

# Chapter 2

## BPSK Radar Waveform

In this chapter, BPSK radar waveforms for spectrum sharing are designed, i.e., the problem of designing MIMO radar BPSK waveform to match a given beampattern in the presence of a cellular system is considered. The classical problem of beampattern matching is modified to include the constraint that the designed waveform should not cause interference to cellular system. So in addition to maximizing the received power at a number of given target locations and minimizing at all other locations this work also seeks to null out interference to cellular system through waveform design. The problem of waveform design for MIMO radars to coexist with a single communication system is considered in [1]. This work extends this approach and designs MIMO radar waveforms that can coexist with a cellular system, i.e., waveforms that support coexistence with many communication systems. Two types of radar platforms are considered. First, radar waveform is designed for stationary maritime MIMO radar that experiences stationary or slowly moving interference channel. Due to tractability of interference channel, null space projection (NSP) is included in unconstrained nonlinear optimization problem for waveform design. Second, radar waveform for moving maritime MIMO radar which experiences interference channel that is fast enough not to be included in optimization problem due to its intractability. For this case, FACE waveforms are designed first and then projected onto null space of interference channel before transmission. The performance of BPSK radar waveform for spectrum sharing is evaluated via numerical examples.

The remainder of this chapter is organized as follows. Section 2.1 discusses BPSK beampattern matching waveform design. Section 2.2 presents the synthesis of Gaussian covariance matrix for beampattern matching design problem. Section 2.3 solves the waveform design optimization problem for spectrum sharing. Section 2.4 discusses simulation setup and results. Section 2.5 concludes the chapter.

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## 2.1 Finite Alphabet BPSK Beampattern Matching

In this section, finite alphabet BPSK waveforms are designed for spectrum sharing by considering a uniform linear array of  $M$  transmit antennas with inter-element spacing of half-wavelength. Then, the transmit signal is given as

$$\mathbf{x}(n) = [x_1(n) \ x_2(n) \ \cdots \ x_M(n)] \quad (2.1)$$

where  $x_m(n)$  is the baseband signal from the  $m$ th transmit element at time index  $n$ . Then the received signal from a target at location  $\theta_k$  is given as

$$r_k(n) = \sum_{m=1}^M e^{-j(m-1)\pi \sin \theta_k} x_m(n), \quad k = 1, 2, \dots, K. \quad (2.2)$$

The above received signal can be represented compactly as

$$r_k(n) = \mathbf{a}^H(\theta_k) \mathbf{x}(n) \quad (2.3)$$

where  $\mathbf{a}(\theta_k)$  is the steering vector defined as

$$\mathbf{a}(\theta_k) = [1 \ e^{-j\pi \sin \theta_k} \ e^{-j2\pi \sin \theta_k} \ \dots \ e^{-j(M-1)\pi \sin \theta_k}] \quad (2.4)$$

Now, the power received from the target at location  $\theta_k$  is given as

$$\begin{aligned} P(\theta_k) &= \mathbb{E}\{\mathbf{a}^H(\theta_k) \mathbf{x}(n) \mathbf{x}^H(n) \mathbf{a}(\theta_k)\} \\ &= \mathbf{a}^H(\theta_k) \mathbf{R} \mathbf{a}(\theta_k) \end{aligned} \quad (2.5)$$

where  $\mathbf{R}$  is the correlation matrix of the transmitted signal. The desired beampattern  $\phi(\theta_k)$  is formed by minimizing the square of the error between  $P(\theta_k)$  and  $\phi(\theta_k)$  through a cost function defined as

$$J(\mathbf{R}) = \frac{1}{K} \sum_{k=1}^K \left( \mathbf{a}^H(\theta_k) \mathbf{R} \mathbf{a}(\theta_k) - \phi(\theta_k) \right)^2. \quad (2.6)$$

It is important to realize that  $\mathbf{R}$  can not be chosen freely since it is a covariance matrix of the transmitted waveform and thus it must be positive semidefinite. In addition, the interest is in constant envelope waveform, i.e., all antennas are required to transmit at same power level which translates to same diagonal elements of  $\mathbf{R}$ . Thus,  $\mathbf{R}$  is subject to two constraints, namely,

$$\begin{aligned} C_1 : \mathbf{v}^H \mathbf{R} \mathbf{v} &\geq 0, & \forall \mathbf{v} \\ C_2 : \mathbf{R}(m, m) &= c, & m = 1, 2, \dots, M. \end{aligned}$$

Thus, under the given constraints, a constrained nonlinear optimization problem can be setup to solve beampattern matching problem

$$\begin{aligned} \min_{\mathbf{R}} \quad & \frac{1}{K} \sum_{k=1}^K \left( \mathbf{a}^H(\theta_k) \mathbf{R} \mathbf{a}(\theta_k) - \phi(\theta_k) \right)^2 \\ \text{subject to} \quad & \mathbf{v}^H \mathbf{R} \mathbf{v}, \quad \forall \mathbf{v} \\ & \mathbf{R}(m, m) = c, \quad m = 1, 2, \dots, M. \end{aligned} \quad (2.7)$$

For radar waveform design, this constrained nonlinear optimization problem can be transformed into an unconstrained nonlinear optimization problem by bounding the variables using multidimensional spherical coordinates [2]. Once  $\mathbf{R}$  is synthesized, the waveform matrix  $\mathbf{X}$  with  $N_s$  samples defined as

$$\mathbf{X} = [\mathbf{x}(1) \ \mathbf{x}(2) \ \dots \ \mathbf{x}(N_s)]^T \quad (2.8)$$

can be realized from

$$\mathbf{X} = \mathcal{X} \mathbf{\Lambda}^{1/2} \mathbf{W}^H \quad (2.9)$$

where  $\mathcal{X} \in \mathcal{C}^{N_s \times M}$  is a matrix of zero mean and unit variance Gaussian random variables,  $\mathbf{\Lambda} \in \mathcal{R}^{M \times M}$  is the diagonal matrix of eigenvalues and  $\mathbf{W} \in \mathcal{C}^{M \times M}$  is the matrix of eigenvectors of  $\mathbf{R}$  [3]. Due to the distribution of  $\mathcal{X}$ , the distribution of the random variables in the columns of  $\mathbf{X}$  is also Gaussian but the waveform produced is not guaranteed to have the CE property.

## 2.2 Gaussian Covariance Matrix Synthesis for Desired Beampattern

An algorithm to directly synthesize covariance matrix of Gaussian random variables to generate finite alphabet constant envelope binary phase shift keying (BPSK) waveform for a desired beampattern was proposed by Ahmed et al. [2]. Using the same approach, the Gaussian random variables with zero mean and unit variance,  $x_m$ , can be mapped onto BPSK symbol,  $z_m$ , through a simple relation

$$z_m = \text{sign}(x_m), \quad m \in \{1, 2, \dots, M\}. \quad (2.10)$$

Using results from [2], we have

$$\begin{aligned} \mathbb{E}(z_p z_q) &= \mathbb{E}(\text{sign}(x_p) \text{sign}(x_q)) \\ &= \frac{2}{\pi} \sin^{-1}(\mathbb{E}(x_p x_q)) \end{aligned} \quad (2.11)$$

where  $x_p$  and  $x_q$  are Gaussian random variables and  $z_p$  and  $z_q$  are BPSK random variables. Therefore, the relation between real covariance matrix of beampattern  $\mathbf{R}$  and Gaussian covariance matrix  $\mathbf{R}_g$  is given by

$$\mathbf{R} = \frac{2}{\pi} \sin^{-1}(\mathbf{R}_g). \quad (2.12)$$

The Gaussian covariance matrix  $\mathbf{R}_g$  is generated by the matrix  $\mathbf{X}$  of Gaussian random variables using (2.9). Then BPSK random variables are generated directly by

$$\mathbf{Z} = \text{sign}(\mathbf{X}). \quad (2.13)$$

In [3], the authors propose to synthesize  $\mathbf{R}_g$  as  $\mathbf{R}_g = \mathbf{U}^H \mathbf{U}$  which transforms Eq. (2.7) as

$$\min_{\psi_{ij}} \frac{1}{K} \sum_{k=1}^K \left( \frac{2}{\pi} \mathbf{a}^H(\theta_k) \sin^{-1}(\mathbf{U}^H \mathbf{U}) \mathbf{a}(\theta_k) - \phi(\theta_k) \right)^2 \quad (2.14)$$

where  $\psi_{ij}$  are the variables of the optimization problem and  $\mathbf{U}$  is given by

$$\mathbf{U} = \begin{pmatrix} 1 & \sin(\psi_{21}) & \sin(\psi_{31}) & \sin(\psi_{32}) & \cdots & \prod_{m=1}^{M-1} \sin(\psi_{Mm}) \\ 0 & \cos(\psi_{21}) & \sin(\psi_{31}) & \cos(\psi_{32}) & \cdots & \prod_{m=1}^{M-2} \sin(\psi_{Mm}) \cos(\psi_{M,M-1}) \\ 0 & 0 & \cos(\psi_{31}) & \ddots & & \vdots \\ \vdots & \vdots & & \ddots & \cdots & \sin(\psi_{M1}) \cos(\psi_{M2}) \\ 0 & 0 & \cdots & \cdots & \cdots & \cos(\psi_{M1}) \end{pmatrix} \quad (2.15)$$

### 2.3 BPSK Waveform Design for Spectrum Sharing

This section considers the design of MIMO radar waveforms for spectrum sharing. Two waveform design approaches are considered: one includes spectrum sharing constraint in the optimization problem and the other does not. The motivation and reasons for these two approaches and their impact on radar waveform performance is discussed in the following sections.

We design MIMO radar waveform with the additional constraint of waveform being in null space of interference channel. In addition, we design the waveform without this constraint but project the designed waveform onto the null space of the interference channels.

The MIMO radar is sharing spectrum with a cellular system which has  $N_{\text{BS}}$  base stations, thus, there exist  $N_{\text{BS}}$  interference channels, i.e.  $\mathbf{H}_i$ ,  $i = 1, 2, \dots, \mathcal{K}$ , between the MIMO radar and the cellular system. In this chapter, we consider a MIMO radar that has less transmit antennas than the receive antennas of communication systems,



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**Algorithm 3** Modified NSP and Waveform Design
 

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if  $\mathbf{H}_i$  received from Algorithm 2 then
  Perform SVD on  $\mathbf{H}_i$  (i.e.  $\mathbf{H}_i = \mathbf{U}_i \boldsymbol{\Sigma}_i \mathbf{V}_i^H$ )
  if  $\sigma_j \neq 0$  (i.e.  $j$ th singular value of  $\boldsymbol{\Sigma}_i$ ) then
     $\dim[\mathcal{N}(\mathbf{H}_i)] = 0$ 
    Use pre-specified threshold  $\delta$ 
    for  $j = 1 : \min(N_{BS}, M)$  do
      if  $\sigma_j < \delta$  then
         $\dim[\mathcal{N}(\mathbf{H}_i)] = \dim[\mathcal{N}(\mathbf{H}_i)] + 1$ 
      else
         $\dim[\mathcal{N}(\mathbf{H}_i)] = 0$ 
      end if
    end for
  else
     $\dim[\mathcal{N}(\mathbf{H}_i)] =$  The number of zero singular values
  end if
  Send  $\dim[\mathcal{N}(\mathbf{H}_i)]$  to Algorithm 2.
end if
if  $\check{\mathbf{H}}$  received from Algorithm 2 then
  Perform SVD on  $\check{\mathbf{H}} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}$ 
  if  $\sigma_j \neq 0$  then
    Use pre-specified threshold  $\delta$ 
     $\sigma_{\text{Null}} = \{ \}$  {An empty set to collect  $\sigma$ s below threshold  $\delta$ }
    for  $j = 1 : \min(N_{BS}, M)$  do
      if  $\sigma_j < \delta$  then
        Add  $\sigma_j$  to  $\sigma_{\text{Null}}$ 
      end if
    end for
     $\check{\mathbf{V}} = \sigma_{\text{Null}}$  corresponding columns in  $\mathbf{V}$ .
  end if
  Setup projection matrix  $\mathbf{P}_{\check{\mathbf{V}}} = \check{\check{\mathbf{V}}}\check{\check{\mathbf{V}}}^H$ .
  Get NSP radar signal via  $\mathbf{Z}_{\text{NSP}} = \mathbf{Z}\mathbf{P}_{\check{\mathbf{V}}}^H$ .
end if

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where  $\mathbf{U}$  is the complex unitary matrix,  $\boldsymbol{\Sigma}$  is the diagonal matrix of singular values, and  $\mathbf{V}^H$  is the complex unitary matrix. If SVD results in non-zero singular values, we calculate null space numerically via Algorithm 3. A threshold is defined and all the vectors in  $\mathbf{V}^H$  corresponding to singular values below the threshold are collected in  $\check{\mathbf{V}}$ . Then, the projection matrix is formulated as in [5, 6]

$$\mathbf{P}_{\check{\mathbf{V}}} = \check{\check{\mathbf{V}}}\check{\check{\mathbf{V}}}^H. \quad (2.16)$$

### 2.3.1 BPSK Waveform for Stationary MIMO Radar

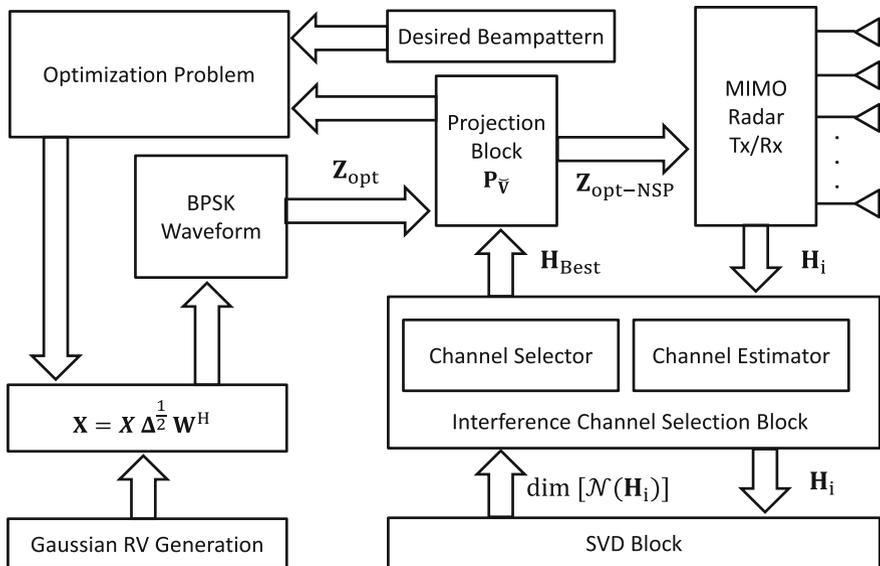
Consider the case of a maritime MIMO radar when a ship is docked or is stationary and thus radar platform is stationary. In this case, interference channels have little

to no variations and thus it is feasible to include the constraint of NSP into the optimization problem. The new optimization problem is formulated by the combination of projection matrix, Eq. (2.16), into the optimization problem in Eq. (2.14) as

$$\min_{\psi_{ij}} \frac{1}{K} \sum_{k=1}^K \left( \frac{2}{\pi} \mathbf{a}^H(\theta_k) \mathbf{P}_{\tilde{\mathbf{V}}} \sin^{-1}(\mathbf{U}^H \mathbf{U}) \mathbf{P}_{\tilde{\mathbf{V}}}^H \mathbf{a}(\theta_k) - \phi(\theta_k) \right)^2. \quad (2.17)$$

This optimization problem does not guarantee to generate constant envelope radar waveform but guarantees that the designed waveform is in the null space of the interference channel or the designed waveform does not cause interference to the communication system. In addition, it is an evaluation of the impact of the NSP on the CE radar waveforms. The waveform generation process is shown using the block diagram of Fig. 2.1. The waveform generated by solving the optimization problem in Eq. (2.17) and then using Eq. (2.9) is denoted by  $\mathbf{X}_{\text{opt}}$ . The corresponding BPSK waveform is denoted by  $\mathbf{Z}_{\text{opt}}$  which is obtained using Eq. (2.13). Then, the output NSP waveform is given by

$$\mathbf{Z}_{\text{opt-NSP}} = \mathbf{Z}_{\text{opt}} \mathbf{P}_{\tilde{\mathbf{V}}}^H. \quad (2.18)$$



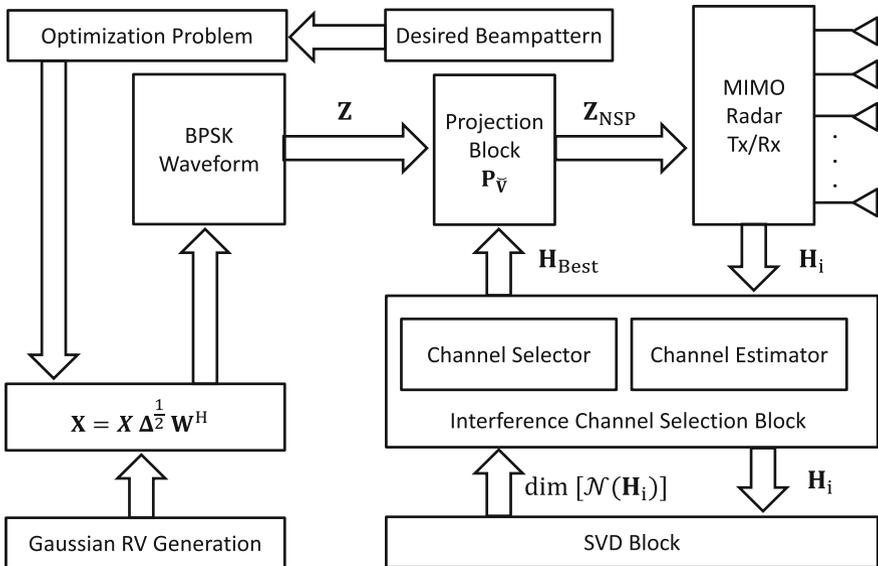
**Fig. 2.1** Block diagram of the transmit beampattern design problem for a stationary maritime MIMO radar. The desired waveform is generated by including the projection matrix  $\mathbf{P}_{\tilde{\mathbf{V}}}$ , for the candidate interference channel  $\mathbf{H}_{\text{Best}}$ , in the optimization process. For this waveform constant envelope property is not guaranteed. The candidate interference channel is selected by Algorithms 2 and 3

### 2.3.2 BPSK Waveform for Moving MIMO Radar

Consider the case of a maritime radar which is moving and thus experiences interference channels that change too fast. In this case, it is not feasible to include the NSP in the optimization problem. Alternately, we can design CE waveforms by solving the optimization problem in Eq. (2.14) and then projection the waveform onto the null space of interference channel. The waveform generation process is shown using the block diagram of Fig. 2.2. Thus, the CE waveform is generated and then projected onto the null space of the interference channel according to

$$\mathbf{Z}_{\text{NSP}} = \mathbf{Z} \mathbf{P}_{\bar{\mathbf{V}}}^H. \quad (2.19)$$

This formulation projects the CE waveform onto the null space of the interference channel. In the next section, the impact of projection on the radar waveform performance is studied.



**Fig. 2.2** Block diagram of the transmit beampattern design problem for a moving maritime MIMO radar. The desired waveform is generated with constant envelope property and then projected onto the candidate interference channel  $\mathbf{H}_{\text{Best}}$  via projection matrix  $\mathbf{P}_{\bar{\mathbf{V}}}$ . The candidate interference channel is selected by Algorithms 2 and 3

## 2.4 Numerical Examples

This section provides numerical examples discussing BPSK waveforms for spectrum sharing. A uniform linear array (ULA) of ten elements, i.e.,  $M = 10$ , is considered with an interelement spacing of half-wavelength. In addition, all antennas transmit at the same power level which is fixed to unity. Each antenna transmits a waveform with  $N_s = 100$  symbols and the resulting beampattern is the average of 100 Monte-Carlo trials of BPSK waveforms. The mean-squared error (MSE) between the desired,  $\phi(\theta_k)$ , and actual beampatterns,  $P(\theta_k)$ , is given by

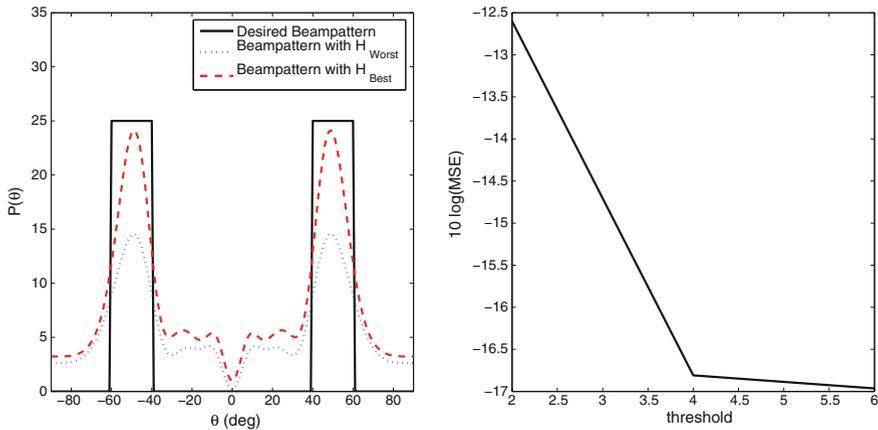
$$\text{MSE} = \frac{1}{K} \sum_{k=1}^K \left( P(\theta_k) - \phi(\theta_k) \right)^2.$$

The interference channels,  $\mathbf{H}_i$ , are modeled as a Rayleigh fading channels with Rayleigh probability density function (pdf) given by

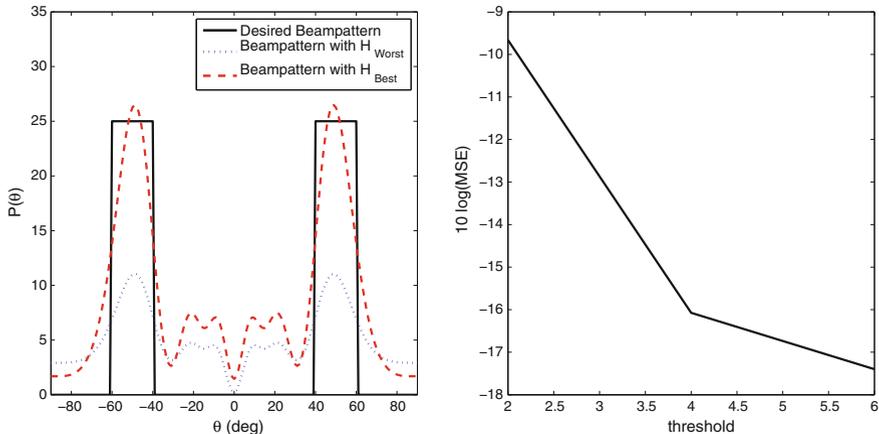
$$f(h|\rho) = \frac{h}{\rho^2} e^{-\frac{h^2}{2\rho^2}}$$

where  $\rho$  is the mode of the Rayleigh distribution. The candidate interference channel,  $\mathbf{H}$ , for waveform design is selected using Algorithm 2 and its null space is computed using SVD according to Algorithm 3.

In Fig. 2.3, the desired beampattern has two main lobes from  $-60^\circ$  to  $-40^\circ$  and from  $40^\circ$  to  $60^\circ$ . It is the beampattern for stationary maritime MIMO radar obtained



**Fig. 2.3** Transmit beampattern and its MSE for a *stationary* maritime MIMO radar. The figure compares the desired beampattern with the average beampattern of BPSK waveforms for null space projection *included* in beampattern matching optimization problem for candidate interference channel  $\mathbf{H}_{\text{Best}}$



**Fig. 2.4** Transmit beampattern and its MSE for a *moving* maritime MIMO radar. The figure compares the desired beampattern with the average beampattern of BPSK waveforms for null space projection *after* optimization for candidate interference channel  $\mathbf{H}_{\text{Best}}$

by solving the optimization problem in Eq. (2.17). The resulting waveform covariance matrix is given by

$$\mathbf{R}_{\text{opt-NSP}} = \frac{1}{N_s} \mathbf{Z}_{\text{opt-NSP}}^H \mathbf{Z}_{\text{opt-NSP}}$$

Note that the desired beampattern and the beampattern obtained by including the projection matrix inside the optimization problem match closely for interference channel  $\mathbf{H}_{\text{Best}}$  than  $\mathbf{H}_{\text{Worst}}$ .

In Fig. 2.4, the desired beampattern has two main lobes from  $-60^\circ$  to  $-40^\circ$  and from  $40^\circ$  to  $60^\circ$ . It represents the beampattern of a moving maritime MIMO radar. Since interference channels are evolving fast, beampattern is obtained by solving the optimization problem in Eq. (2.14) and then projecting the resulting waveform onto the null space of  $\mathbf{H}$  using the projection matrix in Eq. (2.16). The resulting waveform covariance matrix is given by

$$\mathbf{R}_{\text{NSP}} = \frac{1}{N_s} \mathbf{Z}_{\text{NSP}}^H \mathbf{Z}_{\text{NSP}}.$$

Note that the desired beampattern and the beampattern obtained by projecting the waveform onto the null space of interference channel match closely for interference channel  $\mathbf{H}_{\text{Best}}$  than  $\mathbf{H}_{\text{Worst}}$ .

In Figs. 2.3 and 2.4, MSE of beampattern matching design problem is shown. It shows that interference channel with the largest null space have the least MSE. This is in accordance with the methodology to select  $\mathbf{H}_{\text{Best}}$  among  $k$  interference channels using Algorithms 2 and 3.

In Figs. 2.3 and 2.4, the desired beam pattern match closely the actual beam pattern when interference channel  $\mathbf{H}_{\text{Best}}$  is used. Thus, by careful selection of interference channel using Algorithms 2 and 3, when sharing spectrum with a cellular system, we can obtain a beam pattern which is very close to the desired beam pattern and in addition do not interfere with the communication system

## 2.5 Conclusion

In this chapter, we considered the MIMO radar waveform design from a spectrum sharing perspective. We considered a MIMO radar and a cellular system sharing spectrum and we designed radar waveforms such that they are not interfering with the cellular system. A method to design MIMO radar waveforms was presented which matched the beam pattern to a certain desired beam pattern with the constraints that the waveform should have constant envelope and belong to the null space of interference channel. We designed waveform for the case when the MIMO radar is stationary and thus NSP can be included in the optimization problem due to the tractability of interference channel. We also designed waveform for the case when the MIMO radar is moving and experiences rapidly changing interference channels. This problem didn't consider the inclusion of NSP in the optimization problem due to the intractability of interference channel but rather constructed a CE radar waveform and projected it onto null space of interference channel. The interference channel was selected using Algorithms 2 and 3 and results showed that for both type of waveforms the desired beam pattern and NSP beam patterns matched closely.

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