Preface

This book is not a treatise on Lebesgue spaces, since this would not be a feasible work due to the extension of their usage, e.g., in physics, probability, statistics, economy, engineering, among others. The objective is more realistic, being the introduction of the reader to the study of different variants of Lebesgue spaces and the common techniques used in this area. Since the Lebesgue spaces measure integrability of a function, they can be seen as the father of all integrable function spaces where more fine properties of functions are sought.

We can find many books where the subject of Lebesgue spaces is touched upon, for example, books dealing with measure theory and integration. In the literature, we can also find some books dealing with Sobolev spaces and they dedicate, in general, not more than one chapter to Lebesgue spaces. A book dedicated solely to Lebesgue spaces is unknown to the authors. With this in mind, we decided to write a book devoted exclusively to Lebesgue spaces and their *direct* derived spaces, viz., Marcinkiewicz spaces, Lorentz spaces, and the more recent variable exponent Lebesgue spaces and grand Lebesgue spaces and also to basic harmonic analysis in those spaces. We think this will be a welcome to any serious student of analysis, since it will give access to information that otherwise is spread among different books and articles, as well as more than two hundred problems.

For example, one of the attractiveness of Lorentz and weak Lebesgue spaces is that the subject is sufficiently concrete and yet the spaces have fine structure and importance in applications. Moreover, the area is quite accessible for young people, leading them to gain sophistication in mathematical analysis in a relatively short time during their graduate studies. These features, among others, make the subject particularly interesting.

We think it is appropriate to comment on the choice of the writing style and some peculiarities. In the first part dealing with function spaces, we tried to be as thorough as possible in the proofs, although this could sound prolix for some readers. Another aspect is the inclusion of proofs of classical results that deviate from the standard ones, e.g., in the proof of the Minkowski inequality, we used the classical approach via Hölder's inequality for the Lebesgue sequence space and the less-known direct approach for the Lebesgue spaces. We also decided to include a chapter briefly touching upon the so-called nonstandard Lebesgue spaces, namely, on variable exponent Lebesgue spaces and on grand Lebesgue spaces since these are areas where intense research is being made nowadays. The topic of variable exponent spaces became very fashionable in recent years, not only due to mathematical curiosity, but also to the wide variety of their applications, e.g., in the modeling of electrorheological fluids as well as thermorheological fluids, in the study of image processing, and in differential equations with nonstandard growth. Grand Lebesgue spaces attracted the attention of many researchers and turned out to be the right spaces in which some nonlinear equations in the theory of PDEs have to be considered, among other applications.

This text is addressed to anyone that knows measure theory and integration, functional analysis, and rudiments of complex analysis.

Part of the content of this book has been tested with the students from *Universidad Nacional de Colombia* and also from the *Pontificia Universidad Javeriana* in the classes of advanced topics in analysis and also in measure theory and integration.

Since many of the results were collected in personal notebooks throughout the years, a considerable number of exact references were lost. We want to emphasize that the content is NOT original of the authors, except maybe the rearrangement of the topics and some (hopefully a small number) mistakes. If the reader finds misprints and errors, please let us know.

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