Preface and Introduction

> Though this be madness, yet there is method in't.
>
> *Hamlet*, William Shakespeare

Matrix mechanics was introduced in 1925 by the German physicist Werner Heisenberg[1] [13]. However, the American Nobel laureate Steven Weinberg[2] writes about this publication [26]:

> If the reader is mystified at what Heisenberg was doing, he or she is not alone. I have tried several times to read the paper that Heisenberg wrote on returning from Helgoland, and, although I think I understand quantum mechanics, I have never understood Heisenberg's motivations for the mathematical steps in his paper.

For years, Heisenberg and his colleagues had been struggling with a problem that had been raised in 1913 by Niels Bohr[3] in his atomic theory: why do electrons in atoms occupy only certain permitted orbits with certain well-defined energies? Heisenberg took a completely new approach to this question. Because the trajectory of an electron in an atom cannot be directly observed, he decided to deal only with measurable quantities (namely the allowed energies for the quantum states of all electrons in the atom, as well as the frequencies with which an atom spontaneously jumps from one of these quantum states to a different state while emitting a light particle, i.e., a photon). Heisenberg introduced a sort of "table" of these frequencies. He performed mathematical operations on it, which led to new tables for the various physical quantities such as position, velocity, or the square of the velocity of an electron.

To be more precise, the table entries were the so-called transition amplitudes, that is, quantities whose squares specify a transition probability. When returning

[1] Werner Heisenberg, 1901–1976, German physicist, Nobel Prize 1932.

[2] Steven Weinberg, * 1933, American physicist, Nobel Prize 1979.

[3] Niels Bohr, 1885–1962, Danish physicist, Nobel Prize 1922.

from Helgoland (where he first had this crucial idea) to Göttingen, Heisenberg found out that the operations he applied to these tables were well known to mathematicians. The tables were called *matrices,* and the operations that he used to get from the table representing the electron velocity to the table representing the square was named *matrix multiplication.* Starting from the known dependence between the energy of a particle and its velocity and position in a simple system, Heisenberg could calculate a table of the system's energies in its different quantum states, similar to Newton's[4] calculation of the energy of a planet based on its position and velocity.

At the time, Heisenberg was constantly in touch with some influential theoretical physicists, including the German researchers Max Born[5] and Pascual Jordan[6] and the English physicist Paul Dirac.[7] Until the end of the year 1925, they transformed Heisenberg's ideas into a comprehensive and systematic version of quantum mechanics, which today we refer to as *matrix mechanics.* With the help of the new matrix mechanics, the German physicist Wolfgang Pauli[8] managed in the following January to solve the paradigmatic problem in atomic physics, namely the calculation of the energy levels of the hydrogen atom. His calculations proved the earlier ad hoc results of Bohr.

H.S. Green, an employee of Max Born, writes in [12]:

> Most books on quantum theory emphasize the wave mechanical approach (of Schrödinger[9]), probably because it is supposed to be easier to understand for those who already have a solid knowledge on differential equations.

In this book, however, we restrict ourselves to the algebraic method using matrices and only briefly describe Schrödinger's wave mechanics in order to show the equivalence with Heisenberg's matrix mechanics. By implementing numerical algorithms in standard software such as MAPLE or MATHEMATICA, matrices and matrix equations are easy to handle these days [23].

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[4] Isaac Newton, 1642–1727.

[5] Max Born, 1882–1970, German physicist, Nobel Prize 1954.

[6] Pascual Jordan, 1902–1980, German physicist.

[7] Paul Dirac, 1902–1984, English physicist, Nobel Prize 1933.

[8] Wolfgang Pauli, 1900–1958, German physicist, Nobel Prize 1945.

[9] Erwin Schrödinger, 1887–1961, Austrian physicist, Nobel Prize 1933.

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