Preface

Methods for solving global optimization problems with or without constraints find application in several disciplines like Game theory, Biology, Statistics, Engineering, Mathematics, Management Science, Economics, Physics, and in virtually every task that can be modeled by parametric systems. Many excellent methods have been developed so far, but several of them assume certain conditions about functions to be processed, as convexity or differentiability. However, when tackling real-world problems, we often get into situations in which objective functions to be optimized are not differentiable, convex, or even continuous. In such settings, many traditional methods are not so helpful, being necessary to find more general ways to get adequate results. In the past decades a large number of new global optimization methods were idealized aiming at reaching that more general objective, and a substantial part of them belong to the category of metaheuristic methods, being also known generically as metaheuristics. Many are based on probabilistic foundations, that is to say, use probability theory results to get to their objectives. Knowledge of the capabilities and limitations of these algorithms leads to a better understanding of their reach over various applications and indicates the way to future research on improving and extending algorithms' theoretical foundations and respective implementations. In another direction, almost all of them were developed to solve problems in linear spaces, not being able to deal with more general domains, like manifolds, for example. The main goal of this book is to present certain techniques for solving global optimization problems on manifolds by means of evolutionary algorithms, introducing certain applications chosen to complement the central presentation. In addition, the results will serve as a basis for constrained optimization in Euclidean spaces as well. By presenting detailed examples of applications we will try to stimulate the reader's intuition and make use of the proposed ideas easier to all wishing to solve constrained optimization problems on linear spaces or more general manifolds. In addition, applications tend to be concentrated on Game Theory, in particular Nash equilibrium problems related to several interesting real-world situations—they are reformulated as constrained global optimization problems and are solved with the help of Fuzzy ASA.

More abstract examples include minimization of well-known functions, in order to illustrate in detail the utilization of the proposed ideas. In order to offer usable material, the presented methods and examples use the Fuzzy ASA method, although many other paradigms could be adequate as well.

The insertion of an introductory chapter about metaheuristics is pertinent and justifiable because the algorithms presented in the book propose the use of existing evolutionary techniques when optimizing on manifolds. In this fashion, although Fuzzy ASA is used as the model optimizing paradigm, the suggested methods may be coupled to other metaheuristics. One big advantage of this viewpoint is that a great deal of tested knowledge may be applied in the new, expanded scope with almost null adaptation effort. The chapter also works as an indication and suggestion for future works, based on some of the fundamental ideas contained in the book.

Algorithms able to optimize functions defined on manifolds are important in that they may present lower computational complexity and frequently exhibit better numerical properties, as not getting caught in local minima attraction basins, for example. This becomes very clear when dealing with constrained optimization subject to equality constraints, given the inherent difficulty associated with such a type of restriction, due to its characteristic "bouncing" effect. This is so because it is not simple to keep evolving points exactly inside feasible regions using the conventional methods, giving rise to undesired oscillations along the optimization process. So, despite being important in more general contexts, even in traditional optimization tasks they may be quite effective. In this book it is presented a method for global optimization of functions defined on finite dimensional manifolds, which may be loosely described as configuration spaces that locally "look like" Euclidean spaces and, in truth, include them as particular cases, that is to say, $\mathbb{R}^n$ is a manifold as well. Pertinent elements of General and Differential Topology needed to develop the proposed algorithms are presented and it is possible to see that many already developed evolutionary paradigms can be applied almost directly, when faced and used in the adequate way. As many real-life problems can be naturally regarded as models whose defining parameters evolve on manifolds, like constrained optimization ones with equality constraints, for instance, new results in that direction are always welcome.

Prerequisites for reading this book include some knowledge of Linear Algebra, introductory Numerical Analysis and basic Probability Theory. Many necessary definitions and fundamental results are provided and formal mathematical requirements are kept to a minimum. The focus will be kept on continuous problems. This book can be used in courses related to optimization as well as by researchers and practitioners, and is adequate for self-study too.

The work is divided into three parts:

- Part I presents basic information about optimization algorithms, describing some well-known metaheuristics, their main characteristics, and overall architecture;
- Part II exposes fundamental facts about Topology, the Fuzzy ASA global optimization method along with its overall structure, well-known results about manifold theory, and the proposed methods themselves;

- Part III contains some important applications of (constrained) global optimization, with special emphasis on solutions of Generalized Nash equilibrium problems (GNEPs).

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