# Preface

Networked systems consist of distributed nodes that sense their surroundings, exchange information with other nodes, and perform actuation to change either their own internal states or their environment. Such systems play an ever-increasing role in modern transportation, energy, and health applications. One of the emerging areas of networked systems is cyber-physical systems (CPS), in which the cyber and physical components of the system are tightly coupled and deeply interrelated, thus making computing, networking, and control interact closely. In order to provide needed guarantees on stability, performance, and availability, these networked systems must be controlled via external inputs. Their immense scale and heterogeneity, however, renders individual control of each network node impractical. A widely studied and implemented alternative approach is to directly control a subset of nodes (often called leaders, drivers, or input nodes), which steer the remaining nodes to their desired states via local interactions.

Two motivating applications for this approach are as follows. First, consider a network of unmanned vehicles, whose goal is to navigate an environment and reach a desired destination while maintaining formation. The trajectory of the vehicles is specified in real time by human operators. Rather than having a different operator for each vehicle, which would not scale to large formations and would increase the possibility of human error, a subset of the vehicles are controlled directly while the remainder are steered by predetermined control laws. As a second application, consider a social network in which each user's opinion is shaped by the opinions of the user's neighbors. In order to influence the opinions of all users in the network, an external entity could steer the opinions of a subset of well-connected users, who influence the remainder of the network.

In these and other applications, the choice of input nodes is known to determine critical properties of the networked system, including the rate of convergence to the desired state, robustness to disturbances, and controllability. The input nodes should be chosen in order to provide certifiable guarantees on these properties. Selecting a subset of input nodes, however, is inherently a discrete optimization problem, making continuous optimization techniques for control synthesis inapplicable.

This monograph presents a submodular optimization framework for selecting input nodes in networked systems. Submodularity is a diminishing returns property of discrete functions, analogous to concavity of continuous functions that enables efficient optimization algorithms with provable optimality guarantees for problems that would otherwise be computationally intractable. While submodular optimization has been applied to a variety of machine learning problems, submodularity has so far received little attention in the control-theoretic literature. This monograph demonstrates submodular structures that are inherent in a variety of networked system properties, including robustness to noise, smooth convergence, synchronization, and controllability, leading to a unifying, computationally efficient design framework. Moreover, the monograph describes new techniques for distributed submodular optimization that are motivated by the distributed nature and resource constraints of networked systems.

The contents of this monograph are summarized as follows. Part I, entitled "Submodular Functions and Optimization," describes submodular functions and algorithms for submodular maximization, and consists of three chapters. Part II, entitled "Submodularity in Dynamics and Control," presents a submodular optimization framework for design of networked systems, and consists of seven chapters.

Chapter 1 defines submodular functions and describes their relevant properties. The chapter also presents examples of submodular functions. The concept of a matroid, which is a set system that generalizes linear independence in vector spaces, acyclic properties of graphs, and partition and matching constraints, is defined. The concepts of matroid rank, basis, and closure are introduced, along with dual matroids and matroid union.

Chapter 2 focuses on centralized algorithms for submodular maximization, in particular how the diminishing returns property of submodular functions enables provable optimality guarantees from simple greedy and local exchange algorithms. Robust submodular maximization algorithms are also presented. The chapter discusses matroid-constrained submodular maximization using greedy and continuous greedy algorithms, followed by online submodular maximization.

Chapter 3 is devoted to distributed submodular maximization. Two algorithms for distributed cardinality-constrained submodular maximization, namely a greedy algorithm and an exchange-based algorithm, are presented. Theoretical and empirical analysis of the optimality and scalability of both techniques is given, followed by distributed algorithms for matroid-constrained submodular maximization. Techniques for the related problem of parallel submodular maximization using multiple processors are also presented.

Chapter 4 gives background on control of networked systems. The chapter introduces needed concepts from graph theory, including basic definitions, algebraic properties of graphs, and spectral graph theory. Control strategies for consensus of first- and second-order integrators, distributed estimation, and opinion dynamics of social networks are presented. The effect of input nodes on the network dynamics is described.

Chapter 5 discusses a submodular optimization framework for smooth convergence in networked systems. Smooth convergence ensures that the networked nodes converge to their desired states with minimal delay and with minimal error in their intermediate states. The convergence rate of networked system dynamics is analyzed. A submodular optimization approach for smooth convergence is presented that is based on identifying connections between the system dynamics and the statistics of a random walk on the network. The submodular optimization approach is developed for static and dynamic networks. Two cases of dynamic networks are considered, namely networks where the topology evolves according to a random process with distribution that is known during the design phase, and networks where the topology dynamics have unknown distribution, and hence must be learned over time.

Chapter 6 focuses on synchronization in complex networks with nonlinear dynamics. Synchronization is essential in systems including power grids, biological neural networks, and vehicle formations, and in some cases must be guaranteed by pinning a subset of catalyst nodes to a desired frequency and phase. Background on the classical Kuramoto model for synchronization of coupled oscillators is given, followed by existing techniques for analyzing synchronization under the Kuramoto model in homogeneous and heterogeneous networks. Threshold-based conditions for synchronization with external inputs are presented, as well as efficient algorithms for verifying that the conditions are satisfied. A submodular optimization approach for selecting catalyst nodes to guarantee the existence of stable synchronized states, as well as ensuring that synchronization is reached from almost any initial state, is developed.

Chapter 7 is devoted to minimizing the impact of link noise, which corrupts the information exchanged by the nodes and leads to errors in the node state updates. Network coherence metrics for quantifying the impact of noise in networked systems are defined. Convex relaxation techniques for minimizing errors due to link noise are discussed. A submodular optimization approach to minimizing errors due to link noise is introduced, based on demonstrating equivalence between the error in the node states and the commute time of a random walk on the network, which is shown to be submodular. The submodular optimization framework is developed for static networks, networks with random link failures, networks with switching topologies, and networks with random mobility.

Chapter 8 focuses on mitigating the security threat of link noise injection by intelligent adversaries. Noise can be injected through false packet insertion or broadcasting interfering signals (jamming). A game-theoretic framework for modeling the interaction between adversaries injecting link noise and the network controller selecting input nodes is presented. The framework is developed for two cases, namely selecting a fixed input set, as well as selecting a time-varying input set. In both cases, the submodularity of the error due to link noise is exploited to develop efficient algorithms for approximating the equilibrium strategies of both the network and adversary.

Chapter 9 presents a framework for input selection based on joint optimization of performance and controllability. Conditions for controllability are given, including matrix pencil, Laplacian, and graph-theoretic conditions. Structured systems, conditions for structural controllability, and existing techniques for selecting input nodes to guarantee structural controllability are presented. The main contribution of this chapter is a formulation of controllability conditions as matroid constraints, enabling joint optimization of performance and controllability as submodular maximization with matroid constraints. A graph controllability index, which characterizes the largest controllable subgraph of the network, is given and its submodular structure is proved. Application of this approach to consensus networks is discussed.

Chapter 10 is devoted to emerging applications of submodularity to control of energy systems. Increasing demand for electricity and reliance on unpredictable renewable energy sources will push power systems close to their stability limits in the coming decades. At the same time, widespread deployment of real-time monitoring systems creates the potential for real-time control of the power system, provided that scalable control algorithms can be developed. This chapter presents possible applications of submodularity toward developing such algorithms. The chapter focuses on three critical problems in power system stability, namely voltage stability, small-signal stability, and transient stability. The focus of this chapter is on future research topics.

Errata for this book can be found at the University of Washington Network Security Lab website, http://www.ee.washington.edu/research/nsl/submodularity_book.

The authors wish to acknowledge the support of colleagues and collaborators, without whom this book would not have been possible. We are delighted to acknowledge discussions with our colleagues, Profs. Mehran Mesbahi, James Ritcey, Arvind Krishnamurthy, Emo Todorov, Jeff Bilmes, Barbara Endicott-Popovsky, Daniel Kirschen, and Baosen Zhang of the University of Washington. Our thanks are also to Prof. Tamer Başar of the University of Illinois, Prof. Mihailo Jovanovic of the University of Minnesota, Dr. Airlie Chapman of the University of Washington, and Dr. Fu Lin of Argonne National Laboratories for many fruitful research discussions. We thank Prof. John Baillieul of Boston University for his insightful comments and for writing the foreword of this book. We acknowledge the efforts of Phillip Lee, Zhipeng Liu, Sean Rice, Hossein Hosseini, Xuhang Ying, and Dr. Kalikinkar Mandal of the Network Security Lab (NSL@UW) at the University of Washington. We would also like to thank our editor, Oliver Jackson, for his support and guidance, as well as the anonymous reviewers of the manuscript for their time and invaluable comments.

None of these efforts would have come to fruition without the support of our sponsors who recognized the need for new directions in performance and control. We gratefully acknowledge the support provided by the National Science Foundation Grants #CNS-CPS-1446866 and #CNS-CPS-1544173, the Office of

Naval Research under Grant #N00014-14-1-0029, and the King Abdulaziz City for Science and Technology. We would like to thank Dr. David Corman of the National Science Foundation and Dr. J. Sukarno Mertoguno of the Office of Naval Research, who have been two of the strongest supporters of this effort. The funding support helped in a sequence of results in one area that culminated in the book that you hold in your hands.

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http://www.springer.com/978-3-319-26975-7

Submodularity in Dynamics and Control of Networked Systems
Clark, A.; Alomair, B.; Bushnell, L.; Poovendran, R.
2016, XVII, 210 p. 63 illus., 15 illus. in color., Hardcover
ISBN: 978-3-319-26975-7