Chapter 2
Theory of Modelling

2.1 Models Are Everywhere

Models are rather frequent tools in science and even daily life. Models are a representative of the reality with certain purposes: in children's games, model cars represent real cars and thus break down the concept of street traffic to a scope that is manageable for children. There is an obvious difference between the use of the word 'model' in this example and that in the term 'climate model'. There is a confusingly wide range of models; indeed, the same word is used for a confusingly wide range of concepts. Clarifying what is referred to as a model in this book is thus necessary first step in our discussion of archaeological modelling. For this purpose, we will provide some examples from geography and archaeology.

In 1798, Malthus [6, 14, 35, 57] proposed his theory of population growth. According to his idea, populations will double every 25 years if provided with perfect conditions. Nonetheless, Malthus also described certain limits; for instance, war and disease reduce the population. Moreover, the food supply is limited and if the population increases a certain extent called carrying capacity, a part of the population will starve. The model postulates an exponential population growth with a threshold at the carrying capacity. Verhulst's logistic equation represents this model [14]:

$$\frac{\partial d}{\partial t} = rP(1 - \frac{P}{C}) \tag{2.1}$$

In this equation, C stands for carrying capacity, r is the rate of population growth and P represents the population.

Innovations can increase the carrying capacity. Although this model has been criticised over the last two centuries, it proves a good basis for predicting population dynamics.

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In the carrying capacity model, the location of people is important: if the population size exceeds the carrying capacity, the population will cease to grow further. Trade can compensate this effect to a certain degree, with urbanization reflecting a good example. Nonetheless, urbanisation also reveals another flaw in the model, namely that people are mobile and attracted to certain places. This problem is addressed by the gravity model, which was adapted from physics to geography and finally to archaeology. In 1686, Newton [39] defined the law of gravitation with which the mutual influence of bodies of matter in motion can be explained and predicted. Newton discovered that two bodies attract each other and that the force of attraction is determined by their mass and the distance between them. For the geographical problem of migration, Ravenstein [44] developed a similar model, formulated as a proper socio-geographical law of gravitation by Stewart [47] in 1948. The idea is that two populations attract each other according to the number of individuals belonging to the populations (p_1, p_2) and the distance (d) between them, with the attraction F.

$$F = \frac{p_1 p_2}{d^2} \tag{2.2}$$

Gravitational laws cover a wide range of applications in geography-related fields of research. Reilly's [45] retail gravitational law is an example for economics, while the work from Diachenko and Menotti [13] is an example of an archaeological application.

There are many other models dealing with location and distance; for instance, Christaller's [11] central place model or Thünen's model [51] is also interested in distances. V. Thünen's does not investigate the attraction but rather the influence of land use in relation to the distance of the market places (Fig. 2.1). The land use depends on the costs of bringing the products to the markets. For each place, the landowner decides for the land use that allows the highest land rate. The land rate (R) is defined by the equation

$$R = Y(p - c) - YTd \tag{2.3}$$

where Y is the yield per unit of land, p is the market price per unit of commodity, c is the production expenses per unit of commodity, T the freight rate and d the distance to the market. For each product with specific yields, prices and costs within a certain range of distances can be found, where the rents are optimal. For example, market gardening has an optimal distance to the market that is lower than the optimal distance for field crops. If the characteristic land use for different distances are on a map like a graph, the famous Thünen rings emerge.

In contrast to the explicit models mentioned above, many implicit or latent models are also used, whereby we will provide some further examples from archaeology and geography. Latent models are used as models, although they are rather hidden. They are very powerful and influential but explicit discussions can seldom be found. We will mention the four main latent models in archaeology [38].

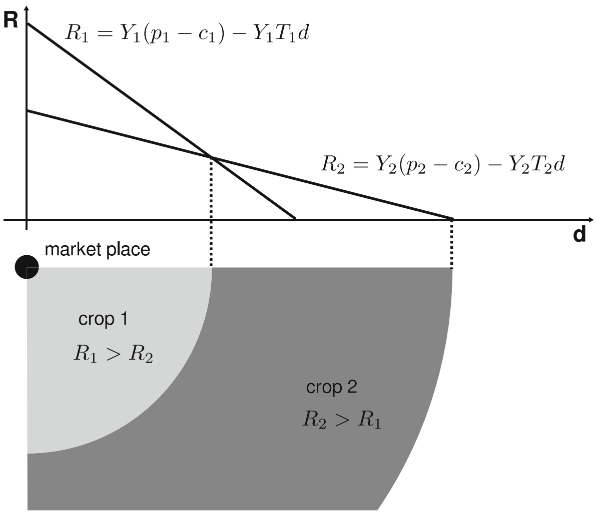

Fig. 2.1 Land rent (R) and land (crop 1 and crop 2) use in relation to the distance of the market place (d), based upon Thünen

Objects with similar diagnostic features are subsumed under a certain type. This classification allows making general statements like 'all finds of type 1 date to phase 4' or 'all finds of type 2 served function b'. However, a type is more than a class; rather, it implies a certain interpretation, such as a chronological or functional interpretation. For all objects of a certain type we assume a certain dating, function, provenance or meaning. This is a simple yet powerful conceptual model.

Upon first glance, it does not seem to involve quantitative aspects. However, this model is a very general one, used to develop other models with more specific elements. For example, the types can be defined using numerical classification. Subsequently, we can reduce the range of interpretations to chronological categories, for example. Finally, we obtain the latent typo-chronological model, which assumes that objects of the same type date to the same time and that similar objects date to phases close to the original one. This model implies a metric for object similarity and time. The measurement of similarity is based upon common features and allows constituting a scale, a typological series. The most simple temporal metric uses relative phases and the number of phases between two points in time as a measure of the temporal distance. The different phases are defined by the presence of different diagnostic types. Using the diagnostic-type-based chronology, the features of the objects condensed in types can be plotted against time. The typo-chronological model is nothing but a regression model with ordinal numbers, which maps the relationship of two parameters. This model, which was developed in the nineteenth century, has subsequently proved very successful—then uses the description of finds only. It is a very sophisticated concept of how to turn formal descriptions of material culture into history. The resulting chronologies proved very good, although some modifications of the relative chronology were necessary. A formalisation of the process of developing typo-chronologies was developed by Sir William Matthew Flinders Petrie [41], known as seriation.

A brief excursion will bring us to a once latent model of geography made explicit by Tobler [52]. He states in his 'first law of geography' that

Everything is related to everything else, but near things are more related than distant things.

Obviously, this is an analogy to the typo-chronological model: rather than 'similar' he uses 'related' and instead of 'time' he uses 'space'. This model is the implicit foundation of interpolation—for example—where related also means similar.

Now we turn to 'culture' which is another important concept in archaeology. In fact, culture is an important term for many disciplines, including geography, ethnology, sociology and cultural studies, although, the traditional archaeological concept of culture is different from that in cultural studies. The much criticised, yet still pervasive, traditional archaeological concept of culture states that people living in a certain area possess the same ideas, the same types of artefacts and the same style of decorations. The area delimited by the cultural border is thought to be homogeneous. The traditional archaeological concept is obviously a simplification that does not fit to reality and is incompatible with the term 'culture' in some other disciplines. Despite this problem, this model can serve certain purposes; for example, it allows observing a pattern in a vast amount of archaeological data and connecting it to certain research questions. One example is the concept of Kossinna [26] and Childe [7], who perceive cultures as actors of history. If you accept some theoretical assumptions of this approach—although we do not—the traditional archaeological concept of culture is a useful model. Another example is Lüning's [32] concept, which interprets cultures as zones of validity for chronological systems. In this concept, the very simple model of culture makes sense; thus, the degree of simplification fits perfectly to the task.

Despite being problematic, the traditional archaeological concept of culture is implicitly present in many archaeological works, even of those who explicitly reject the concept. It is very tempting to structure archaeological observations using this concept as a first step of research. Sometimes the traditional concept is modified to serve certain purposes; for instance, Clarke [9] accepts the cultural model as a mere classification of a set of archaeological observations, although he insists upon heterogeneity and fuzzy borders. While the traditional archaeological concept of culture assumes homogeneity within cultural borders and particularly in small areas and single settlements, another latent model assumes heterogeneity.

The social rank model assumes that the social rank in a certain community corresponds to the wealth expressed by artefacts in a grave. Likewise, as a simplification, this model allows exploring prehistoric societies by constructing hypothetical social structures.

The latent models mainly comprise relationships between basic categories. These relationships are described in a very simplified way which involves unrealistic

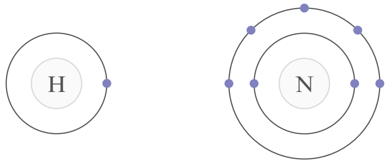

Fig. 2.2 Bohr model of hydrogen and nitrogen

assumptions and does not fit reality. However, the latent models are useful and powerful due to the simplicity of the model, allowing a preliminary structuring of the data. The threat lies in the fact that we mostly use latent models without being aware of it and hence we cannot be cognizant of the natural limits of these models.

Explicit models of different disciplines form a contrast to the latent and implicit models. Physics provides very instructive examples for understanding explicit models. There are two models used to deal with light: one model describes light as an electromagnetic wave, thus explaining interference, for instance; whereas the other model describes light as a particle and is able to explain reflection and diffraction. While these models can be used for different purposes, according to quantum mechanics both particles and waves are possible at the same time, called the wave-particle-duality of light. Hence, a universal model of light involves two complementary parts, each of which describes certain properties of the phenomenon but fails to describe others. It has recently been possible to observe both effects of light at the same time [42].

Another well-known type of model in physics is atom models; for instance, the Dalton model expresses that the element comprises similar small particles that are called atoms and cannot be sub-divided any further. By contrast, the Bohr model shows that atoms comprise of electrons, protons and neutrons and that protons and neutrons form the nucleus, while the electrons are on certain orbits surrounding the nucleus, representing different energy levels (Fig. 2.2). A change of orbit means a change in energy level.

This rather simple model can explain many properties of matter. Despite the fact that models with a higher predictability exist, it remains in use owing to its simplicity.

In chemistry, the mechanical molecule model—introduced by Hofmann [23]—is an example of a very influential chemical model. This model connects to the atom model yet has its focus on another scale. Molecules are visualised using balls and sticks: the balls represent the single atoms—which are coloured differently according to the different elements—while the sticks represent the bonds. Structure formulae like the balls-and-sticks model and chemical formulae in the same manner as simple empirical formulae represent certain aspects of chemical compounds. Different graphical representations of benzene—including the balls-and-sticks model—exemplify models of chemical compounds (Fig. 2.3).

While the atom and the molecule models are focused on small details, climate models deal with objects ranging between the regional and global [24, 29]. In fact, this is a rather marginal difference. More importantly, climate models do not comprise the description of simple relationships of some elements that allow defining a structure very precisely. Climate models involve an enormous amount

C_6H_6

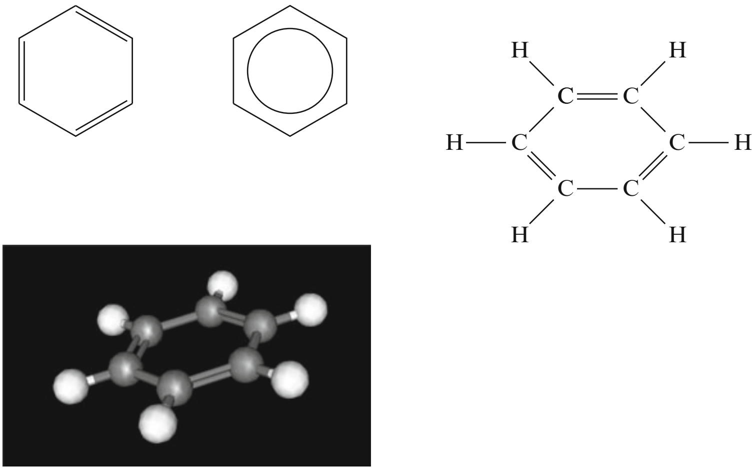

Fig. 2.3 Chemical benzene models

of data and some rules that define the behaviour of the climate system. Even with this amount of data, climate models are less dependable than molecular models. Nonetheless, this does not mean that molecular models are more precise. Simple molecular models do not care for scales and angles, but they are suitable for describing the chemical bonds between different types of atoms.

Differential equations define the interrelationship between different parameters in climate models. Hence, it is possible to calculate the effect of a changing parameter. A set of differential equations and certain initial conditions allow calculating the state of the climate system for any point in time. Numerical simulations are used in climate models, whereby the result depends on the initial conditions. In complex models like climate models, non-linear relationships, emergence from certain patterns and *butterfly effects*—or more precisely, the sensitive dependence on initial conditions—are possible. This means that the probability of the simulation result declines with the increasing temporal distance to the starting point in time. We know this effect from the daily weather forecast, which is not convincing for a longer period, owing to butterfly effects. Initial conditions differing in small details can cause completely different states of the system after a short period. One example of climate models is atmospheric general circulation models (AGCM), which comprise a number of differential equations.

Now we move on from the climate component of environment to a geological component. Process-based models of coasts model the morphological change of coastal areas [36]. These models couple different components concerned with currents, waves and sediment transport. The models are simulations based upon initial states and differential equations and are used similarly to the climate models.

Another example of simulations is *cellular automata*. The idea is to define a grid and rules for the behaviour of the grid cells, whereby the value of a cell depends on the value of other cells. In an iteration, the values of all cells are calculated. The complex interrelationship between cells can lead to a dynamic pattern, which changes in every step of iteration. The famous ‘game of life’ is an application of cellular automata developed by J. H. Conway in 1970 [10, 18]. Cells can be dead or alive, whereby the state of a cell depends on the state of the surrounding cells. During the simulation, characteristic patterns can emerge that cannot be deduced from the rules. This is a characteristic of a complex system.

Agent-based modelling [16, 40] is related to cellular automata, though they allow agents that preserve their identity while moving. We can think of cellular automata as being a special kind of agent-based models where the agents are not allowed to move but need to have different states. Agent-based models are defined by the rules of behaviour of the individual agents and can produce emerging patterns. In case of cellular automata as well as agent-based models, the study of complex systems is the most interesting application. The bottom-up approach allows connecting certain system properties and patterns to individual behaviour. In the case of simulations that use system parameters, this is much more difficult.

The rules defined within an ABM are not a statement about the goals of the agent; rather, they concern how the agent acts to reach a goal under specific conditions. The *homo economicus* will purchase the cheapest good from a set of similar goods. This principle is applied even if the goal is getting some rain by offering goods to certain gods: if the gods possess the same power, the homo economicus will offer to the god with the most moderate demands.

Despite the fact that the homo economicus is rather unrealistic as a general concept, it remains in use for understanding economic mechanisms such as markets. Indeed, certainly the homo economicus certainly remains a component of the description of more realistic actors.

In biology, the development of populations is a common topic in which modelling approaches are applied. In particular, the interrelationship between the populations of two species is addressed by models and simulations. In this case, the behaviour of coupled systems is estimated using differential equations. The most famous example is the predator–prey model to which the Lotka–Volterra equations [31, 55] are applied. The idea is that there is a growing population of prey, mostly exemplified as rabbits, and a growing population of predators, such as wolves. The growth of rabbits depends on the number of wolves, since the rabbits are the wolves’ food. On the other hand the wolves’ growth depends on the number of rabbits: the more wolves there are, the lower the growth of the rabbit population, while the more rabbits there are, the higher the growth of the wolf population. This interrelationship can be modelled with two differential equations:

$$\frac{\partial x}{\partial t} = ayx - bx \tag{2.4}$$

$$\frac{\partial y}{\partial t} = cy - dxy \tag{2.5}$$

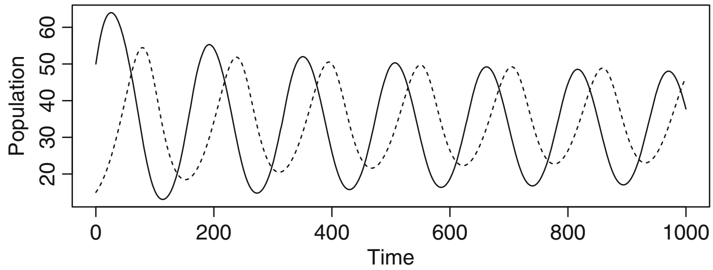

Fig. 2.4 Typical development of the population of two species in the predator–prey model. Prey: *solid line*; predator: *dashed line*

In the equations, x is the population of the predator and y the population of its prey. The other parameters control the behaviour of the system. A usual system behaviour is an alternating up and down of the populations where the predator shows a certain delay (Fig. 2.4).

Epidemic models [50, 53] can serve as an example from medicine. The models of animal epidemics have two main components: first, the process of the disease, which includes—for instance—infected, infectious and diagnosed as characteristics and second, the network of contact, which includes the paths of spread.

The final example is taken from social science and has been transferred to many other disciplines. The concept of *ideal types* was proposed by Max Weber [56]. Ideal types are models of certain facts or objects that represent the extreme or pure application of a single property or principle. Ideal types do not exist in reality but are useful to understand the mechanisms and relationships of some phenomena. They are based upon the one-sided accentuation of a certain point of view.

It seems that the term 'ideal type' is something very similar to the term 'model'. Ideal types appear to be a special type of model embedded in another terminological framework.

While this brief collection of some models is naturally incomplete it offers a notion of the range of different types of models. Obviously, there are many terms of models in use. Is 'model' a fancy term without real content or a fuzzy 'term' covering a field of meanings rather than one actual meaning, or is there a general term of 'model' that is varied in the different disciplines? The choice of successful and traditional examples indicates that fanciness is at least not the major aspect of the term 'model'. After providing an overview of some models from different disciplines, it is time to explore the meaning of the term 'model'. Subsequently, we will be able to apply the modelling concept in a scientific framework and discuss basic methods used in landscape modelling.

2.2 What Is a Model?

The term ‘modelling’ is becoming increasingly fashionable. An exploration of the keyword ‘modelling’ in the Library of Congress reveals that modelling became popular in the 1960s. We should mention that ‘model’ was introduced into mathematics (mathematical logic) in 1933 by the Polish logician Alfred Tarski [48]. His theory became important in the 1950s during the course of the work on artificial languages (for example, programming languages). Tarski’s English papers from the 1950s certainly helped to distribute his ideas. In addition, one related process increased the importance of the term ‘modelling’, namely the quantitative revolution that took place in the 1950s and 1960s, which was enabled by the development of computer systems.

In many disciplines, a peak in the 1970s marked the first hype of quantitative modelling. In the 1980s, there was a break induced by post-modern theories. Since the 1990s, the importance of ‘modelling’ has continuously increasing again.

We now know that the term ‘model’ is very fashionable, although we do not know what a model is. Accordingly, we have to explore the meaning of ‘model’. Tarski [48, pp. 11–12] defines it as:

> *A possible realisation in which all valid sentences of a theory T are satisfied is called a model of T.*

This is a definition from the mathematical logic and it hardly applies to all disciplines. A very pragmatic and useful characterisation of ‘model’ comes from Herbert Stachowiak [46], who claims that a model:

- is a mapping;
- is a reduction and
- is pragmatic.

We can use this as a first simple standard definition of model which might be sufficient for some contexts and can be replaced by a more sophisticated definition, if required:

Definition 2.1. A **model** is a simplified mapping for a special purpose.

The entire concept of Stachowiak—including his definition of ‘model’—is embedded in system theories. Stachowiak explicit definition [46, pp. 322–323] is:

O_1 and O_2 are objects and O_2 is a model of O_1 for k in the time interval t regarding a certain purpose Z if in t k:

1. is L-rational
2. performs a description P_1 of O_1
3. performs a description P_2 of O_2
4. performs a mapping of P_1 on P_2
5. performs a transcoding of P_1 in P_2
6. performs the substitution of O_1 by O_2

7. performs certain operations on O_2 in order to fulfil the purpose Z and which transfer O_2 to O_2*
8. performs a description P_2* of O_2*
9. performs the reverse mapping of P_2* on P_1*
10. accepts P_1* as the description of O_1*
11. accepts the substitution of O_1* by O_2*
12. performs a recoding of P_1 in P_2 regarding P_1* and P_2*

Obviously, his characterisation seems more comprehensive than the formal definition with the twelve conditions of determination.

Balzer [5] formulated a definition related to Tarski's original definition from the field of philosophy of science. Accordingly, a model:

- is a structure of a theory;
- complies with a set of hypotheses and
- is not a simple statement.

For Richard Chorley and Peter Haggett, a model is a 'simplified and intelligible picture of the world' [8, p. 22]. This picture allows an overview by decomposing the original into details and merging selected details again for a certain purpose. Due to the selection, models have a limited range of applications. Chorley and Haggett characterise models with some key terms. We have already mentioned the selectiveness of models, which allows focusing on certain aspects, while they are not valid for others. In addition, models are structured because they exploit the connection between significant aspects of the real world. This also means that models are an approximation of reality and an analogy of the real world. They are suggestive because they suggest extension of the model and re-applicable because they can be applied to other subsystems of the real world. The main function of models can be seen in bridging between observation and theory. The validation of models is based upon the fact that models are able to predict reality. Certain models with a very high probability are called laws. Since models have a limited probability and a limited range of application, validation based upon prediction becomes a key feature.

In archaeology, David Clarke [9] was the first to deal with models explicitly. He followed the ideas of his friend—the geographer Peter Haggett [8]—but focused particularly on the models' ability to predict. Clarke claims that models are best characterised by:

- comprehensiveness;
- predictiveness;
- efficiency and
- accuracy.

It is obvious that the origin of this characterisation is the quantitative revolution of the 1960s. In this context, models are a mapping of real-world processes, which allows predicting system states for another moment.

In the concept of Mahr [33, 34], the usage of models is the key to understanding their nature. Mahr states that any object can be used as model if it is assumed that it represents another object in terms of certain aspects. He speaks of 'models for' if the models are used as a template for another object and 'models of' if they map or picture an other object. The virtual model is represented by one or more model objects. In addition, this distinguishes between the perspective of producing and applying a model. Mahr connects the viewpoint of induction with that of deduction. The cargo of a model is the information, which is transferred from the inductive component to the deductive component.

Frigg and Hartmann [17] distinguish between representational models of phenomena and data and models of theory. Rather than providing a definition, they describe certain types of models, among which are fictional objects, physical objects and equations. It seems that these types of models are completely different things. The conclusion of Frigg and Hartmann's article [17] might serve as an example of wide-spread opinion about the confused nature of models:

> Models play an important role in science. But despite the fact that they have generated considerable interest among philosophers, there remain significant lacunas in our understanding of what models are and of how they work.

Björn Kralemann and Claas Lattmann [27, 28] adopt a semiotic approach, claiming that models are a specific kind of sign, namely the icon. Charles Peirce classifies signs into icons, symbols and indices: while an icon refers to its object through similarity, an index refers through factual connection and a symbol through a norm. This concept picks up the mapping function from Stachowiak.

According to Thalheim and Nissen [49], *models are artefacts, representing a part of the real world*. These artefacts are used as tools for a certain purpose. The relationship between the model and original is an analogy. A model is connected to a community of practice, which developed and uses the model. This practice-based concept of a model was developed with a bottom-up approach, using a large number of specific modelling approaches in Kiel. The Kiel term of a model is intended to cover a wide range of interdisciplinary applications. The process of developing models and negotiating their use and meaning is perceived as something like Wittgenstein's language games. The community of practice defines the criteria that enable the model to be accepted and used. The model as an instrument has to be adequate, dependable and well-formed, whereby the latter term means that an artefact serving as a model should obey some formal rules. Adequate models serve the purpose defined by the objective. The analogy used in a model is significant for the objective and the representation is more simple, clear, abstract, focused, usable or intelligible than the original. For dependable models we have valid reasons to believe that they will serve the purpose, whereby empiric evidence, coherency, fallibility and stability are criteria for dependability.

Models can have different functions, most prominently including description, explanation, optimisation, verification, representation, reflection and documentation. Application scenarios are distinguished from function, although they are obviously related. Possible scenarios are explanation, designing, prediction and description. The background of a model is formed by paradigms, theories and principles, which are called bases, on the one hand, as well as principles, culture and common sense—called foundations—on the other. The science and art of modelling is divided into three facets. A model itself—to repeat the definition—is an artefact used as tool for a certain purpose. To model is an activity that covers the model development, including the optimisation, merging, specialisation, generalisation and presentation of models, as well as their usage. This facet is the practice embedded in a certain culture. Modelling is a technique based upon certain principles and tightly connected to the purpose. Although, this concept cannot be condensed in one definition sentence, we will use this as the second—more elaborated—standard concept of model.

2.3 Types of Models

There is a multitude of model classifications; accordingly, we can only discuss a small, yet very important selection of classifications here. We will start with a dichotomy that runs like a thread through the history of science: theory and empiricism. It is necessary to distinguish between theoretical and empirical models (Fig. 2.5). Theoretical models—also known as *models for*, ideal models, constructions or templates—apply principles. Observations are only used to set up the model's outline. The model is formed based upon theoretical considerations. Theoretical models are related to the method of deduction. A theoretical model allows determining what would be good, rather than what is really the case.

Empirical models map observations and they are also known as *models of*, real models, reconstructions or mapping. Empirical models are related to the method of induction and they allow us to describe what is, while we not understand why.

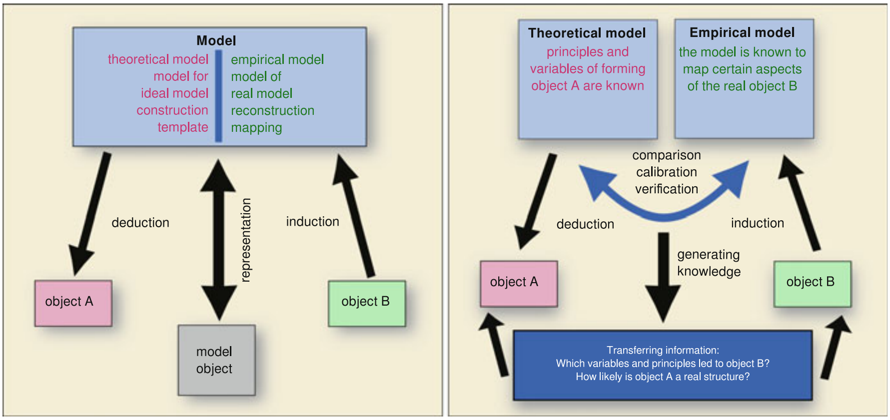

Fig. 2.5 Two main types of models and model comparison

In most cases, a model is represented by a model object. This is a material representation like a model railway, a constructional drawing or a person who sits for a painter. Sometimes the model object simplifies the decision concerning whether it is a theoretical or empirical model. In some cases, this is not so easy because it depends on the degree of empirical and theoretical input, as well as the purpose of the model. A model object can be used as a theoretical model for one purpose and an empirical model for another. Regardless, we have to establish that our model can be used in the intended way and thus serve the intended purpose. This is very important because a misclassification leads to misinterpretation. A model that has to be used as theoretical model due to the limited input of empirical data cannot be used as an empirical model. We would misinterpret the model as a description of an observation. A classical example is the usage of Thiessen polygons to find borders, which is not possible since the method allows establishing where borders should be drawn to minimise distances between related points, but not to establish the position of real borders.

At this point, it seems that models have a very limited account of explanation. Theoretical models are only a mapping of some ideas and empirical models do nothing but duplicating the original with reduced characteristics. Knowledge emerges when models are compared. If we find that a theoretical model fits to an empirical model, then we can transfer the knowledge about principles and variables from the theoretical model to the empirical model. We can denote the resulting model as *conclusive model* or *interpreted model*. An interpreted model provides us with the requirements to understand the object that we have modelled. This step of comparison can be realised as direct comparison, calibration, verification or validation.

We will only mention some of the other classifications. Models can be static or dynamic, depending on time, as possible a variable. Moreover, models can be discrete or continuous. Stochastic models deal with probability and contingency, while deterministic models aim to be exact or unique. Models can aim at optimisation or equilibrium. Models can be classified by mathematical techniques, namely linear equations, non-linear equations or differential equations.

Kai Velten [54] developed the more elaborated three-dimensional SQM classification of models, whereby S, Q and M represent dimensions of the model classification. We only mention the criteria of the dimensions:

S: system

- physical–conceptual
- natural–technical
- stochastic–deterministic
- continuous–discrete
- dimension
- field of application

Q: objectives

- phenomenological–mechanistic
- stationary–instationary

- lumped–distributed
- direct–inverse
- research–management
- speculation–design
- scale

M: mathematical structure

- linear–non-linear
- analytical–numerical
- autonomous–non-autonomous
- continuous–discrete
- differential equations
- integral equations
- algebraic equations

Finally we have to deal with complexity. Models involve mapping a part of a complex reality. Developing complexity is a research process stepping towards more complex models. Higher complexity means:

- a better mapping of complex cases;
- sensitive measures of correspondence of empiric and theoretic models;
- an increasing amount of new knowledge and
- a pretentious demand for data and methods.

Reducing complexity is the concept of extracting relevant information from models (Fig. 2.6). Models with reduced complexity can represent the significant relations of a complex reality and hence have an analogical meaning as eigenvectors for a square matrix.

We can distinguish between five types of models in a chain of developing complexity.

1. Mapping models do nothing other than map an object. An example is archaeological documentation.
2. Emergent models allow producing new knowledge by comparing models.
3. Quantitative models introduce mathematical methods.
4. Cybernetic models allow reciprocal effects.
5. Complex models allow non-linear interdependency, memory-effects, emergent phenomena in systems and other features.

Which degree of complexity is appropriate for a model depends on the research question as well as the data. We have to find this optimal degree of complexity for each application. In principle, one would develop a high degree of complexity first and subsequently reduce the complexity until the optimal degree is reached. In practice, the effort involved for this is often too great, whereby the researcher stops at the assumed optimal degree of complexity.

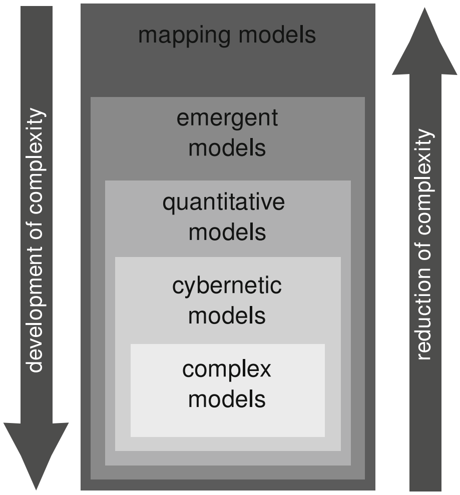

Fig. 2.6 Developing and reducing complexity

2.4 Usage of Models

Modelling provides us with a specific terminological and conceptual framework that is able to significantly reduce the complexity of the research process. Moreover, models offer further benefits:

- increasing knowledge;
- transferring of knowledge;
- practising interdisciplinary research;
- handling complex data and problems and
- working at the cutting edge of research.

For the usage of models there are some rules of thumb that we denote as directions of modelling. These guidelines aim to avoid pitfalls and increase the quality of modelling, stating that one should consider:

- the purpose of models;
- the limit of model types;
- the complementarity of model types;
- the empirical verification of theoretical models and
- purposive developing and reducing complexity.

2.5 Models Between Theory and Method

Sometimes modelling is understood as the mere application of certain techniques, although, we reject this perspective. Like all other methods—to keep this restriction for a moment—a model is based upon a certain theory that serves to solve a problem.

The concept of Thalheim and Nissen [49] makes clear that models include certain theories, methods (for development and application) and objectives. In many cases, data also has to be considered.

Theories are necessary to determine the parameters of the method and interpret the results. The method connects objective, theory and data. The method has to be appropriate for the specific data and the theory of the investigation and has help answering the research question. The data has to be helpful for solving the problem based upon theory and method. A fine-tuning of the four elements of objective, theory, method and data is essential for the successful usage of methods, as well as quantitative methods in general [43].

While many issues can be discussed without a reference to models and modelling, the term 'model' provides a terminological framework that makes it more comprehendible and efficient to discuss some points. In particular, the Kiel term of model [49] offers a concept that can be used to develop a consistent background and structure of an application.

2.6 Examples

We started this chapter by providing some examples to offer a potential notion of models. Now we end this chapter with some examples to show the potential of models in archaeology and geography. Some examples are directly linked to those at the beginning of this chapter, although, by now the reader should have some background knowledge on modelling and hence be able to interpret the examples in a different way.

One of the most virulent topics in archaeology is the process known as neolithisation. From a local to regional perspective, this is the transition from the mesolithic to the neolithic way of life [22], which can be caused by external and internal factors or a combination of both. From a supra-regional perspective, the relationship between the time and location of the transition can be investigated and modelled. The process is perceived as a diffusion. Diffusion models can describe the movement of both people and cultural traits and they are used to describe different phenomena. Fisher [15] developed the concept of *the wave of advance* for modelling the spread of advantageous genes. Accordingly, Fisher published an equation that has to be satisfied:

$$\frac{\partial p}{\partial t} = k\frac{\partial^2 p}{\partial x^2} + mp(1-p) \tag{2.6}$$

In this equation, p is the frequency of a mutant gene, t the time, x a spatial coordinate, m the intensity of selection and k the diffusion coefficient. A similar equation was developed by Kolmogoroff et al. [25] and hence the equation is known as the Fisher–Kolmogoroff–Petrovsky–Piscounoff or FKPP equation. Many other authors have used this concept and in archaeology Ammerman and Cavalli-Sforza [2, 3] adapted the idea to model neolithisation, proposing a similar equation:

$$\frac{\partial p}{\partial t} = kp(1 - \frac{p}{c}) \tag{2.7}$$

In this equation, k is the growth rate and c the maximal p-value, also known as the *carrying capacity*.

The mentioned works allow an analytical solution that provides the frequency of the new feature at a certain place and time. More recent articles employ simulations, Bayesian inference or additional information like variable coefficients [1, 4, 30].

A similar diffusion process was developed by the geographer Torsten Hägerstrand [19, 20], who dedicated some of his work to modelling the diffusion of innovation. Although the basic mechanism was similar to the wave-of-advance model, Hägerstrand focused on other aspects. In particular, he does not assume a uniform wave; rather, he also considers diffusion in networks. His work comprises three major parts: empirical models of the diffusion of innovation, Monte Carlo simulations and conceptual models of the process of diffusion. In contrast to the wave-of-advance model, Hägerstrand is not interested in the arrival or dominance of a certain innovation, but rather in the dynamics of the spread of innovation. Which factors influence the spatial and temporal patterns of the diffusion of innovation? Hägerstrand's diffusion model still serves as the basis for further development [12].

The next example is based upon the predator–prey model and in fact it is an extension of this classical model. The HANDY (human and nature dynamics) model [37] aims to understand crises of populations and cultures. It starts with a re-interpretation of the original predator–prey model. The human population is interpreted as a predator while nature is interpreted as prey. In addition to this ecological component, HANDY has an economical component, whereby the population is divided into two parts: commons and elites. The elites are defined by wealth and both groups have different consumption rates. In contrast to the predator–prey model, HANDY comprises four differential equations:

$$\frac{\partial x_C}{\partial t} = \beta_C x_C - \alpha_C x_C \tag{2.8}$$

$$\frac{\partial x_E}{\partial t} = \beta_E x_E - \alpha_E x_E \tag{2.9}$$

$$\frac{\partial y}{\partial t} = \gamma y(\lambda - y) - \delta x_C y \tag{2.10}$$

$$\frac{\partial w}{\partial t} = \delta x_C y - c_C - c_E \tag{2.11}$$

In these equations, x is the human population, y the nature, w the wealth and c the consumption. The indices E for elites and C for commons can qualify x and c. Different scenarios can be applied to the model; for example, an egalitarian society, equitable society and unequal society. These models—particularly when applied in simulations—teach us about the relationships between the parameters used under conditions of crises as well as stability.

We return to the Thünen model to exemplify some of the details discussed above. According to Stachowiak, a model is a reduced mapping for a certain purpose. The purpose is obviously to discuss the main parameters that influence the land use in relation to the distance to the market. The relationship between the land use and distance is mapped and mainly comprises some equations concerning the rent for different products and the maximisation of the rent. The reduction is the selection of only a few parameters that seem to be significant for the topic. For instance, topography which influences the transportational costs is excluded as well as other markets.

From Haggett and Clarke's perspective the Thünen model allows predicting land use. Owing to the reduction, the accuracy of this model is rather low. The quality of the Thünen model for prediction also depends on the scale employed. It might produce a rough picture that is sufficient for some purposes, but it will definitely not be useful for other purposes. From this perspective, the Thünen model seems to be a weak model. However at this point we are making a mistake; namely, the purpose of this model is not prediction but rather to 'discuss the main parameters which influence the land use in dependence of the distance'. We would be misusing the model in applying it for prediction.

We can learn from Mahr [34] that it is a theoretical model. It might be based upon empirical observations, although the model itself comprises a simple theoretical rule concerning how to optimise land use. If we find that the theoretical model matches the empirical observations, we learn that the theoretical rules also apply to the empirical case; otherwise, parameters not included in the model might be important or the optimisation is not intended, which is rather strange. In addition, Mahr shows us that a model is represented by a model object. In the Thünen case, the equations or the ideas that they express are the model, while the map with the Thünen rings is the model object. It should be mentioned that using another metric that uses another definition of distance is also possible; for instance, we could define a distance that respects topography. The model would be the same, as would the map of Thünen rings in the space spanned by the metric in use. However, we could also plot the rings on a geographical map, whereby they would no longer be circles. This example stresses the importance of the definition of space; indeed, we will connect to this idea at several places in this volume.

From Thalheim and Nissen's [49] perspective, the model is an artefact that involves a set of equations. This artefact is used to represent the relationship between land use and distance. From the above discussion, we have learned that it is an adequate and well-formed model. Moreover, it is also dependable because the usage in many textbooks shows that it successfully serves the aforementioned purpose. Obviously, the Thünen model is more simple, clear, abstract, focused, usable and intelligible than the original relationship of parameters of land use. We have already seen that the function of a model matters, whereby the Thünen model holds limited use for prediction. The major critique of this model can be explained by a shift in function: while the original model was developed to explain a relationship, most of the more recent geographical literature implicitly assumes that the model should predict. The Thünen model is not accepted for use in prediction by the community of practice. At the same time the model is used for explaining a relationship, albeit likewise not explicitly. The Thünen model is mostly described as the history of research, serving in a hidden way to explain some basics and it is finely criticised as being insufficient for prediction [21]. This phenomenon shows that models are created in a kind of language game by a community of practice. Function, paradigms, evaluation and other parameters can change, and a new version of a model can be completely different to the original one. Sometimes it is useful to return to the original version, given that the changing perspective may offer new perspectives of the original model, which can help to better understand the topic.

2.7 Problems

2.1. Please discuss:

(a) Which is your preferred definition of model and why?
(b) What is the difference between an empirical and theoretical model?

2.2. Please classify some models that you already know according to the following classes:

(a) empirical model, theoretical model
(b) SQM classification
(c) stochastic model, deterministic model, simulation
(d) five degrees of complexity

2.3. Please identify the models among the following list of objects and classify them according to the classes from the last problem:

(a) chronological scheme with relative and absolute dates
(b) plate of types of ceramic vessels
(c) thin section of ceramic sherds
(d) gypsum reconstruction of a ceramic vessel
(e) digital 3D reconstruction of the same vessel
(f) digital 3D reconstruction of a landscape (elevation)
(g) seriation matrix of finds
(h) drawing of a profile of an excavation trench
(i) photography of the same profile
(j) soil sample from the same profile
(k) stratigraphic Harris-matrix of the layers of the same profile
(l) map of the ancient road system of a certain region
(m) agent-based model of ancient trade

2.4. Please outline the benefits of models from the examples in this chapter from your perspective.

2.5. Please list some properties of complex systems.

2.6. Can you find an example of the need for balancing theory, method, data and research question based upon your own experience.

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