What is Born–Jordan quantization? And what is it supposed to be good for? Well, it might very well be that Born–Jordan quantization and the associated operator calculus provide us with the only physically correct quantization scheme, as opposed to the Weyl quantization commonly used in physics. Already this fact is sufficient to motivate a comprehensive mathematical study of the topic. Another motivation is that very recent and ongoing work shows that the Born and Jordan approach provides better results in the study of spectrograms in time–frequency and signal analysis, by damping unwanted interference effects. To understand what Born–Jordan operators are about one has to go back to the early years of quantum mechanics, where a rule for quantizing monomials was proposed by Max Born and Pascual Jordan in 1925 following Werner Heisenberg’s paper which inaugurated what is nowadays called matrix mechanics; in this paper Heisenberg proposed the idea of a quantum theoretical reinterpretation of the notion of classical observable. One year later, Erwin Schrödinger proposed his eponymous equation describing the time evolution of de Broglie’s wavefunctions, and showed that his approach led to the same predictions as Heisenberg’s matrix mechanics, thus proving the uniqueness of quantum mechanics as a new theory. The quantum rule proposed by Born and Jordan was quickly superseded, mainly for mathematical reasons, by another rule due to Hermann Weyl, which became de facto the preferred quantization in physics, and thus leading to two different quantum mechanics: Heisenberg’s matrix on the one side, and Schrödinger’s wavemechanics on the other side.

For all these reasons we believe that this new pseudo-differential calculus deserves to be studied; the present work is an introduction to the topic, which is still in its infancy. We do hope that it will trigger interest among researchers and students. One more word: this book was primarily written for quantum physicists and mathematicians interested in quantum mechanics. However, it might also be of interest to specialists working in signal theory and time–frequency analysis: it suffices to replace everywhere $\hbar$ with $1/2\pi$ and $x$ with $t$.

I wrote the draft of this book in one week (sometimes during the spring 2015), it was actually a skeleton, and a rickety one, but it contained, with a few exceptions,
the main lines of the final version. But fine tuning and putting flesh on the bones was a harder task. So it is both my duty and great pleasure to thank the following mathematicians and physicists for encouragement and stimulating conversations: Paolo Boggiatto (Turin), Elena Cordero (Turin), Glen Dennis (London), Hans Feichtinger (Vienna), Serge de Gosson (Stockholm), Basil Hiley (London), Franz Luef (Trondheim), Fabio Nicola (Turin), Luigi Rodino (Turin), Michael Ruzhansky (London) and Ville Turunen (Esbo).

Special thanks to my beloved wife, Charlyne de Gosson, who has rearranged and corrected the Bibliography, and to my colleague and friend, Glen Dennis, for having read the manuscript with great attention and pointing out numerous typos and errors.

This work has been funded by the grant P27773-N23 of the Austrian *Fonds zur Förderung der wissenschaftlichen Forschung* (FWF).

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Summer 2015
Born-Jordan Quantization
Theory and Applications
de Gosson, M.A.
2016, XIII, 226 p., Hardcover
ISBN: 978-3-319-27900-8