Preface

We revisit the Callias index formula in connection with supersymmetric Dirac-type operators H of the form

$$H = \begin{pmatrix} 0 & L^* \\ L & 0 \end{pmatrix}$$

in odd space dimensions n, originally derived in 1978, and prove that

$$\operatorname{ind}(L) = \left(\frac{\mathrm{i}}{8\pi}\right)^{(n-1)/2} \frac{1}{[(n-1)/2]!} \lim_{\Lambda\to\infty} \frac{1}{2\Lambda} \sum_{i_1,\dots,i_n=1}^{n} \varepsilon_{i_1\dots i_n} \times \int_{\Lambda S^{n-1}} \operatorname{tr}_{\mathbb{C}^d}(U(x)(\partial_{i_1}U)(x)\dots(\partial_{i_{n-1}}U)(x))x_{i_n}\,\mathrm{d}^{n-1}\sigma(x), \tag{1}$$

where

$$U(x) := |\Phi(x)|^{-1}\Phi(x) = \operatorname{sgn}(\Phi(x)), \quad x \in \mathbb{R}^n.$$

Here the closed operator L in $L^2(\mathbb{R}^n)^{2^{\hat{n}}d}$ is of the form

$$L = \mathcal{Q} + \Phi,$$

where

$$\mathcal{Q} := Q \otimes I_d = \left(\sum_{j=1}^{n} \gamma_{j,n}\partial_j\right) I_d,$$

with $\gamma_{j,n}$, $j \in \{1, \dots, n\}$, elements of the Euclidean Dirac algebra, such that $n = 2\hat{n}$ or $n = 2\hat{n} + 1$. Here Φ is identified with $I \otimes \Phi$, satisfying

$$\Phi \in C_b^2\big(\mathbb{R}^n; \mathbb{C}^{d\times d}\big), \quad d \in \mathbb{N},$$
$$\Phi(x) = \Phi(x)^*, \quad x \in \mathbb{R}^n,$$

there exists $c > 0$, $R \geqslant 0$ such that

$$|\Phi(x)| \geqslant cI_d, \quad x \in \mathbb{R}^n \backslash B(0, R),$$

and there exists $\varepsilon > 1/2$ such that, for all $\alpha \in \mathbb{N}_0^n$, $|\alpha| < 3$, there is $\kappa > 0$ such that

$$\|(\partial^\alpha \Phi)(x)\| \leqslant \begin{cases} \kappa(1 + |x|)^{-1}, & |\alpha| = 1, \\ \kappa(1 + |x|)^{-1-\varepsilon}, & |\alpha| = 2, \end{cases} \quad x \in \mathbb{R}^n.$$

These conditions on Φ render L a Fredholm operator, and to the best of our knowledge, they represent the most general conditions known to date for which the Callias index formula (1) has been derived.

We also consider a generalization of the index formula (1) to certain classes of non-Fredholm operators L for which (1) represents its (generalized) Witten index (based on a resolvent regularization scheme).

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