A Game Theory Model for Self-adapting Traffic Flows with Autonomous Navigation

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Abstract It is widely believed that road traffic as a whole self-adapts to the current situation to make travel times shorter, if the navigation devices exploit real-time traffic information. A novel theoretical approach to study this belief is the online routing game model. This chapter describes the model of online routing games in order to be able to determine how we can measure and prove the benefits of online real-time data in navigation systems. Three different notions of the benefit of online data and two classes of online routing games are defined. The class of simple naive online routing games represents the current commercial car navigation systems. Simple naive online routing games may have undesirable properties: stability is not guaranteed, single flow intensification may be possible and the worst case benefit of online data may be bigger than one, i.e. it may be a “price”. One of the approaches to avoid such problems of car navigation is intention propagation where agents share their intention and can forecast future travel times. The class of simple naive intention propagation online routing games represents the navigation systems that use shortest path planning based on forecast future travel times. In spite of exploiting intention propagation in online routing games, single flow intensification may be possible, the traffic may fluctuate and the worst case benefit may be bigger than one. These theoretical investigations point out issues that need to be solved by future research on decision strategies for self-adapting traffic flows with autonomous navigation.

Keywords Performance • Games • Online routing games • Benefit of online data

1 Introduction

This chapter investigates the properties of autonomous car navigation devices with access to real-time data. If all the information about the road network, the cars on the roads and the destination of the cars could be collected by a centralized system, then
it would be able to create an optimal plan for the trips of the cars. Optimality may be measured in several ways, but usually we assume that the goal is to optimize some “global” parameter of the traffic, like the sum of the travel times. We also assume that the goal is to assure some kind of fairness for all traffic participants, for example, none of the cars pays with some extra long travel time to achieve the global optimum of the whole traffic. Everyday traffic is not coordinated by a centralized system, and even if the traffic was coordinated by such centralized system, there would be the question whether the individual traffic participants would conform to its instructions. In reality, the traffic participants make their own autonomous decisions based on their intentions and the information available for them locally. This means that instead of centralized decision making, we have a set of autonomous distributed decision makers, i.e. a multi-agent system. In this aspect autonomy refers to the autonomous route planning by the navigation devices in the individual cars instead of following the instructions of some centralized planner.

Another aspect of the autonomous behaviour of navigation systems is related to the ability of the traffic as a whole to self-organize and adapt to the current situation, which is a kind of autonomicity. The major trends that became more and more accomplished in the history of computing are ubiquity, interconnection, intelligence, delegation and human orientation [14]. The current wave of this progress is marked by the widespread availability of online real-time data. The navigation devices in cars can get up-to-date information on the current status of the traffic, like the current travel time on each road, indicating the current situation of the traffic that needs to be adapted to. The routing algorithms implemented in the navigation devices must be able to utilize this real-time data to self-heal the global traffic; for example, if a road becomes congested, then the navigation devices autonomously tell the individual cars how to adapt to the current traffic situation and send the cars to less congested roads. Although current navigation devices are already able to utilize real-time data for route planning, these systems were implemented without clear understanding of the impact of real-time data on traffic as a whole and how real-time data affects the above-mentioned self-adaptation aspects of the traffic flows. Note that in this chapter we focus on online-data-based self-adaptation which is different from self-adaptation based on previous experiences, like in the case of route selection from home to work based on the experience of the previous day.

Two well-known examples of real-time information based navigation systems are Google Maps and Waze. The planning in these systems is done on central server(s) which may play similar role to the virtual environment in the anticipatory vehicle routing of Claes et al. [3]. There are other traffic management systems that combine central planning and local freedom, like the PLANETS system [8] in which global control strategy is provided from a Traffic Management Centre, but traffic participants have a freedom to make decisions autonomously. In our view, self-interested agents will not conform to a central strategy if it is not individually rational, so the global strategy must emerge from the autonomous agents’ decision. Therefore, we believe that the basic theoretical model of the online route planning problem should not have an explicit concept of a central planner or a virtual environment even if the agents use the services of these abstractions.
It is widely believed and intuitively we might think that traffic route planning is able to self-adapt to the current situation and plan trips with shorter travel time if we take into account real-time traffic information; however, the classical theoretical models do not have definite answer if online-data-based car navigation is able to self-adapt and produce better traffic or not. There is need for new theoretical studies like the one described in this chapter, because autonomous cars are being designed and the usage of online-data-based navigation systems is spreading and we do not even know how to measure their benefit, not to mention how to optimize their self-healing behaviour.

This chapter has four main sections. In Sect. 2 we shortly describe the classical non-adaptive game theory model for the routing problem, which does not handle real-time information. In Sect. 3 we describe the new online game theory model, which is able to model self-adaptation to real-time information. In Sect. 4 we show the properties of the class of simple naive online routing games, which model the behaviour of traffic using the currently available commercial online-data-based navigation devices. In Sect. 5 we show the properties of the class of simple naive intention propagation (SNIP) online routing games, which model the prediction utilizing anticipatory vehicle routing systems proposed by researchers to improve the currently available commercial online-data-based navigation devices. Finally in Sect. 6 we summarize the main messages of the chapter. The description of these models and the analysis results are based on formal proofs of previous papers [11, 12], but here we present them in an easier understandable way and put them in the context of autonomy and self-adaptation.

2 Classic Non-adaptive Approach

In this section we highlight the main theoretical findings of routing games and online mechanism design, because the model of autonomously self-adapting navigation is based on them and the research in the new field of online routing games\(^1\) has to answer similar questions.

**Routing Games** Algorithmic game theory studies networks with source routing (Sect. 18 in [9]), in which end users simultaneously choose a full route to their destination and the traffic is routed in a congestion sensitive manner. Two models are used: non-atomic selfish routing and atomic selfish routing. Non-atomic routing is meant to model the case when there are very many actors, each controlling a very small fraction of the overall traffic, so a traffic flow from a source to a destination can be divided among several routes. Atomic routing is meant to model the case when each actor controls a considerable amount of traffic, so a single traffic flow is not divided among several routes. Both models are studied in detail and showed

\[^1\]In the generic form joint resource utilization games [11].
similar properties. The main difference is that the non-atomic model basically has continuous functions having unique extreme values, while the atomic model has discrete functions which can approximate extreme values at several points. The algorithmic game theory model of the routing problem is based on the triple \((G, r, c)\), where \(G\) is the road network given by a directed graph, \(r\) is the total traffic flow given by a vector of \(r_i\) traffic flows from source to destination vertices and \(c\) is the throughput characteristic of the road network given by a cost function.

A flow distribution is optimal if it minimizes the cost of the total traffic flow over all possible flow distributions. A flow distribution is an equilibrium flow distribution if none of the actors can change its traffic flow distribution among its possible paths to decrease its cost. The equilibrium flow distribution is a rational choice for every actor, because deviating from the equilibrium would increase the cost for the actor.

It is proven (Sect. 18 in [9]) that every non-atomic routing problem has at least one equilibrium flow distribution and all equilibrium flow distributions have the same total cost. The price of anarchy is the ratio between the cost of an equilibrium flow distribution and the optimal flow distribution. If the cost functions are linear, then the price of anarchy in any non-atomic routing problem is not more than \(4/3\). If the cost functions can be non-linear, then one can create cost functions to exceed any given bound on the price of anarchy of non-atomic routing problems.

In atomic routing problems, the existence of equilibrium flow distribution is not always guaranteed. Atomic routing problems have equilibrium flow distribution if every traffic flow \(r_i\) has the same value or if the cost functions are linear. If there are more than one equilibrium flow distributions, then their total costs may be different. If the cost functions of an atomic routing problem are linear, then the price of anarchy is at most \((3 + \sqrt{5})/2\). If the cost functions of an atomic routing problem are linear and in addition every traffic flow \(r_i\) has the same value, then the price of anarchy is at most \(5/2\).

It is known that if the routing problem has an equilibrium and the actors try to minimize their own cost (best-response), then the traffic flow distribution converges to an equilibrium.

The algorithmic game theory investigations of the routing game revealed important properties; however, the algorithmic game theory model contains the following assumptions: (a) The throughput characteristic of the network does not change with time, and the drivers can compute this characteristic or learn it by repeatedly passing the road network. (b) The drivers simultaneously decide their optimal route. (c) The outcome travel time for a given driver depends on the choice of all the drivers and the characteristic of the network, but not on the schedule of the trip of the drivers. These assumptions are not valid for car traffic where the drivers use a navigation device exploiting online data.

The issue of traffic dynamism is studied in the field of dynamic traffic assignment [5], but there they investigate the time-varying properties of traffic flow, whereas here we assume that the traffic flow is basically constant and only the cost functions may change. In our investigations the critical issue is the adaptive sequential decision making of the agents. In the classical game theory approach, the issue of sequential decision making of agents is studied in online mechanism design.
**Online Mechanisms** Online mechanism design problem is a multi-agent sequential decision-making problem [10]. When agents participate in the mechanism, they report to a central planner for a given period their request for certain resources at given valuations (which may be different from their private values). The central planner decides which resources at which cost are allocated to which agent in each time step. All agents are trying to maximize their utility. The *model of the online mechanism problem* is the five tuple \((t, \theta, k, c, u)\), where \(t\) is a sequence of time periods, \(\theta\) is the set of agent types where each agent type is characterized by the arrival/departure time of the agent and its valuation of goods, \(k\) is a sequence of decision vectors in each time period by each agent, \(c\) is the cost function of the decisions and \(u\) is the utility function of the agents.

In this model the \(\theta\) set of agent types may be model-free when no probabilistic information is known about the agents or may be model based if probabilistic information is known. The agents may report values different from their private agent type, but only for the time period when they are present, at the beginning of the reported time period and without knowing the reports of the other agents (closed direct revelation). Usually the goal is to design online mechanisms where the truthful revelation is the dominant strategy. The *effectiveness of online mechanisms* is measured similarly as that of online algorithms: the performance of the online mechanism is compared with that of an offline mechanism that has the complete information about all future agent types.

The dynamic nature of online mechanisms is a good starting point to model the online-data-based adaptive routing problem; however, the differences are considerable: in contrast with online mechanisms, in the online-data-based adaptive routing problem, there is no central planner (agents make their own plan), the arrival and departure times are not flexible (agents want to start their plan when they arrive) and the actual cost is determined not at the decision time but at utilization time.

### 3 Game Theory Model for the Online-Data-Based Self-adapting Routing Problem

The online-data-based self-adapting routing problem is a challenging application, because in this problem autonomous agents have access to real-time data, and based on this information, they autonomously try to self-organize themselves by creating adapted plans to achieve their individual goals in an environment where they jointly utilize resources that become more costly as more agents use them. In this problem agents are dynamically arriving and departing after completing their plans. The plans are created by exploiting online data that describe the current status and the current cost of the resources. There is uncertainty about the feasible decision of an agent, because the cost of the resources will change by the time the agent starts to use them: departing agents will release the resources as they complete their plans, agents simultaneously creating their plans will influence each other’s
costs and agents arriving later may also influence the costs of the resources used by agents already executing their plans. This is somewhat similar to typical game theory problems, where the outcome of the action of the agent depends on its own decision plus the decisions of the other agents; however, in the self-adapting routing problem, the outcome depends on even more circumstances as written above. This type of applications are called online joint resource utilization games [11] which is derived from algorithmic game theory [9] and online mechanisms [10]. Note that these games are different from resource allocation or minority games [7] which are simultaneous one shot or repeated simultaneous games where there might be some coordination among some of the agents. In contrast, online joint resource utilization games are continuous and non-cooperative games exploiting real-time data.

Adaptive car navigation using real-time data is a special case of online joint resource utilization games, because the allowed order of the resource utilization in the plan of the agents is restricted by the structure of the road network. From theoretical point of view, online-data-based car navigation applications are called online routing games [11]. Note that in this approach each driver makes an individual online-data-based decision at the time of entering the network, whereas in other approaches [2] drivers learn the best route to select, based on past experiences.

3.1 The Model of Online Routing Games

In order to have a generic model, the model of the online joint resource utilization game was defined [11] as an extension of the algorithmic game theory model of the routing problem and the online mechanisms. The model resembles the algorithmic game theory routing game model in the concepts of flow, cost and resource, and it resembles the model of online mechanisms in the sequences of time periods and decisions. Time unit $T$ is introduced in order to be able to compute the rate of resource utilization. The model of online routing games [11] is like the model of online joint resource utilization games but with a restriction on the allowed plans represented by a graph and with somewhat different cost functions. The formal model is described in this subsection, and there are a few examples in Sects. 4 and 5.2.

The model of the online routing game is the sextuple $(t, T, G, c, r, k)$, where

- $t = \{1, 2, \ldots\}$ is a sequence of equal time periods.
- $T$ is a natural number with $T$ time periods giving one time unit (e.g. 1 min).
- $G$ is a directed graph $G = (V, E)$ with vertex set $V$ and edge set $E$ where each $e \in E$ is characterized by a cost function $c_e$ which is equal to the utilization time of the edge.
- $c$ is the cost function of $G$ with $c_e : R^+ \rightarrow R^+$ for each edge $e$ of $G$ mapping the incoming flow at each time period to the travel time on that edge, which is
never less than the remaining cost of any other agent currently utilizing that edge increased with the time gap of the flow.\(^2\)

- \( r \) is the total flow given by a vector of \( r_i \) flows with \( r_i \) denoting the flow aiming for a trip \( P_i \) from a source vertex \( s_i \) of \( G \) to a target vertex \( t_i \) of \( G \).
- \( k = (k^1, k^2, \ldots) \) is a sequence of decision vectors with decision vector \( k^t_i = (k^t_i, k^t_2, \ldots) \) made in time period \( t \) and \( k^t_i \) the decision made by the agent of the flow \( r_i \) in time period \( t \).

In this model, the graph \( G \) may contain parallel edges. The cost functions are non-negative, continuous and non-decreasing. The cost functions have a constant part which does not depend on the flow on the edge and a variable part which depends on the flow on the edge. The variable part is not known to any of the actors of the model until an agent exits an edge and reports it. The flow \( r_i \) is given by \( T / \alpha \cdot n_i \) where \( n_i \) is a natural number constant, meaning that the following distance of the units of the flow \( r_i \) are \( n_i \) time periods.\(^3\) The \( k^t_i \) decision is how the trip \( P_i \) is routed on a single path of the paths leading from \( s_i \) to \( t_i \). The actual cost of a path \( (e_1, e_2, e_3, \ldots) \) for a flow starting at time period \( t \) is the sum of the cost of the edges, and the actual cost of an edge is determined at the time when the flow enters the edge.

The actual cost of the edges becomes known for the agents only when an agent reports its actual cost. Because agents do not report cost values in each time step, the agents interested in the cost values must decrease the last reported value by taking into account the time elapsed since the last reporting event (it is similar to the pheromone evaporation in [4]).

The online routing game model can accommodate changes of the cost function \( c \) over the sequence of time periods \( t \), because the agents can get information about the actual cost only from the cost reported by the agents exiting an edge.

**Routing Strategy** The critical point in the online routing game is how to determine the best decision vector \( k \). The algorithmic game theory approach assumes that the agents have full information about the cost functions, and the theory tells what the best strategy is in the case of simultaneous decisions but does not tell how the agents can achieve this. In online mechanisms a central planner decides which resources at which cost are allocated to which agent. In online routing games there is no central planner. The agents in online routing games will have to apply algorithms similar to online algorithms [1]. At this time we are not investigating how the agents of online routing games determine their strategy; instead, we are investigating the performance of the strategies of current navigation devices.

**Simple Naive Strategy** Typical navigation software currently installed in cars use simple shortest path search in the road network, possibly modifying the distances with the online information about the actual traffic delay. We call this decision

\(^2\)In this model cars cannot overtake the cars already on the road and there is a time gap, i.e. minimum “following distance”.

\(^3\)So if \( T = 6 \) time steps and \( n_i = 2 \), then one car enters the network every second time step and the intensity of the flow is 3, because 3 cars enter the network in a time unit.
strategy *simple naive strategy*. This strategy is investigated because of its practical importance. Note that the simple naive strategy is by definition deterministic; thus it is a pure strategy.

### 3.2 The Benefit of Online Real-Time Data

We would like to be able to tell if the agents are better off by autonomously trying to self-adapt to the observed online real-time data or not. In order to be able to compare the costs of the agents using online data with the costs of the agents not using online data, we have to know what we are going to compare with what.

If we take the approach of online algorithms, then we would compare the results of the online routing game with the results of an oracle that has all the information needed. One might think that in our case the oracle with all information would be the central planner, because the central planner has all the information and can tell each agent which route to take.

The central planning oracle might be good to measure the global effectiveness of the agents in the online routing game model; however, it evaluates not only the benefits of making decisions based on online data, but in addition, it evaluates the different decision-making strategies as well. In the online routing game model there is no coordination among the agents and the agents make decisions using, for example, the simple naive strategy, while in the central planning and the algorithmic game theory approaches, the agents are coordinated and they exploit their knowledge about the cost functions. Therefore, if we want to evaluate only the benefits of autonomous self-adaptation using online real-time data, then we want to compare the results with an “oracle” using the same decision-making strategy.

In the algorithmic game theory model, there is equilibrium and the price of anarchy concept is the ratio between the equilibrium and the optimum. Later in this chapter (in Theorem 1), we will see that there are simple naive strategy online routing games which do not have equilibrium at some flow values. If there is no equilibrium, then we must have different measures for the best, worst, and average cases (which are guaranteed to exist if there is finite sequence of time periods). Depending on the type of application, we are interested in the different types of benefits. The most important is the worst case, because it can be used to provide a guarantee in critical applications. The best case can be used in applications, where we have to make sure that a certain value is achieved at least once. The average case is seldom useful in itself; usually we have to consider statistical distribution parameters as well.

The above discussion is concluded with the definition of the different benefits of online real-time data [11]. If these benefits are below 1, then the agents have a benefit, because their costs (travel times) are reduced. If these benefits are greater than 1, then they are in fact a “price” like the price of anarchy.
Definition 1  The worst/best/average case benefit of online real-time data at a given flow is the ratio between the cost of the maximum/minimum/average cost of the flow and the cost of the same flow with an oracle using the same decision-making strategy and only the fixed part of the cost functions.

Classes of Online Routing Games  Online routing games using the same type of decision strategies belong to the same class of online routing games. Each class needs to be evaluated how much benefit they make out of online real-time data, in order to be able to determine the type of application where they are suitable. The evaluation should include formal proofs. In this paper we discuss the formal analysis of the class of simple naive strategy online routing games and the class of SNIP online routing games.

Although the simple naive decision strategy is often applied in real world, it is not the best, because it does not alternate the agents of a flow among two or more paths, whereas the optimal central planning and the algorithmic game theory approach use several paths for the same flow. Further research is needed to study different online routing game decision strategies derived from other related games like resource allocation or minority games [7] and the El Farol Bar problem in [6].

4 Simple Naive Strategy Online Routing Games

The simple naive strategy was introduced in Sect. 3.1 and now we discuss properties of simple naive strategy online routing games [11]. The first property states that if the agents of the car navigation system use simple naive strategy to autonomously adapt to the current situation of the traffic, then at some flow values they may make the traffic fluctuate.

Theorem 1  There are simple naive strategy online routing games which do not have equilibrium at certain flow values.

Proof  The proof [11] is informally illustrated in Fig. 1. The traffic will fluctuate between the roads $e_1$ and $e_2$ if at some flow value the non-congested travel time on $e_2$ is smaller than the non-congested travel time on $e_1$, which is smaller than the

![Fig. 1 Simple naive online routing game with fluctuation](image-url)
congested travel time on $e_2$, which is smaller than the congested travel time on $e_1$. In the beginning the traffic starts to flow on $e_2$, so the travel time on $e_2$ starts to increase, and when the travel time on $e_2$ exceeds the non-congested travel time on $e_1$, then the traffic at vertex $v_1$ switches to $e_1$, and then the travel time on $e_2$ starts to decrease, and when the travel time on $e_2$ drops below the travel time of $e_1$, the traffic switches to $e_2$, so the travel time on $e_2$ starts to increase and the cycle starts again.

The second property is the possibility of single flow intensification: if the agents of the navigation system use simple naive strategy to autonomously adapt to the current situation of the traffic and only a single flow enters the road network, then at some flow value at some time there may be a road somewhere in the network, where the flow is bigger than the flow that entered the network. The formal statement is the following:

**Theorem 2** There are simple naive strategy online routing games, where the total traffic flow has only one incoming flow, i.e. $r = (r_1)$; however, the flow on some of the edges of the road network $G$ sometimes may be more than $r_1$.

**Proof** The proof [11] is informally illustrated here with the network of Fig. 2, where road $e_2$ is not susceptible to congestion, the non-congested travel time on road $e_1$ is smaller than the travel time on $e_2$, which is smaller than the congested travel time on $e_1$ at some flow value, which is smaller than 1.5 times the travel time on $e_2$. In addition, the travel time on $e_2$ is more than 2 time units. In this network it may happen that a platoon of the full incoming flow going on $e_1$ is caught up by some agents that go on $e_2$ and arrive at vertex $v_2$ at the same time, so a bigger flow will go into $e_3$ than the one that enters the network.

The third property is that the online information may have a “price”: if the agents of the car navigation system use simple naive strategy to autonomously adapt to the current situation of the traffic, then sometimes they may be worse off than without exploiting information about the current situation. The formal statement is the following:

**Theorem 3** There are simple naive strategy online routing games where the worst case benefit of online real-time data is greater than one, i.e. in these games the worst case benefit is a “price”.

![Fig. 2 Simple naive online routing game with “single flow intensification”](image-url)
Proof The proof [11] is informally illustrated here with the network of Fig. 2 if road $e_3$ is susceptible to congestion. Without online information all the agents would select the path $(e_1, e_3)$; however, if the agents exploit online information, then in accordance with Theorem 2, at some flow value at some time, the incoming flow of $e_3$ will be a platoon of the full incoming flow of the network from $e_1$ plus some other flow from $e_2$. The result is that the travel time on path $(e_1, e_3)$ in this case will be longer than the travel time without online information. □

5 Simple Naive Intention Propagation Strategy Online Routing Games

As we have seen, if the agents of car navigation systems use the simple naive strategy to autonomously adapt to the current situation of the traffic, then the traffic may have properties that we are not happy with. These findings are in line with the simulation results, like the simple scenario consisting of two parallel routes investigated in [13]. The simulations also showed that online information often leads to oscillations in the number of cars on the routes, the velocity and the travel times, which lead to worse overall performance. In the discussion the authors conclude that one of the reasons for the oscillations is that the real-time travel information reflects the state of the network some time ago. Another reason for the oscillation is that the agents do not coordinate their actions. In order to improve these, the authors advise the usage of anticipatory traffic forecast based on the broadcast route choice of the agents, which basically means that the agents share or propagate their intentions. In order to improve the simple naive strategy, the approach of intention propagation was proposed in the anticipatory vehicle routing system using delegate multi-agent systems [3]. In this section we discuss how the online routing game model [11] is used to investigate some of the properties of the usage of intention-propagation-based prediction in autonomously self-adapting car navigation.

5.1 Intention Propagation

The anticipatory vehicle routing proposed in [3] uses the individual planned routes of the agents to forecast future traffic density. Every vehicle is represented by a vehicle agent running on a smart device inside the vehicle. Vehicle agents communicate with the delegate multi-agent system. The delegate multi-agent system represents the traffic environment and is able to make forecast of future traffic density based on the current traffic situation and the planned routes of the vehicles. The delegate multi-agent system provides the traffic forecast back to the vehicle agents which use this information to plan their trip.
The delegate MAS can predict future travel times based on the intention notifications that it has received from all vehicle agents. The delegate MAS has a parametrized model that describes the relationship between the travel time and the intention notifications. The parameters are continuously updated based on both historical and real-time data, so basically the delegate MAS computes the cost functions of the online routing game model with the ability to handle adapting cost functions.

If the predicted future travel times show that a new travel route is preferable, then the vehicle agent is free to change its route plan. If the vehicle agent changes its route plan, then it notifies the delegate MAS of its change of intention. The old intention is then invalidated and the new intention is registered in the delegate MAS.

Although the vehicle agent could use several strategies to revise its intention, we assume that vehicle agents always select the shortest travel time which is called simple naive decision strategy in the online routing game model.

5.2 Properties of Intention-Propagation-Based Prediction in Online Routing Games

A slightly modified version of the above anticipatory vehicle routing system is used to define and formally analyse the class of online routing games that use intention-propagation-based prediction in their decision mechanism [12]. This class of online routing games are called SNIP online routing games.

**Definition 2** Simple naive intention propagation online routing games (SNIP online routing games) are online routing games where the decision-making agents of the flows $r_i$ are the vehicle agents of the anticipatory vehicle routing system; the vehicle agents use the delegate MAS as described in the previous section to predict the travel time for each path $p_j$ of their trip $P_i$; and their decision $k^j_i$ is to select the path with the shortest travel time among the predicted travel times on the different paths of their trip $P_i$. The vehicle agent notifies the delegate MAS of its selected path, and the delegate MAS remembers this selection while the vehicle agent is in the network and invalidates it when the vehicle agent exits the road network.

Note that SNIP online routing games are a little bit different from the anticipatory vehicle routing system of Claes et al. [3], because the SNIP vehicle agents select their route when they enter the road network, and in accordance with the online routing game model, they do not revise it during their trip.

The agents receive a prediction of future traffic in SNIP online routing games, so we would expect that this additional information can be used to improve the properties of simple naive online routing games. Unfortunately intention propagation does not solve the “single flow intensification” problem, as the next Theorem 4 says.
Theorem 4 There are SNIP online routing games where the total traffic flow has only one incoming flow, i.e. \( r = (r_1) \); however, the flow on some of the edges of the network \( G \) sometimes may be more than \( r_1 \).

Proof In this paper we are informally highlighting the essence of the proof [12] of the theorem with the SNIP online routing game \( SN_{5,1} \). The network of \( SN_{5,1} \) is shown in Fig. 3. The cost functions are \( c_{e_1} = 10 + x \), \( c_{e_2} = 10.5 + x \) and \( c_{e_3} = 1 + x \), where \( x \) is the total incoming flow on the edge. The network has only one incoming flow from \( v_1 \) to \( v_3 \). Because the flow receives predictions, normally it alternates the flow between the roads \( e_1 \) and \( e_2 \). Because the cost functions of \( e_1 \) and \( e_2 \) are different, the flow fluctuates on road \( e_3 \). As a result, the flow on \( e_3 \) will be bigger, for a short time, than the incoming flow.

The above Theorem 4 shows that “single flow intensification” may happen in SNIP online routing games, but it does not happen the same way as in simple naive online routing games. The proof of the above theorem cannot be continued to prove that the worst case benefit of online data in SNIP online routing games may be more than one the same way as it was done in [11]. However, there is an additional alternative proof in the next Theorem 5, and this proof points out another reason for possible worst case benefit above one.

Theorem 5 There are SNIP online routing games where the worst case benefit of online real-time data is greater than one.

Proof We are informally highlighting the essence of the proof [12] of the theorem with the SNIP online routing game \( SN_{5,2} \). The network is shown in Fig. 4. The cost functions are \( c_{e_1} = 1 \), \( c_{e_2} = 1 \), \( c_{e_3} = 10 + x \) and \( c_{e_4} = 10.5 + 10 \times x \), where \( x \) is the total incoming flow on the edge. The total traffic flow is \( r = (r_1, r_2) \) with flow \( r_1 = 1 \) from the source \( v_0 \) to the target \( v_3 \) and flow \( r_2 = 1 \) from \( v_1 \) to \( v_3 \). Without online data, both flow would select the road \( e_3 \), so the cost of both flow would be 13 and the total cost 26. With online data, the flows realize at some time that the cost of \( e_3 \) will go above the cost of \( e_4 \). This happens at the same time for both flows, and they are not aware that the other flow is going to change to \( e_4 \) at the same time, so they do not take into account the additional cost on \( e_4 \). This is because the traffic forecaster is only aware of the intention propagations before the current time step, but does not know and cannot forecast the decisions at the current time step.
Fig. 4 The network of the SNIP online routing game $SN_{5,2}$

step. Because $e_4$ is more susceptible to congestion than $e_3$, the cost on $e_4$ will be more than on $e_3$, so the total cost may go above 26.

The situation is even worse than in the theorem above, because the worst case benefit of online data can be arbitrarily large as the next theorem shows. Note that Theorem 6 is not specific to intention propagation, just this property was not investigated for simple naive online routing games.

**Theorem 6** Given any arbitrarily large number $\alpha$, there are SNIP online routing games with linear cost functions, where the worst case benefit of online real-time data is bigger than $\alpha$.

**Proof** Basically the proof [12] of this theorem is based on the SNIP online routing game $SN_{5,2}$ of the above Theorem 5. In short and informally, if the cost function of the road $e_4$ is steep enough, then the flows can incur big enough cost when they change to $e_4$ at the same time.

The last question is whether intention propagation can help to avoid fluctuation? Unfortunately the answer is not positive, as the next theorem shows.

**Theorem 7** There are SNIP online routing games which do not have equilibrium at certain flow values.

**Proof** In short and informally, the proof [12] of this theorem is the continuation of the scenario of Theorem 5. Once the two flows change to $e_4$ at the same time, they immediately realize from the prediction that this has high cost, so they revert to $e_3$, but after a while the cost of $e_4$ drops below the cost of $e_3$, so both flows change to $e_4$ again at the same time. Then this fluctuating cycle continues.

6 Conclusions

Information and communication technologies allow that modern car navigation devices utilize live online data from road traffic networks to optimize the route of vehicles. The navigation devices in cars are autonomous agents, because they plan their route based on their intentions and local information instead of following the
instructions of some centralized planner. The routing algorithms implemented in the navigation devices must be able to utilize real-time data to self-heal the global traffic and autonomously tell the individual cars how to adapt to the current traffic situation. Although current navigation devices are already able to utilize real-time data for route planning, these systems were implemented without clear understanding of how real-time data affects the autonomous and self-adaptation aspects of traffic flows.

In order to be able to measure and prove properties of autonomous traffic routing based on online data, the formal model of online routing games was developed. This model is an extension of the models of routing games of the algorithmic game theory approach and the online mechanisms. Different classes of online routing games are foreseen, and two of them were discussed here. One is the class of simple naive online routing games, which models the currently available commercial real-time-data-based navigation devices. The other is the class of SNIP online routing games, which models the prediction utilizing anticipatory vehicle routing systems proposed by researchers to improve the currently available commercial online-data-based navigation devices.

Several properties of these two classes of online routing games were proved in [11, 12]. Here we informally presented these proofs, discussed them in an easily understandable way and highlighted the critical phenomena that are behind these properties. In the class of simple naive online routing games, stability is not guaranteed, so it makes sense to talk about worst, average and best case benefit of online data. Simple naive online routing games may have the “single flow intensification” property. The result of this is that the worst case benefit of online data may be bigger than 1, which means that sometimes some of the autonomous cars are worse off with utilizing online data for the self-adaptation of traffic flows, than without utilizing online data.

The class of SNIP online routing games may also have the “single flow intensification” property. The worst case benefit of online data may also go above 1 and the traffic may fluctuate. We have pointed out that one of the reasons of this surprising result is the “simultaneous decision” problem: the traffic forecaster predicts future traffic conditions based on the intentions of the vehicles already on the road, but it does not predict the intentions of the vehicles currently making decisions. If many vehicles make decisions at the same time, then they may try to take the same alternative route to avoid the already predicted congestion and cause congestion on the alternative route. Obviously, intention propagation helps the vehicles to detect the possibility of congestion formation before the congestion is actually formed, and thus there is smaller “time window” to make the same “wrong” decision to head towards the newly forming congestion than in the case of the simple naive online routing games. The technique of intention propagation and traffic forecast is therefore an important improvement to the simple naive online strategy.

The issues discussed here point out notions and characteristics that can become the basis to guide future research. These issues also challenge future research to develop online routing game decision strategies that have worst case benefits of online data below 1 or prove that it is not possible to develop such strategy.
If such strategies are possible, then we expect that the application of these new strategies will be individually rational choice, and therefore the decision strategies can be implemented in the navigation devices themselves instead of the centralized planning approaches like those of Google Maps and Waze, because some users are reluctant to provide private data for the centralized approach.

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References

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