Chapter 2
Aerodynamics

2.1 Relative Air Flow

Aerodynamics refers to the study of forces applied to a solid object such as an airplane wing by a gas (typically air) flowing around it. Such a flow can be created either by moving the solid object through the atmosphere, or by blowing the air past a stationary object in a wind-tunnel. Since it is only the motion of the air relative to the object which creates the aerodynamic forces, it is unimportant how the flow has been generated as long its velocity and acceleration relative to the object are the same. For this reason, aerodynamic behavior of an airplane in actual flight can be understood by studying a model of a similar shape placed in a wind-tunnel. The relationship of the actual airplane’s dimensions with those of the model is called scaling, which also affects the flow conditions (density, velocity, temperature, etc.) required for simulating the actual aerodynamic properties in a wind-tunnel test.

The fluid being an infinite medium can have its properties changing from point to point. Let us define the flow properties as those of a tiny volume of fluid called a fluid element. This element can move by translation and rotation, and its shape can also deform due to internal stresses. It is useful to define the properties of the flow far upstream of the object. These are called the freestream properties. The relevant freestream properties which dictate the magnitudes of the aerodynamic forces experienced by an airplane wing are the relative airspeed, the atmospheric density, and air temperature.

When the relative airspeed is quite low, the local variations in the density caused by the air flow can be neglected, and we have what is called an incompressible flow. This is very much like the flow of a liquid which cannot be compressed by applying any amount of pressure, and transmits any pressure changes (called pressure waves) almost instantaneously everywhere. This concept is called Pascal’s law and is the principle behind the working of hydraulic machines such as an automobile jack, where a small force can be used to lift a much heavier object by quickly transmitting the applied pressure through an incompressible hydraulic fluid. Since pressure is force per unit area (see Chap. 1), the force transmitted to the loaded end is much
larger due to its greater area. However, it is important to maintain the applied pressure. If the hydraulic fluid were a compressible gas, some pressure would be inevitably lost in compressing the medium, therefore a smaller pressure would be transmitted for lifting the load. In addition to being inefficient, such a jack would also be a sluggish device, because the spring-like compression of the fluid would significantly delay the transmission of the pressure wave from one end to the other. Similarly, much smaller pressure differences would be created on an airplane wing by a flowing air, if the air behaves as if it is a compressible medium. Furthermore, the pressure waves crossing the air as a compressible medium would be significantly delayed.

### 2.1.1 Compressibility

The speed at which small pressure changes can be transmitted across a medium is called the *speed of sound*. The pressure waves are transmitted by making the particles of the medium vibrate along the direction of motion. When the material particles are tightly bound as in a solid medium, the energy comprising the pressure wave is quickly transmitted, and hence the speed of sound is high. On the other hand, when the medium particles are separated by large distances as in a gas, a small pressure wave would travel much more slowly due to the time taken by the particles to cross the intervening distances. Therefore, a denser medium has a higher speed of sound. The magnitude of the speed of sound would also depend upon the size and structure of the particles, which have to do with its chemical structure. Hence the speed of sound is related to the physical and chemical properties of the medium. In a perfectly incompressible medium, there would be hardly any change in the speed of sound caused by the flow of the medium because its density and chemical composition everywhere are the same. However, if the medium were compressible, it would experience density variations due to the flowing medium, and hence different points in the medium would have different values of the speed of sound. Since greater density variations are produced by a higher flow speed, it is natural to expect that a faster air flow would have larger changes in its local density. If the flow speed were to be so large that in addition to significant density variations, large changes in the temperature are also present which can affect the chemical properties of the medium, then we have an extreme case of compressible flow. Such flows are experienced by objects entering the atmosphere from the space due to their very high speeds. However, the speeds in normal airplane flight are seldom so high as to cause chemical changes in the air, and the latter can be regarded as a *chemically perfect* gas.

From the foregoing discussion, it is clear that the aerodynamic compressibility effects depend upon the flow speed relative to the speed of sound. To classify the flow regimes, it is thus important to define a parameter called the *Mach number*.
as the ratio of the freestream flow speed and the freestream value of the speed of sound. There are the following three categories of the flow, entirely depending upon the Mach number:

(a) Incompressible flow, where the Mach number is negligible (approximated to be zero).
(b) Compressible flow of a chemically perfect gas, where the Mach number is significant, but not so large as to cause variations in the chemical composition of the gas. This category is further divided into the following:
   - Subsonic flow, where the Mach number is less than 1.
   - Supersonic flow, where the Mach number is greater than 1.
   - Transonic flow, where the Mach number is very close to 1.
(c) Hypersonic flow, where the Mach number is very much greater than 1. For air, this regime is defined by Mach numbers larger than 5. The hypersonic regime is usually accompanied by high temperatures, which cause the gas to depart from its constant (perfect) chemical properties found at lower Mach numbers.

In this chapter we will only consider the subsonic flow for simplicity, whereas the essential features of the hypersonic flow will be discussed in Chap. 6 in the context of atmospheric entry vehicles.

### 2.2 Lift and Drag

Lift and drag are the basic aerodynamic forces created whenever a solid object moves in the atmosphere in a fixed direction. As we have seen previously, air consists of a large number of tiny particles called molecules. These molecules are always in a random motion (called Brownian motion) even though a volume of the air (such as in a room) is at rest. The particles being tiny cannot be observed, but their effects are felt by us two distinct ways. One way in which we can feel the air particles is by the atmospheric pressure exerted by them on the skin. This is the force per unit area acting normal to our body surface at a given point. If the object is at rest, all points on its surface have the same pressure, because the random motion of molecules causes them to strike the stationary object in the same way at all points. However, this changes as soon as motion takes place relative to the air. For example, when we move a hand through the air, we immediately feel the increased pressure on the hand in a direction opposite to that of the motion. This is the aerodynamic drag force which arises due to the motion of the object relative to the air, and opposes the motion. We can also feel aerodynamic drag when we stand in a breeze, or directly in front of a fan, in the form of a light brush of air against the part of the body surface which is tangential to the flow of air. This is the second source of drag caused by the brushing (friction) of the molecules against our skin, and is quite different from the impact of the molecules normal to the skin. Thus we have two sources of aerodynamic drag, namely, the pressure and the skin friction.
Lift is the name given to the net aerodynamic force acting normal to the direction of the relative motion to the air which takes place in a constant plane. It is the force which makes airplane flight possible, because it can be made to counter gravity by devising a proper lifting mechanism (wing). As in the case of the drag, the lift also has two fundamental sources, viz. the pressure and the skin friction.

When the motion of an object through the air is not only confined to a constant plane, but also involves a sideways (or lateral) motion, it also experiences a third force called the sideforce which acts in a lateral direction, normal to the flight path. The lift, the drag, and the sideforce are thus mutually perpendicular forces.

In addition to the aerodynamic forces of lift, drag, and sideforce, an object moving through the air also experiences an external torque, that can be resolved into moments about three mutually perpendicular axes, $Oxyz$, fixed to the body (see Fig. 1.5b). These are the rolling moment about $Ox$, the pitching moment about $Oy$, and the yawing moment about $Oz$.

From the foregoing discussion, it is clear that all the aerodynamic forces and moments are caused by the following two basic effects:

- Air particles striking the surface of the solid object, such that an exchange of momentum takes place in a direction normal to the surface. This rate of change of momentum per unit area normal to the surface is called the pressure.
- Air particles sticking to, or dragging along the solid surface, such that an exchange of momentum takes place in a direction tangential to the surface. This rate of change of momentum per unit area tangential to the surface is called the shear stress. The skin friction force experienced by a solid object is the integration (summation) of the shear stress acting at all the points on the surface.

Every object experiences both of these effects, and the magnitude of each effect depends upon the following factors:

- The shape presented by the object to the oncoming airflow.
- Properties of the atmosphere prevailing at that point.
- The flight speed of the object relative to the atmosphere.
- The size of the object indicated by a reference length and a reference area.

For any flight object such as an airplane, the shape is selected to maximize the lift to drag ratio in the normal cruising flight. Of course, there are other design requirements, such as the volume required to house the payload, the engines, the structure, the fuel, and the miscellaneous components required for the airplane’s operation, which prevent the selection of the ideal shape purely from lift and drag considerations.

When the orientation of the vehicle is changed with respect to the flight direction, it presents a different shape to the oncoming airflow, and thus experiences a change in the aerodynamic forces and moments. The orientation of an object of an arbitrary shape can be described by a maximum of three angles relative to the specified flight direction. However, if the object has a plane of symmetry (which almost all flight vehicles have) then it is necessary to use only two angles to represent its orientation relative to the flight direction. As discussed later, these two angles are the
angle-of-attack and the angle of sideslip. The sideslip angle gives the relative velocity component normal to the plane of symmetry, whereas the angle-of-attack is used to resolve the flow into two mutually perpendicular directions (forward and upward) in the plane of symmetry.

The atmospheric properties governing the lift, drag, sideforce, and the aerodynamic moments are the density, the pressure (or temperature), and the viscosity of the air prevailing at the flight altitude. Everything else remaining the same, the aerodynamic forces and moments are directly proportional to the atmospheric density. The atmospheric properties are modeled by assuming a standard variation with the altitude, which is briefly explained in Appendix.

For a given shape and size of the object, its orientation relative to the freestream, and for a specified set of atmospheric properties, the aerodynamic forces and moments are directly proportional to the square of the flight speed.

The reference length and area indicating an object’s size determine the magnitude of the aerodynamic forces and moments experienced by it. The reference area is usually selected to be the wing’s planform area defined as the area of the wing as seen from the top. The reference length is usually a characteristic dimension of the object along the expected flow direction. This length is called the characteristic length. For example, the characteristic length of an airplane wing’s cross section at any spanwise location taken parallel to the freestream is its chord, which is the straight line joining the foremost point (the leading edge) to its rear-most part (the trailing edge). For the whole wing, the characteristic length is selected to be either an average (mean) chord of the wing, or its span measured from tip to tip.

The aerodynamic forces are often rendered non-dimensional by dividing by the product of the density of the freestream, the square of the freestream speed, and the reference area. Usually a factor of \(1/2\) is also used in the division, because it produces the dynamic pressure when multiplying the density and the square of the airspeed. The dynamic pressure has a special place in incompressible flows. Hence, the non-dimensional aerodynamic force is derived by dividing the force by the product of the freestream dynamic pressure and a reference area. Similarly, a non-dimensional aerodynamic moment is obtained through the division by the product of the freestream dynamic pressure, the reference area, and the reference length.

2.2.1 Viscous Flow

The stickiness of a fluid is called its viscosity. A governing parameter for viscous flows is the Reynolds number, which is defined as the ratio of the flow’s momentum per unit cross-section area to its viscosity. Thus the Reynolds number of a flow is related to the shear stress created by the flow. A highly viscous fluid such as
the molasses flowing over an object at a very low speed has a small Reynolds number and hence large shear stress, whereas a gas flowing rapidly over the same object would have a high Reynolds number and a small shear stress. The Reynolds number is directly proportional to the density and the speed of the flow, but inversely proportional to its viscosity. For example, water is 1000 times denser, but only about 100 times more viscous than the air. Hence, the Reynolds number of an object moving with the same speed in water is about ten times higher than that in air. For a given speed, density, and viscosity, the Reynolds number is proportional to the characteristic length. The Reynolds number based upon the mean chord of a typical airliner while cruising is of the order of ten million, whereas that of a small insect can be only 100.

### 2.2.2 Streamlined Shapes

Since air is a gas with a low viscosity, the magnitude of shear stresses on the wing of an airplane is typically much smaller than the exchange of momentum normal to the flow (pressure), because of its high Reynolds number. Therefore, the shape of an airplane wing can be designed to produce the maximum possible lift due to the pressure variations over it. However, even a small viscous shear stress results in a drag that must be balanced by the thrust of the engines, and determines the fuel consumption (hence flight efficiency). The streamlined shape is derived from the technical term streamline, which is defined as a curve in a steady flow whose tangent at any point gives the local flow direction. The streamlines help us in discussing what eventually happens to a steady flow which is uniform far upstream of an object such as a wing. The pattern of a steady flow can be shown in a diagram by a set of streamlines passing around the object under study. The uniform flow far upstream of the wing is depicted (as in Fig. 2.1) by parallel straight lines. When passing a smooth solid surface such as the wing, the streamlines must be parallel to the local surface, because the flow cannot go through it.

When the streamlines faithfully follow the external contours of the flight object, there is the maximum possible exchange of momentum between the flow and the solid object. Such a flow is called an attached flow. By designing the vehicle with flat and thin surfaces like the wings and tails, and slender shapes like the fuselage and nacelles, it is possible to keep the flow largely attached when essentially flying along the longer vehicle dimension (the longitudinal axis). Such a shape is referred to as a streamlined shape, and the generally small angle made by the longitudinal axis with the flight direction is called the angle-of-attack. For a wing, the longitudinal axis is the chord, while that of a fuselage or a nacelle is its center-line dimension. The pressure variations on the wings and tails then give rise to a force normal to the flight direction, which is called the lift. This is the beneficial part of the aerodynamic effects, and enables heavier-than-air flight. By increasing the angle-of-attack at a given speed, it is possible to increase the magnitude of the lift. However, there is a limit on the extent up to which lift can be increased with the angle-of-attack.
Fig. 2.1 A steady and uniform freestream flow passing a streamlined solid object (a wing) can encounter separation from the object due to a steady loss of momentum of fluid elements caused by skin friction. The shear deformation and eventual rotation of the fluid elements sap energy from the flow, thereby creating the skin friction drag and the pressure (or form) drag, respectively.

Unfortunately, even the most well-designed streamlined shape experience an adverse phenomenon caused by the pressure variations along the body surface. This effect called the pressure drag (or form drag) is due to the continuously decelerating flow near the surface caused by the skin friction. As a fluid element adjacent to the body decelerates, it eventually reaches a point where its speed has fallen below that required to provide the necessary centripetal acceleration for hugging a convex part of the body. (Recall from Chap. 1 that the centripetal acceleration around a curve at a constant speed is proportional to the square of the speed.) Therefore, a streamline is no longer able to follow a convex contour, and is said to separate from the body, which is depicted in Fig. 2.1 as an example of the flow past a wing at a large angle-of-attack. The separation of the flow usually happens near the aft part of a streamlined shape, but the point of separation can move upstream if the body presents a higher inclination (angle-of-attack) to the flight direction. As the flow completely separates from the body, it is no longer able to exchange any momentum with the body, and hence the magnitude of pressure variation (thus the lift) caused by the flow falls abruptly. The steep fall in the lift due to the separated flow is accompanied by an increase in the drag. This is the stall condition experienced by all wing-like shapes at a large angle-of-attack.

In order to understand how the pressure drag is created, let us focus on the fluid element depicted in Fig. 2.1, the lower part of which was initially in contact with the body and thus stationary with respect to it. However, the upper part is unrestrained by friction, thus an internal stress is created in the element which tends to deform its shape by a mechanism called shear. Energy is lost in this process, which results in the skin friction drag. The situation is similar to a piece of rubber whose lower extreme is at rest, but the upper extremity is being moved tangentially, causing it to deform as shown in the figure. As soon as the element separates from the body, its lower part is no longer being restrained by a contact with the body. In the process,
its internal shear stress is relieved and its shape springs back to the original one. However, the element now starts to rotate in response to the moment created by the different forces applied at the top and the bottom. This is like a person slipping on ice and tipping over, because the ice cannot provide the necessary tangential force to balance the motion of the upper body. A similar slowing down and eventual separation takes place in all the layers of fluid adjacent to the layer which actually comes into contact with the surface. The rotary motion (called the vorticity) of the separated fluid elements takes away a large part of tangential momentum from the flow, which appears as an increase in the net drag on the body called the pressure drag.

The explanation given here is a simple way of understanding an important phenomenon, where the fluid elements are treated as if they were solid bodies. Actually, fluid mechanics is governed by the considerations of mass and energy conservation, in addition to the momentum conservation (Newton’s laws) applied not to individual particles, but to a large collection of them. A detailed computation of the aerodynamic forces and moments thus requires the solution of the mathematical equations governing fluid mechanics.

2.3 How Is Lift Created?

A simple way to understand the creation of lift is by considering the flow past an airplane wing. Suppose the airplane is moving at a constant speed in a straight line relative to a stationary atmosphere. Since the relative motion takes place at a constant speed in a fixed direction, the air flow experienced by the wing is steady and uniform at every point located far upstream of the wing, as shown in Fig. 2.2. For simplicity, we also assume the wing has a constant chord and an infinite span. In such a case the flow cannot go around the wing tips, hence it is forced to remain in the two-dimensional plane formed by the page. In that case, the aerodynamic characteristics are the same everywhere along the span because there is no spanwise component of the flow. Such a flow is called a two-dimensional flow, and is depicted in Fig. 2.2. The cross section of the wing taken parallel to the freestream (thus

![Fig. 2.2](image-url) A steady and uniform freestream is deflected downward by a positively cambered airfoil, and hence experiences an upward lift force that is equal and opposite to the rate of change of momentum of the flow
normal to the span) is called an airfoil. As shown in Fig. 2.2, an airfoil has a particular shape, which is essentially the same throughout the span. Its front part (the leading edge) is rounded, the top part is curved downward, whereas the bottom part is relatively flatter, and the rear portion ends in a sharp point (the trailing edge). Such a cross-sectional shape of the wing is said to have a positively cambered shape. If both the upper and the lower surfaces are symmetrically curved about the chord plane, the airfoil is said to be symmetrical. In a rare case, one could have a negatively cambered airfoil in which the lower surface is curved upward more than the upper surface.

### 2.3.1 Downwash

For simplicity, we assume that the flow is entirely subsonic, i.e., its speed everywhere is less than the speed of sound. In a subsonic flow, the pressure disturbance caused by the wing can travel upstream, thus the flow can begin to deviate from uniform flow far upstream in such a way as to accommodate the wing. This is evident in Fig. 2.2, where the steady streamlines begin to diverge much before reaching the wing. If this were not the case (i.e., if the flow were supersonic), the flow would experience a “shock” on arriving at the leading edge of the wing. We will discuss supersonic flow a little later. Since the subsonic streamlines smoothly deviate so as to follow the local curvature of both the upper and the lower surfaces of the airfoil, a positively cambered airfoil will produce a downward deflection of the flow as shown in Fig. 2.2.

The net change in the flow direction caused by merely passing the wing can be resolved in two mutually perpendicular directions, as shown by the vector triangles of Fig. 2.3. Let us first take the ideal situation when there is no friction (viscosity) in the fluid medium; thus there is no change in the magnitude of the flow’s velocity...
(i.e., the relative speed) in passing the wing. It is depicted in Fig. 2.3 by the dashed arrows that even in this “ideal” case of inviscid flow, there are both a downward change (downwash) and a forward change (i.e., its decrease in the freestream direction) of the flow’s velocity due to the wing.

The downwash created by the wing causes a net change of momentum of the flow in the downward direction. Because a fluid particle passing the wing experiences a downward (normal) force causing a change in its direction, it applies an equal and opposite reaction to the wing in the upward direction, normal to the freestream by Newton’s third law of motion (see Chap. 1). The summation of the forces normal to the freestream exerted by all the fluid particles is the net lift force on the wing. The magnitude of the lift can be calculated from Newton’s second law of motion by considering the net rate of change of momentum of a mass of fluid passing the wing at any given time, in the direction normal to the freestream.

A careful analysis of the vector diagram describing the flow’s velocity components in Fig. 2.3 reveals that the velocity component along the freestream direction has decreased in the magnitude after passing the wing. This change in the flow’s momentum in the forward direction (i.e., against the freestream) creates a net drag—a force in the backward direction (i.e., along the freestream)—by the Newton’s third law. This force is called the lift-induced drag, because it is produced by the very mechanism that generates lift, namely, by rotating the flow’s velocity vector in the downward direction.

When the fluid’s viscosity is also taken into account, the net drag is much larger than the lift-induced drag, because a decrease in the flow’s velocity takes place due to friction. A fluid particle in the neighborhood of the wing is affected by the friction caused by the wing’s surface, and slows down having passed it, resulting in a smaller velocity of the flow far downstream as compared to that far upstream. This loss of fluid’s momentum results in both the velocity components of Fig. 2.3 decreasing in their magnitudes, which increases the drag and slightly decreases the lift. The decrease in the lift happens because the downwash has a smaller magnitude due to viscosity. Consequently, viscous effects are detrimental to flight as they decrease the aerodynamic efficiency of a wing, which is measured by the lift-to-drag ratio.

2.3.2 Pitching Moment

Apart from producing the lift and drag, the steady flow past a wing also generates a pitching moment about any arbitrary point, O. The reason for this is once again the net rotation of the flow in the downward direction normal to the freestream. The generation of a rotational flow component (called the angular velocity) which induces a downwash far downstream of the wing, causes a rate of change of the flow’s angular momentum in a clockwise direction as we look into the plane of Fig. 2.2. This rate of change of angular momentum of the flow creates a net torque about the wing in the opposite (i.e., counter-clockwise) direction by Newton’s second and third laws, which is the pitching moment. The pitching moment thus
created depends upon the magnitude of the downwash, therefore also on the lift being produced by the wing. If the lift is higher, there is a greater downwash associated with it, and hence the pitching moment is also greater. The variation of the lift, drag, and pitching moment produced by the airfoil in the various flow conditions gives a complete description of its aerodynamic characteristics.

The pitching moment created by the wing depends upon the location of the point, $O$, about which the moment is taken, as well as on the lift. The effect of the drag on the pitching moment is negligible due to its much smaller magnitude, as well as a much smaller moment arm for an airfoil compared to those associated with the lift. The dependence of the pitching moment on the lift can be understood by considering that the lift is independent of the arbitrary location of the point $O$. However, the contribution of the lift to the net pitching moment is different about each location of $O$, due to a change in the moment arm of the lift from that point. Reasoning in this manner, we can surmise that there must be a location of the point $O$ about which the net pitching moment is independent of the lift, its moment arm being zero at that point. Such a point is called the aerodynamic center, and its location is such that there is no variation in the net pitching moment about it, even if there is a change in the downwash (thus the lift). Since the lift for a given freestream properties (airspeed, density, etc.) can change only if there is a change in the angle-of-attack, the pitching moment about the aerodynamic center is also independent of the angle-of-attack. For a thin airfoil in a subsonic flow, the aerodynamic center is located near the quarter chord point measured from the leading edge, and moves near the mid-chord location in a supersonic flow.

The presence of the aerodynamic center implies that in the zero-lift condition, the airfoil creates a pure couple resulting in the pitching moment having exactly the same value about any arbitrary point, $O$. For a positively cambered airfoil, the zero-lift pitching moment tends to lower the nose of the aircraft. This is called a nose-down pitching moment. Since a nose-up pitching moment is considered to be positive by convention, the pitching moment produced by a positively cambered airfoil is negative. A symmetrical airfoil does not have a zero-lift pitching moment, whereas a negatively cambered airfoil has a positive (nose-up) value of the zero-lift pitching moment. Figure 2.4 depicts the lift, drag, and the zero-lift pitching moment on a cambered airfoil.

The location of the aerodynamic center and the value of the zero-lift pitching moment are important aerodynamic parameters when the aircraft’s stability and control are considered. We will return to them in Chap. 3.

2.3.3 Pressure Distribution

In the ideal case of an inviscid flow, the lift is entirely created by the variation of the local pressure exerted by the flow over the wing. Since the pressure is the force per unit area, the summation of the forces acting on the lower surface normal to the freestream direction must exceed that on the upper surface for a positive lift.
Clearly, the lift, the net difference in the pressure distribution between the lower and the upper surface, and the downwash created by the wing are mutually related. An increase in any one of these quantities causes an increase in the other two. For example, an airfoil with a larger camber will generate a larger downwash, thereby increasing both the pressure difference and the lift for the same flow conditions (freestream speed, density, etc.). This is the very mechanism of increasing the lift with the help of movable flaps during take-off and landing. The deflection of leading-edge and trailing-edge flaps increases the effective camber of the airfoil, which produces a larger downwash, and thus a greater lift. The example of a trailing-edge flap is shown in Fig. 2.4, whose deflection increases the lift, the drag, and the zero-lift pitching moment due to an increase in the effective camber of the airfoil. A similar effect is obtained by a leading-edge flap. Large airplanes such as the airliners, and high-speed fighter type airplanes employ both leading-edge and trailing-edge flaps. The increased lift due to the deflection of the flaps helps while taking-off and landing, because it decreases the airspeed required for flight. A similar increase in the lift also helps during maneuvering. In the normal cruising condition, the flaps are retracted, because the same lift can be now provided by the increased flight speed with a smaller effective camber of the airfoil.

The lift and the pitching moment of an aircraft are calculated from the pressure distribution on the wing and the tail. The viscous effects modify the pressure distribution computed by assuming an inviscid flow. Such a change is usually small at the small angles-of-attack required for the normal cruising flight of an aircraft, and can thus be neglected. Therefore, the lift and pitching moment in most cases can be estimated from an inviscid pressure distribution. However, at the larger
angle-of-attack required during take-off, landing, and slow maneuvers (see Chap. 3), the viscous effects can cause an extensive flow separation (as previously explained), and must be taken into account while calculating the pressure distribution.

### 2.4 Vorticity and Circulation

Let us consider what happens to the flow in the two-dimensional case when it is deflected downward by the wing. We can appreciate this by the example of a skier, who was traveling in a straight line at a constant speed before being turned by a ski pole briefly stuck in the snow. The resulting force applied normal to the direction of motion causes a turn in the direction of the force. The skier briefly describes a curve as long as the force is being applied, but returns to a straight-line motion (in a new direction) as soon as the pole is lifted from the snow and the force is removed. By a similar process, the fluid elements passing a wing are turned downward as shown in Fig. 2.2. In the process of being turned by the wing, the elements describe curved paths depicted by the streamlines in the wing’s vicinity.

Returning to the analogy of the skier, when the pole is used to apply a force normal to the direction of motion, a torque is also applied on the skier’s body. This happens because the pole hits the snow a little away from the body of the skier. The torque would cause the skier to trip and rotate about a vertical axis passing through her center of mass, unless another torque is quickly applied by the other pole in the opposite direction.

A fluid element is like a skier with only one pole, because it can interact with the solid body (wing) on only one of its sides. In addition to the force applied to deflect the particle’s trajectory in the normal direction, a tripping action (torque) is also present, which causes the particle to start spinning about an axis passing through its center of mass (as indicated for the separated fluid element in Fig. 2.1). Therefore, the wing supplies both a downward velocity and an angular velocity to all the fluid elements in its vicinity. If the fluid is inviscid, there can be no opposing torque applied by the neighboring elements, and the fluid element continues to spin at a constant angular rate as it deflects downward. This is called an *irrotational flow* where the net vorticity (or angular velocity) of the flow is conserved.

By what exact mechanism can the rotation of an air element affect the lift of a wing in the attached flow situation? As discussed earlier, there are only two ways a fluid can exchange momentum with a solid surface: (a) a normal pressure and (b) a tangential shear stress exerted on the surface. By Newton’s third law, the surface exerts equal and opposite forces on a contacting fluid element. If there is no viscosity in the fluid medium, the elements away from the solid surface can only be affected by the changes in the pressure transmitted to them by the intervening particles. However, all real fluids have some viscosity, thus a shear stress is also applied by one fluid particle to the other along the direction of motion. The magnitude of shear stress experienced by two contacting surfaces is proportional to the difference in the speed (relative speed) of the surfaces. When two adjacent fluid particles are
moving at nearly the same speed, they experience a negligible shear stress. This is the situation in a typical air flow far away from the wing. But very close to the wing, we have the opposite situation. Here the viscous effects are dominant, because large velocity differences exist between adjacent layers of fluid. A fluid element is slowed down very abruptly in contact with the wing’s surface, as if brakes were applied to it. This is due to a large shear stress on the contacting particle because of its large initial speed relative to the wing.

The velocity difference between the fluid layer that has almost come to rest after contacting the wing and an adjacent moving layer causes a shear stress on the moving layer. This shear stress is slightly smaller than what was experienced by the layer which contacted the wing. In this manner, a shear stress of a successively smaller magnitude is experienced by fluid layers farther and farther away from the wing. Due to the difference in the shear stress magnitudes on the vertically opposite sides of a fluid element, a torque is applied on it, which rapidly decreases in the magnitude as the vertical distance of the element from the wing increases. As seen earlier, this torque causes the elements in the vicinity of the wing to begin rotating, while those elements which are a little away from the wing do not experience any appreciable torque, and consequently do not rotate. This is the only mechanism of producing vorticity (or angular velocity) in the flow. In this discussion, we have arrived at the heart of an important idealization. Far away from the solid boundary, viscous effects of an air flow are negligible, therefore it can be treated in the same manner as if it were inviscid. But very close to a solid boundary, the inviscid flow approximation is invalid, and the rotation of the fluid elements under shear stresses must be considered.

It is now clear that lift generation cannot be completely understood unless the rotation of the fluid elements due to the wing is also described. Consider a closed ring formed of specific fluid elements moving with the flow as shown in Fig. 2.5. Far upstream of the wing, the ring is merely translating, i.e., all the elements constituting...
it are moving together with the same velocity. If one were to sum up the flow velocity along (i.e., tangential to) the closed ring, then the result is called circulation. Due to the symmetrical flow pattern far upstream, the circulation is exactly zero there. As the wing is approached, the shape of the ring is distorted, because the fluid elements now have different velocities. In crossing the wing in the two-dimensional (infinite span) case, the ring must break up, because the fluid elements cannot travel in the spanwise direction to go around the wing.

Now consider an instant when the ring of the specific fluid elements is re-formed (shown in Fig. 2.5) and encloses the wing. The circulation in this case will have a non-zero value, because of the asymmetrical flow pattern around the wing. This circulation must have been supplied during the process of breaking-up and re-formation of the ring. Furthermore, the fluid elements enclosed by the ring are themselves rotating due to the viscous shear stress imparted by the wing, as discussed previously. Therefore, there is a relationship between the circulation around a closed curve and the net angular velocity (or vorticity) of the elements enclosed by the curve.

If there is no downwash produced by the wing, the lift is zero due to a symmetrical flow pattern around the wing, and hence the circulation vanishes along a closed curve enclosing the wing. Consequently, we have a direct relationship between the lift produced by a wing and the flow circulation around it.

Termed briefly, vorticity must be created by the wing in order to turn the flow downward. If there were no mechanism of producing vorticity (thus a circulation around a closed curve) by the wing, there would be no lift generated by it. The only mechanism of producing circulation in an attached flow condition is by the viscous interaction of the fluid layer adjacent to the wing. Due to the airfoil shape of the wing, just the right amount of circulation is created by it that leads to the lift without causing a flow separation.

The magnitude of circulation required to produce lift is practically calculated by assuming that the flow leaves tangentially to the airfoil’s chord at the trailing edge. The basis for this assumption—called the Kutta-Joukowski condition—is the visualization of the flow as it leaves the airfoil. The physical process producing this condition is nothing but the viscous shear on the fluid layer adjacent to the wing. If there is no viscosity in the fluid, there can be no circulation, nor any “tripping” of the fluid elements to create vorticity for generating the downwash.

### 2.4.1 Effects of Angle-of-Attack

When examining the effect of the angle-of-attack on the lift, drag, and pitching moment on a given wing, we assume that the freestream conditions (the airspeed, density, temperature far upstream of the wing) remain constant. This implies that we are actually interested in the variation of the non-dimensional force and moment coefficients with the angle-of-attack. Once again we confine the discussion to the subsonic case, and only consider the wings of moderate to large aspect ratios for
simplicity. The circulation, the downwash, the lift coefficient, and the pitching moment coefficient all increase linearly with the angle-of-attack in the attached flow condition. However, as the angle-of-attack is increased to a moderately large value, flow separation takes place, causing a decrease in all these quantities when compared to the attached flow situation prevailing at the smaller angle-of-attack. Hence, the variation of the lift and the zero-lift pitching moment coefficients become curves, rather than straight lines, reaching their respective maximum values at the stalling angle-of-attack as shown in Fig. 2.6. A further increase in the angle-of-attack causes an abrupt drop of the lift, and a variation in the zero-lift pitching moment due to a complete flow separation. This is called the stall condition. The stalling angle-of-attack thus determines the maximum lift. When a trailing-edge flap is deflected, the lift curve shifts nearly upward while maintaining the same linear slope (Fig. 2.6a). The lift at any angle-of-attack and the maximum lift coefficient are therefore increased; however, the stall condition is reached at a smaller angle-of-attack due to increased separation. The angle-of-attack corresponding to zero-lift becomes more negative as the flap deflection is increased. The zero-lift pitching moment coefficient also becomes more negative with the deflection of the trailing-edge flap. The variation of the pitching moment at the stall condition is an important flight characteristic of an airplane or glider, which will be discussed in Chap. 3.

The effect of leading-edge flaps (not considered here) is much more complicated, because it depends upon the wing’s spanwise geometry, and the geometrical shape of the leading edge.

The non-dimensional drag coefficient varies linearly with the square of the lift coefficient for the attached flow case, producing a parabolic shape shown in Fig. 2.7. There exists a lift-coefficient, $A$, corresponding to the minimum drag, $B$. The value of the drag when the lift is zero is called the parasite drag. Figure 2.7 also shows the determination of the lift coefficient, $C$, corresponding to the maximum lift-to-drag ratio. This is obtained by drawing a tangent to the drag polar from the point $A$. The lift coefficient (and thus the angle-of-attack) for the maximum lift-to-drag ratio is an important performance parameter for an airplane or glider, which will be discussed in Chap. 3.

At higher angles-of-attack, the flow separation causes a departure from the parabolic shape of the drag polar, leading to a steep rise in the parasite drag (not shown in Fig. 2.7). In the stall condition, the lift being nearly zero does not produce any change in the drag, and a maximum possible, constant value of the parasite drag coefficient is thus obtained.

### 2.4.2 Finite-Wing Effect

The previous discussion was limited to the two-dimensional flow caused by a wing of infinite span, which did not allow a variation of the flow properties along the span. However, since all airplanes have only a finite span, the flow in the spanwise direction is also important. Such a flow is said to be three-dimensional in nature.
2.4 Vorticity and Circulation

The variation of the non-dimensional (a) lift coefficient, and (b) zero-lift pitching moment coefficient with the angle-of-attack for a positively cambered wing equipped with a trailing-edge flap. The stalling angle-of-attack for a given flap deflection (denoted by the point \( B \)) corresponds to the maximum lift coefficient, \( A \), and decreases with an increased flap deflection. The zero-lift angle-of-attack (\( C \)) also varies with flap deflection, as shown. The zero-lift pitching moment is negative for a positively cambered airfoil, and constant with the angle-of-attack until the stall condition is reached.

The effect of finite span can be understood by examining what takes place near the wing tips as the wing generates its lift. The pressure being larger on the lower surface than that on the upper surface of the wing creates a flow from the bottom to the top around the wing tips, as shown in Fig. 2.8. This flow must necessarily be rotating due to its very geometry of curving around the tips. Recall from Chap. 1 that a centripetal acceleration—thus a rotation—of a fluid element is required every time it has to go around a curve. Therefore, strong vortices are created at each wing.
Fig. 2.7 The variation of the non-dimensional drag coefficient with the lift coefficient for a positively cambered wing. The minimum drag coefficient, $B$, is obtained for the lift coefficient at $A$, while the maximum lift-to-drag ratio occurs corresponds to the lift coefficient at $C$.

![Diagram](image_url)

Fig. 2.8 Looking from the rear of an airplane or glider, the effect of a finite-wing span is seen to produce a circulation around each wing tip due to the difference in the pressure acting on the lower and upper surfaces of the wing. This results in wing-tip vortices which impart a normal flow component (downwash) at all points on the wing, thus decreasing its lift, and increasing the lift-induced drag. The leakage of the flow around the tips, the pressure difference between the upper and lower wing surfaces, and hence the lift are naturally decreased. Furthermore, the downwash induced by the wing-tip vortices creates an additional lift-induced drag by the rotation of the flow’s velocity vector (see Fig. 2.3). The net effects of a finite-wing are thus a decrease of the lift and an increase of the lift-induced drag from their respective values in the infinite span (i.e., two-dimensional flow case).

For a given planform area and flow conditions, a wing of a larger span has a higher lift, and a lower drag than a wing of smaller span. Thus the aerodynamic efficiency indicated by the maximum lift-to-drag ($L/D$) ratio increases with the span for a given planform area. The reason why the gliders tend to have the highest values...
of the wing aspect ratio (the square of span divided by the planform area) is that their range depends only upon the maximum lift-to-drag ratio. However, since a higher aspect-ratio wing achieves its maximum $L/D$ ratio at a much higher angle-of-attack than one with a smaller aspect ratio, the airspeed corresponding to the maximum $L/D$ condition becomes smaller. Consequently, an optimum combination of the aerodynamic efficiency and speed are obtained for most airplanes at moderate aspect ratios (less than 10), rather than about 20 employed in the high-performance gliders.

When operating close to the ground during the take-off and landing, a finite-wing’s tip vortices become weaker in strength due to a partial obstruction of their flow from the ground. Therefore, the $L/D$ ratio is increased appreciably from its value far away from the ground. This increase in the aerodynamic efficiency due to the ground is called the ground effect. The ground effect is appreciated by the pilots when trying to make a short landing, because it causes the flight path to stretch out due to the decreased drag and the increased lift. Due to its seemingly advantageous nature, the ground effect has also been employed in designing special airplanes (such as the Russian Ekranoplan) which can fly close to the ground for a better speed or economy. However, such vehicles can only operate over water for obvious reasons.
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