Preface

This book provides a calculus-based introduction to probability and statistics. It contains enough material for two semesters, but with judicious selection, it can be used as a textbook for a one-semester course, either in probability and statistics or in probability alone.

In each section, it contains many examples and exercises and, in the statistical sections, examples taken from current research journals.

The discussion is rigorous, with carefully motivated definitions, theorems, and proofs, but aimed for an audience, such as computer science students, whose mathematical background is not very strong and who do not need the detail and mathematical depth of similar books written for mathematics or statistics majors.

The use of linear algebra is avoided and the use of multivariable calculus is minimized as much as possible. The few concepts from the latter, like double integrals, that were unavoidable are explained in an informal manner, but triple or higher integrals are not used. The reader may find a few brief references to other more advanced concepts, but they can safely be ignored.

Some Distinctive Features

In Chapter 2, events are defined (following Kemeny and Snell, *Finite Mathematics*) as truth sets of statements. Venn diagrams are presented with numbered rather than shaded regions, making references to those regions much easier.

In Chapter 3, combinatorial principles involving all four arithmetic operations are mentioned, not just multiplication as in most books. Tree diagrams are emphasized. The oft-repeated mistake of presenting a limited version of the multiplication principle, in which the selections are from the same set in every stage and which makes it unsuitable for counting permutations, is avoided.

In Chapter 4, the axioms of probabilities are motivated by a brief discussion of relative frequency, and in the interest of correctness, measure-theoretical concepts are mentioned, though not explained.

In the combinatorial calculation of probabilities, evaluations with both ordered and unordered selections are given where possible.

De Méré's first paradox is carefully explained (in contrast to many books where it is mishandled).

Independence is defined before conditioning and is returned to in the context of conditional probabilities. Both concepts are illustrated by simple examples before stating the general definitions and more elaborate and interesting applications. Among the latter are a simple version of the gambler's ruin problem and Laplace's rule of succession as he applied it to computing the chances of the sun's rising the next day.

In Chapter 5, random variables are defined as functions on a sample space, and first, discrete ones are discussed through several examples, including the basic, named varieties.

The relationship between probability functions and distribution functions is stressed, and the properties of the latter are stated in a theorem, whose proof is relegated though to exercises with hints.

Histograms for probability functions are introduced as a vehicle for transitioning to density functions in the continuous case. The uniform and the exponential distribution are introduced next.

A section is then devoted to obtaining the distributions of functions of random variables, with several theorems of increasing complexity and nine detailed examples.

The next section deals with joint distributions, especially in two dimensions. The uniform distribution on various regions is explored and some simple double integrals are explained and evaluated. The notation $f(x, y)$ is used for the joint p.f. or density and $f_X(x)$ and $f_Y(y)$ for the marginals. This notation may be somewhat clumsy, but is much easier to remember than using different letters for the three functions, as is done in many books.

Section 5.5 deals with independence of random variables, mainly in two dimensions. Several theorems are given and some geometric examples are discussed.

In the last section of the chapter, conditional distributions are treated, both for discrete and for continuous random variables. Again, the notation $f_{X|Y}(x, y)$ is preferred over others that are widely used but less transparent.

In Chapter 6, expectation and its ramifications are discussed. The St. Petersburg paradox is explained in more detail than in most books, and the gambler's ruin problem is revisited using generating functions.

In Section 6.4 on covariance and correlation, following the basic material, the Schwarz inequality is proved and the regression line in scatter plots is discussed.

In the last section of the chapter, medians and quantiles are discussed.

In Chapter 7, the first section deals with the Poisson distribution and the Poisson process. The latter is not deduced from basic principles, because that would not be of interest to the intended audience, but is defined just by the distribution formula. Its various properties are derived though.

In Section 7.2, the normal distribution is discussed in detail, with proofs for its basic properties.

In the next section, the de Moivre-Laplace limit theorem is proved, and then used to prove the continuity correction to the normal approximation of the binomial, followed by two examples, one of them in a statistical setting. A rough outline of Lindeberg's proof of the Central Limit Theorem is given, followed by a couple of statistical examples of its use.

In Section 7.4, the negative binomial, the gamma, and beta random variables are introduced in a standard manner.

The last section of the chapter treats the bivariate normal distribution in a novel manner, which is rigorous, yet simple, and avoids complicated integrals and linear algebra. Multivariate normal distributions are just briefly described.

Chapter 8 deals with basic statistical issues. Section 8.1 begins with the method of maximum likelihood, which is then used to derive estimators in various settings. The method of moments for constructing estimators is also discussed. Confidence intervals for means of normal distributions are also introduced here.

Section 8.2 introduces the concepts of hypothesis testing and is then continued in the next section with a discussion of the power function.

In Section 8.4, the special statistical methods for normal populations are treated. The proof of the independence of the sample mean and variance and of the distribution of the sample variance is in part original. It was devised to avoid methods of linear algebra.

Sections 8.5, 8.6, and 8.7 describe chi-square tests, two-sample tests, and Kolmogorov-Smirnov tests.

Preface to the Second Edition

The organization of the book is the same as that of the first edition, except that Section 8.8 is new. It treats simple linear regression in some detail, pulling together and extending the partial strands of earlier discussions in Sections 6.4 and 7.5, which have also been expanded.

We made small improvements in many places to make the text clearer and more precise. This, of course, included the correction of all the known errors.

Many new examples have been added, especially more classical ones, such as the inclusion-exclusion principle, Montmort's problem, the ballot problem, the Monty Hall problem, Bertrand's paradox, Buffon's needle problem, and some new applications, e.g., the Maxwell-Boltzmann and the Bose-Einstein distributions in physics.

In Section 2.2 the previous treatment of the algebra of sets was quite superficial, because we assumed that this material was familiar to most students. Apparently, however, many students needed more, and so we have included a more detailed and rigorous discussion of set operations.

Section 5.3, Functions of Random Variables, was rewritten by adding more examples and omitting the theorems. It seemed to be adequate and pedagogically preferable just to provide brief suggestions for the necessary procedures and to use those in the examples always from scratch, instead of substituting into formulas of theorems.

The first edition had about 370 exercises; we have added about 30 more, especially in sections where their number was inadequate.

The students' online solution manual has been removed, since it was not very useful. Apparently, most students have not even looked at it, and now its removal has created a large number of new exercises available for homework. However, the appendix with brief answers and hints for selected odd-numbered exercises has been retained, and there is a complete online solution manual for instructors.

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