzy optimal solution, obtained by using the Kumar et al.'s method, presented in Sect. 2.5, exactly satisfy the constraints of the fully fuzzy linear programming problems.

(iii) The existing method [1] can be used to find the non-negative fuzzy optimal solution of fully fuzzy linear programming problems (2.1) but cannot be used to find the non-negative fuzzy optimal solution of fully fuzzy linear programming problems (2.8) and (2.9). However, the Kumar et al.'s method, presented in Sect. 2.5, can be used to find the non-negative fuzzy optimal solution of all the fully fuzzy linear programming problems (2.1), (2.8) and (2.9).

Table 2.1 Results of the chosen fully fuzzy linear programming problems

Example	Fuzzy optimal value	
	Existing method [1]	Kumar et al.'s method
2.1	Not applicable	(5, 16, 33, 56)
2.2	Not applicable	(9, 27, 75, 111)
2.3	(1.76, 18.75, 34.86)	(4, 14, 37)

2.8 Comparative Study

The results of the chosen fully fuzzy linear programming problems, obtained by using the existing method [1] and Kumar et al.'s method, presented in Sect. 2.5, are shown in Table 2.1.

The results, presented in Table 2.1 can be explained as follows:

(i) In the problems, chosen in Examples 2.1 and 2.2, all the coefficients are not non-negative triangular fuzzy numbers. So, due to the limitations of the existing method [1], discussed in Sect. 2.3.1, none of these problems can be solved by using the existing method [1]. However, in the problem, chosen in Example 2.3, all the coefficients are represented by non-negative triangular fuzzy numbers. So, as discussed in Sect. 2.3.1, it can be solved by using the existing method [1] but due to the shortcoming of the existing method [1], discussed in Sect. 2.3.2, the obtained results are not exact.

(ii) The Kumar et al.'s method, presented in Sect. 2.5, can be used to find the non-negative fuzzy optimal solution of fully fuzzy linear programming problems with unrestricted coefficients. So, all the problems, chosen in Examples 2.1, 2.2 and 2.3, can be solved by using the Kumar et al.'s method, presented in Sect. 2.5. Also, as discussed in Sect. 2.7, the results obtained by using the Kumar et al.'s method, presented in Sect. 2.5, are exact.

2.9 Conclusions

On the basis of present study, it can be concluded that it is better to use the Kumar et al.'s method [2] as compared to the existing method [1] for solving fully fuzzy linear programming problems with equality constraints.

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