

Contents

1	Quantum Theory Prior to 1925	1
1.1	Bohr–Sommerfeld Quantization Rule	1
1.2	Sommerfeld’s Derivation	2
1.3	Bohr’s Postulates	7
1.4	Atom Sizes	7
2	Heisenberg’s Year 1925	11
2.1	Spectral Lines	11
2.2	Introduction of Matrices	15
2.3	Problems	20
3	Expansion of the Matrix Method	23
3.1	Commutation Relation	23
3.2	Systems with Several Degrees of Freedom	28
3.3	Transformations	29
3.4	Problems	31
4	Observables and Uncertainty Relations	33
4.1	State Vector	33
4.2	The Stern-Gerlach Experiment	34
4.3	States and Postulates	35
4.4	Projection Matrices	36
4.5	Probabilistic Interpretation	38
4.6	Density Matrix	39
4.6.1	Definitions	39
4.6.2	Mixed States	40
4.6.3	Examples	42
4.7	Time Evolution of the Expectation Value	43
4.8	Heisenberg’s Uncertainty Principle	44
4.9	Problems	46

- 5 The Harmonic Oscillator** 47
 - 5.1 Physics of the Harmonic Oscillator 47
 - 5.2 Expectation Values and Variances 56
 - 5.3 Problems 57
- 6 Angular Momentum** 59
 - 6.1 The Matrix Vector of the Angular Momentum 59
 - 6.2 Eigenvalues and Eigenvectors of L^2 and L_3 63
 - 6.2.1 Commutativity of L^2 and L_3 63
 - 6.2.2 Eigenvalues and Eigenvectors 64
 - 6.2.3 Maximum and Minimum Eigenvalues 65
 - 6.2.4 Orientation of the Angular Momentum Vectors 68
 - 6.2.5 The Matrices L^2 and L_3 69
 - 6.2.6 The Matrices L_+ , L_- , L_1 and L_2 71
 - 6.3 Problems 75
- 7 Wolfgang Pauli and the Hydrogen Atom** 77
 - 7.1 Basic Matrices and Matrix Vectors 77
 - 7.2 Introduction of the Matrix Vector \mathfrak{A} 79
 - 7.3 The Hydrogen Spectrum 83
 - 7.4 Problems 84
- 8 Spin** 85
 - 8.1 Magnetic Fields and Light 85
 - 8.2 Derivation of the Zeeman Effect (Without Spin) 85
 - 8.3 Symmetry Considerations 87
 - 8.4 Symmetry and the Spin 91
 - 8.5 Spin- $\frac{1}{2}$ Systems and Spinors 93
 - 8.6 Adding Angular Momenta 94
 - 8.6.1 Clebsch–Gordan Coefficients 94
 - 8.6.2 Clebsch–Gordan Coefficients on the Internet 98
 - 8.7 Spin-Orbit Coupling 98
 - 8.8 Problems 104
- 9 Atoms in Electromagnetic Fields** 105
 - 9.1 Normal Zeeman Effect 105
 - 9.2 Anomalous Zeeman Effect 105
 - 9.2.1 Weak Field Limit 105
 - 9.2.2 Strong Magnetic Field 109
 - 9.3 Problem 109
- 10 Many Particle Systems** 111
 - 10.1 Composed Systems 111
 - 10.1.1 Systems with Two Distinguishable Particles 111
 - 10.1.2 Systems with N Distinguishable Subsystems 113

- 10.1.3 Entangled Systems 114
- 10.2 Indistinguishable Subsystems 115
 - 10.2.1 Interchanging Two Particles 119
 - 10.2.2 Interchanging Three Identical Particles 124
 - 10.2.3 Interchanging N Identical Particles 125
- 10.3 Problems 128
- 11 Equivalence of Matrix and Wave Mechanics 129**
 - 11.1 The De Broglie Wavelength 129
 - 11.2 Operators in the Schrödinger Formalism 130
 - 11.3 Schrödinger’s Wave Mechanics 131
 - 11.4 Equivalence of Heisenberg and Schrödinger Pictures 134
 - 11.5 Example: The Harmonic Oscillator 139
 - 11.6 Problems 140
- 12 Relativistic Quantum Mechanics 141**
 - 12.1 Special Relativity 141
 - 12.1.1 Four-Dimensional Spacetime 141
 - 12.1.2 Lorentz Transformation 142
 - 12.1.3 Velocity and Its Lorentz Transformation 143
 - 12.1.4 Momentum and Its Lorentz Transformation 144
 - 12.1.5 Equation of Motion and Force 144
 - 12.1.6 Energy and Rest Mass 146
 - 12.2 The Dirac Equation 147
 - 12.2.1 The Wave Equation for a Free Particle 147
 - 12.2.2 Invariant Form of the Dirac Equation 150
 - 12.2.3 Solution of the Dirac Equation 151
 - 12.2.4 Dirac’s Interpretation of the Negative Energy 153
 - 12.3 Problems 154
- Appendix A: Solutions to Problems 155**
- Appendix B: The Kronecker Product 183**
- Appendix C: Fourier Decomposition of Periodic Functions 185**
- Appendix D: Laplace–Runge–Lenz Vector 187**
- Appendix E: Permutation 191**
- Appendix F: Determinants 193**
- Appendix G: Dirac’s Bra-Ket Notation 195**
- Appendix H: Proofs of Pauli’s Formulas 197**
- Appendix I: Physical Quantities and Units 207**
- References 209**
- Index 211**



<http://www.springer.com/978-3-319-26364-9>

Quantum Mechanics in Matrix Form

Ludyk, G.

2018, XIII, 214 p. 14 illus., 9 illus. in color., Hardcover

ISBN: 978-3-319-26364-9