A Distance-Based Paraconsistent Semantics for DL-Lite

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Abstract. DL-Lite is an important family of description logics. Recently, there is an increasing interest in handling inconsistency in DL-Lite as the constraint imposed by a TBox can be easily violated by assertions in ABox in DL-Lite. In this paper, we present a distance-based paraconsistent semantics based on the notion of feature in DL-Lite, which provides a novel way to rationally draw meaningful conclusions even from an inconsistent knowledge base. Finally, we investigate several important logical properties of this entailment relation based on the new semantics and show its promising advantages in non-monotonic reasoning for DL-Lite.

1 Introduction

The DL-Lite [2] is a family of lightweight description logics (DLs), the logical foundation of OWL 2.0 QL, one of the three profiles of OWL 2.0 for Web ontology language recommended by W3C [4]. In description logics, an ontology is expressed as a knowledge base (KB). Inconsistency is not rare in ontology applications and can be caused by several reasons, such as errors in modeling, migration from other formalisms, ontology merging, and ontology evolution. In the age of big data, it is becoming impossible to avoid inconsistency of larger scale of KBs. Therefore, handling inconsistency is always considered as an important problem in DLs and ontology management communities [18]. However, DL-Lite reasoning mechanism based on classical DL semantics faces a problem when inconsistency occurs, which is referred to as the triviality problem. That is, any conclusions, that are possibly irrelevant or even contradicting, will be entailed from an inconsistent DL-Lite ontology under the classical semantics.

In many practical ontology applications, there is a strong need for inferring (only) useful information from inconsistent ontologies. For instance, consider a simple DL-Lite KB $\mathcal{K} = (\mathcal{T}, \mathcal{A})$ where $\mathcal{T} = \{Penguin \sqsubseteq Bird, Swallow \sqsubseteq Bird,$

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Bird $\sqsubseteq$ *Fly*} and $\mathcal{A} = \{Penguin(tweety), \neg Fly(tweety), Swallow(fred)\}$. That KB tells us that penguins are birds; swallows are birds; birds can fly; *tweety* is a penguin; *tweety* cannot fly; and *fred* is a swallow. Under the classical DL semantics, anything can be inferred from $\mathcal{K}$ since $\mathcal{K}$ is not consistent (i.e., it has no any model.). Intuitively, one might wish to still infer *Bird*(*fred*) and *Fly*(*fred*), while it is useless to derive both *Fly*(*tweety*) and $\neg$*Fly*(*tweety*) from $\mathcal{K}$.

There exist several proposals for reasoning with inconsistent DL-Lite KBs in the literature. These approaches usually fall into one of two fundamentally different streams. The first one is based on the assumption that inconsistencies are caused by erroneous data and thus, they should be removed in order to obtain a consistent KB [9,16,5,6]. In most approaches in this stream, the task of repairing inconsistent ontologies is actually reduced to finding a maximum consistent subset of the original KB. A shortcoming of these approaches is similar to the so-called *multi-extension problem* in Reiter's default logic. That is, in many cases, an inconsistent KB may have several different sub-KBs that are maximum consistent. The other stream, based on the idea of living with inconsistency, is to introduce a form of paraconsistent reasoning or inconsistency-tolerant reasoning by employing non-standard reasoning methods (e.g., non-standard inference and non-classical semantics). There are some strategies to select consistent subsets from an inconsistent KB as substitutes of the original KB in reasoning [19,8,13,10,7,21]. The Belnap's four-valued semantics has been successfully extended into DL-Lite [14] where two additional logical values besides "true" and "false" are introduced to indicate contradictory conclusions. Inference power of the four-valued semantics is further enhanced by a new quasi-classical semantics for DLs proposed by Zhang et al. [23], which is a generalization of Hunter's quasi-classical semantics for propositional logic. However, the reasoning capability of such paraconsistent methods is not strong enough for many practical applications. For instance, a conclusion ϕ, that can inferred from a consistent KB $\mathcal{K}$ under the classical semantics, may become not derivable under their paraconsistent semantics. We argue that approaches in these two streams are mostly *coarse-grained* in the sense that they fail to fully utilize semantic information in the given inconsistent KB. For instance, when two interpretations make a concept unsatisfiable, one interpretation may be more reasonable than the other. But existing approaches to paraconsistent semantics in DLs do not take this into account usually.

Recently a distance-based semantics presented by Arieli [1] has been proposed to deal with inconsistent KBs in propositional logic, which is inspired from distance-based merging procedures in propositional logic [11]. However, it is not straightforward to generalize this approach to DLs because it directly works on models (it is feasible in propositional logic since a propositional KB has a finite number of finite models) while, in DLs, a KB might have infinite number of models and a model might also be infinite [3]. Additionally, it is also a challenge in adopting distance-based semantics for complex constructors in DLs.

To overcome these difficulties, in this paper we first use the notion of *features* [20] and then introduce a distance-based semantics for paraconsistent reasoning with DL-Lite. Features in DL-Lite are Herbrand interpretations extended with limited structure, which provide a novel semantic characterization for DL-Lite.

In addition, features also generalize the notion of *types* for TBoxes [12] to general KBs. Each KB in DL-Lite has a finite number of features and each feature is finite. This makes it possible to cast Arieli's distance-based semantics to DL-Lite.

The main innovations and contributions of this paper can be summarized as follows. We introduce distance functions on *types* of DL-Lite$^{\mathcal{N}}_{bool}$ KBs, which avoids the problem of domain infiniteness and model infiniteness in defining the distance function in terms of models of KBs. We choose DL-Lite$^{\mathcal{N}}_{bool}$ [2], one of the most expressive members of the DL-Lite family, and define distance-based semantics for DL-Lite$^{\mathcal{N}}_{bool}$ in a way analogous to the model-based approaches in propositional logic. Although our approach is based on DL-Lite$^{\mathcal{N}}_{bool}$, we argue that our technique can easily be adapted to other DLs. Based on the new distance function on types, we develop a way of measuring types that are closest to a TBox and the notion of *minimal model types* is introduced. This notion is also extended to *minimal model features* for KBs. We propose a distance-based semantics for DL-Lite$^{\mathcal{N}}_{bool}$ so that useful information can still be inferred when a KB is inconsistent. This is accomplished by introducing a novel entailment relation (i.e. distance-based entailment) between a KB and an axiom in terms of minimal model features. Our results show that the distance-based entailment is paraconsistent, non-monotonic, cautious as the paraconsistent based on multi-valued semantics. We also show that the distance-based entailment is not over-skeptical in the sense that for a classically consistent KB, the distance-based entailment coincides with the classical entailment, which is missing in most existing paraconsistent semantics for DLs. Due to the space limitation, all proofs are omitted but they are available in an extended technical report in [22].

2 The DL-Lite Family and Features

DL-Lite $^{\mathcal{N}}_{bool}$. A *signature* is a finite set $\Sigma = \Sigma_A \cup \Sigma_R \cup \Sigma_I \cup \Sigma_N$ where Σ_A is the set of atomic concepts, Σ_R the set of atomic roles, Σ_I the set of individual names (or, objects) and Σ_N the set of natural numbers in Σ. We use capital letters A, B, C (with subscripts C_1, C_2) to denote concept names, P, R, S (with subscripts P_1, P_2) to denote role names, lowercase letters a, b, c to denote individual names and assume 1 is always in Σ_N. $\top$ and $\bot$ will not be considered as concept names or role names.

Formally, given a signature Σ, the DL-Lite$^{\mathcal{N}}_{bool}$ language is inductively constructed by syntax rules: (1) $R \leftarrow P \mid P^-$; (2) $B \leftarrow \top \mid A \mid \geq nR$; and (3) $C \leftarrow B \mid \neg C \mid C_1 \sqcap C_2$. We say B a *basic concept* and C a *general concept*. Other standard concept constructs such as $\bot$, $\exists R$, $\leq nR$ and $C_1 \sqcup C_2$ can be introduced as abbreviations: $\bot$ for $\neg\top$, $\exists R$ for $\geq 1R$, $\leq nR$ for $\neg(\geq (n+1)R)$ and $C_1 \sqcup C_2$ for $\neg(\neg C_1 \sqcap \neg C_2)$. For any $P \in \Sigma_R$, $P^{--} = P$.

A TBox $\mathcal{T}$ is a finite set of *(concept) inclusions* of the form $C_1 \sqsubseteq C_2$ where C_1 and C_2 are general concepts. An ABox $\mathcal{A}$ is a finite set of concept assertions $C(a)$ and role assertions $R(a, b)$. Concept inclusions, concept assertions and role assertions are axioms. A KB is composed of a TBox and an ABox, written by $\mathcal{K} = (\mathcal{T}, \mathcal{A})$. $Sig(\mathcal{K})$ denotes the signature of $\mathcal{K}$.

An interpretation $\mathcal{I}$ is a pair $\langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle$, where $\Delta^{\mathcal{I}}$ is a non-empty set called the *domain* and $\cdot^{\mathcal{I}}$ is an interpretation function such that $a^{\mathcal{I}} \in \Delta^{\mathcal{I}}$, $A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$ and $P^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$. General concepts are interpreted as usual. The definition of interpretation is based on the *unique name assumption* (UNA), i.e., $a^{\mathcal{I}} \neq b^{\mathcal{I}}$ for two different individual names a and b.

An interpretation $\mathcal{I}$ is a *model* of a concept inclusion $C_1 \sqsubseteq C_2$ (a concept assertion $C(a)$, or a role assertion $R(a,b)$) if $C_1^{\mathcal{I}} \subseteq C_2^{\mathcal{I}}$ ($a^{\mathcal{I}} \in C^{\mathcal{I}}$, or $(a^{\mathcal{I}}, b^{\mathcal{I}}) \in R^{\mathcal{I}}$); and $\mathcal{I}$ is called a *model* of a TBox $\mathcal{T}$ (an ABox $\mathcal{A}$) if $\mathcal{I}$ is a model of each inclusion of $\mathcal{T}$ (each assertion of $\mathcal{A}$). $\mathcal{I}$ is called a *model* of a KB $(\mathcal{T}, \mathcal{A})$ if $\mathcal{I}$ is a model of both $\mathcal{T}$ and $\mathcal{A}$. We use $Mod(\mathcal{K})$ to denote the set of models of $\mathcal{K}$. A KB $\mathcal{K}$ *entails* an axiom ϕ, if $Mod(\mathcal{K}) \subseteq Mod(\{\phi\})$. Two KBs $\mathcal{K}_1$ and $\mathcal{K}_2$ are *equivalent* if $Mod(\mathcal{K}_1) = Mod(\mathcal{K}_2)$, denoted by $\mathcal{K}_1 \equiv \mathcal{K}_2$. A KB $\mathcal{K}$ is *consistent* if it has at least one model, *inconsistent* otherwise.

Features. Let Σ be a signature. A Σ-*type* (or simply a *type*) is a set of basic concepts over Σ, s.t., $\top \in \tau$, and for any $m, n \in \Sigma_N$ with $m < n$, $R \in \Sigma_R \cup \{P^- \mid P \in \Sigma_R\}$, $\geq nR \in \tau$ implies $\geq mR \in \tau$. As $\top \in \tau$ for any type τ, we omit it in examples for simplicity. T_Σ denotes the set of all Σ-types. Note that if $\exists P$ (or $\exists P^-$) occurs in a general concept C then $\exists P^-$ (or $\exists P$) should be also considered as a new concept independent of $\exists P$ (or $\exists P^-$) in computing types of C respectively. In the rest of the paper, we will use Ξ to denote a set of types $\{\tau_1, \ldots, \tau_m\}$ (called a *type set*) and use Π to denote a set of type sets $\{\Xi_1, \ldots, \Xi_n\}$ (called a *type group*). Then we denote $\cup\Xi = \tau_1 \cup \cdots \cup \tau_m$ and $\cap\Pi = \Xi_1 \cap \ldots \cap \Xi_n$.

A type τ *satisfies* a basic concept B if $B \in \tau$, τ satisfies $\neg C$ if τ does not satisfy C, and τ satisfies $C_1 \sqcap C_2$ if τ satisfies both C_1 and C_2. $T_\Sigma(C)$ denotes a collection of all Σ-types of C. In this way, each general concept C over Σ corresponds to a set $T_\Sigma(C)$ of all Σ-types satisfying C. A type τ *satisfies* a concept inclusion $C \sqsubseteq D$ if $\tau \in T_\Sigma(\neg C \sqcup D)$. And a type τ is a *model type* of a TBox $\mathcal{T}$ iff it satisfies each inclusion in $\mathcal{T}$. *Model type sets* and *model type groups* are analogously defined. If Ξ is a model type set of a TBox $\mathcal{T}$ then $\exists P \in \cup\Xi$ iff $\exists P^- \in \cup\Xi$. This property is called *role coherence* which can be used to check whether a type set is the model type set of some TBox. $\Pi_\Sigma(\mathcal{T})$ denotes the model type group $\{T_\Sigma(\neg C_1 \sqcup D_1), \ldots, T_\Sigma(\neg C_n \sqcup D_n)\}$ of $\mathcal{T}$ where $\mathcal{T} = \{C_1 \sqsubseteq D_1, \ldots, C_n \sqsubseteq D_n\}$ is a TBox over Σ. It appears that $\cap\Pi_\Sigma(\mathcal{T})$ is the collection of model Σ-types of $\mathcal{T}$.

A Σ-*Herbrand set* (or simply *Herbrand set*) $\mathcal{H}$ is a finite set of member assertions satisfying: (1) for each $a \in \Sigma_I$, if $B_1(a), \ldots, B_k(a)$, where $\{B_1, \ldots, B_k\} \subseteq \Sigma_B$ are all the concept assertions about a in $\mathcal{H}$, then the set $\{B_1, \ldots, B_k\}$ is a Σ-type; (2) for each $P \in \Sigma_R$, if $P(a, b_i)(1 \leq i \leq n)$ are all the role assertions about a in $\mathcal{H}$, then for any $m \in \Sigma_N$ with $m \leq n$, $(\geq mP)(a)$ is in $\mathcal{H}$; (3) for each $P \in \Sigma_R$, if $P(b_i, a)(1 \leq i \leq n)$ are all the role assertions in $\mathcal{H}$, then for any $m \in \Sigma_N$ with $m \leq n$, $(\geq mP^-)(a)$ is in $\mathcal{H}$.

We simply write $\tau(a) = \{B_1(a), \ldots, B_k(a)\}$ where $\tau = \{B_1, \ldots, B_k\}$. Moreover, given a set of types $\Xi = \{\tau_1, \ldots, \tau_m\}$, $\Xi(a)$ denotes $\{\tau_1(a), \ldots, \tau_m(a)\}$ without confusion. In this case, we say $\tau(a)$ is in $\mathcal{H}$ if $\{B_1(a), \ldots, B_k(a)\} \subseteq \mathcal{H}$.

A Herbrand set $\mathcal{H}$ *satisfies* a concept assertion $C(a)$ (a role assertion $P(a,b)$ or $P^-(b,a)$) if $\tau(a)$ is in $\mathcal{H}$ and $\tau \in T_\Sigma(C)$ ($P(a,b) \in \mathcal{H}$ or $P^-(b,a) \in \mathcal{H}$). A Herbrand set $\mathcal{H}$ *satisfies* an ABox $\mathcal{A}$ if $\mathcal{H}$ satisfies all assertions in $\mathcal{A}$.

A *Σ-feature* (or simply a *feature*) $\mathcal{F}$ is a pair $\langle \Xi, \mathcal{H} \rangle$, where Ξ is a non-empty set of Σ-types and $\mathcal{H}$ a Σ-Herbrand set, if $\mathcal{F}$ satisfies: (1) for each $P \in \Sigma_R$, $\exists P \in \bigcup \Xi$ iff $\exists P^- \in \bigcup \Xi$ (i.e., Ξ holds role coherence); and (2) for each $a \in \Sigma_I$ and $\tau(a)$ in $\mathcal{H}$, s.t., τ is a Σ-type, $\tau \in \Xi$. A feature $\mathcal{F}$ *satisfies* an inclusion $C_1 \sqsubseteq C_2$ over Σ, if $\Xi \subseteq T_\Sigma(\neg C_1 \sqcup C_2)$; $\mathcal{F}$ *satisfies* a concept assertion $C(a)$ over Σ, if $\tau(a) \in \mathcal{H}$ and $\tau \in T_\Sigma(C)$; and $\mathcal{F}$ *satisfies* a role assertion $P(a,b)$ (resp., $P^-(b,a)$) over Σ, if $P(a,b) \in \mathcal{H}$. A feature $\mathcal{F}$ is a *model feature* of KB $\mathcal{K}$ if $\mathcal{F}$ satisfies each inclusion and each assertion in $\mathcal{K}$. $Mod^F(\mathcal{K})$ denotes the set of all model features of $\mathcal{K}$. It easily concludes that $\mathcal{K}$ is consistent iff $Mod^F(\mathcal{K}) \neq \emptyset$. Given two KBs $\mathcal{K}_1$ and $\mathcal{K}_2$, let $\Sigma = Sig(\mathcal{K}_1 \cup \mathcal{K}_2)$, $\mathcal{K}_1$ *F-entails* $\mathcal{K}_2$ if $Mod^F(\mathcal{K}_1) \subseteq Mod^F(\mathcal{K}_2)$, written by $\mathcal{K} \models^F \mathcal{K}_2$; and $\mathcal{K}_1$ is *F-equivalent* $\mathcal{K}_2$ if $Mod^F(\mathcal{K}_1) = Mod^F(\mathcal{K}_2)$, written by $\mathcal{K} \equiv^F \mathcal{K}_2$. In [20], we conclude that: *(1)* $\mathcal{K}_1 \models \mathcal{K}_2$ iff $\mathcal{K}_1 \models^F \mathcal{K}_2$; *(2)* $\mathcal{K}_1 \equiv \mathcal{K}_2$ iff $\mathcal{K}_1 \equiv^F \mathcal{K}_2$.

3 Distance-Based Semantics for TBoxes

To measure the closeness of two types, we first define a distance function between two types in terms of the symmetric difference for sets.

Definition 1. *Let Σ be a signature, a total function $d : T_\Sigma \times T_\Sigma \to \mathbb{R}^+ \cup \{0\}$ is a* pseudo-distance function *(for short,* distance function*) on T_Σ if it satisfies: (1) $\forall \tau_1, \tau_2 \in T_\Sigma, d(\tau_1, \tau_2) = 0$ iff $\tau_1 = \tau_2$; and (2) $\forall \tau_1, \tau_2 \in T_\Sigma, d(\tau_1, \tau_2) = d(\tau_2, \tau_1)$.*

Given a type $\tau \in T_\Sigma$ and a type set $\Xi \subseteq T_\Sigma$, the distance function between τ and Ξ is defined as $d(\tau, \Xi) = min\{d(\tau, \tau') \mid \tau' \in \Xi\}$.

If $\Xi = \emptyset$, then we set $d(\tau, \Xi) = \mathbf{d}$ where $\mathbf{d}$ is a default value of distance function greater than any value be to considered. This setting is used to exclude all contradictions (e.g., $\top \sqsubseteq \bot$) under our candidate semantics since a contradiction can bring less useful information.

There are two representative distance functions on types, namely, *Hamming distance function* where $d^H(\tau_1, \tau_2) = |(\tau_1 - \tau_2) \cup (\tau_2 - \tau_1)|$ and *drastic distance function* where $d^D(\tau_1, \tau_2) = 0$ if $\tau_1 = \tau_2$ and $d^D(\tau_1, \tau_2) = 1$ otherwise.

An *aggregation function* f is a total function that accepts a multi-set of real numbers and returns a real number, satisfying: (1) f is non-decreasing in the values of its argument; (2) $f(\{x_1, \ldots, x_n\}) = 0$ iff $x_1 = \ldots = x_n = 0$; and (3) $\forall x \in \mathbb{R}^+ \cup \{0\}$, $f(\{x\}) = x$. There exist some popular aggregation functions [15]:

- The *summation* function: $f^s(x_1, \ldots, x_n) = \sum_{1 \leq i \leq n} x_i$;
- The *maximum* function: $f^m(x_1, \ldots, x_n) = max_{1 \leq i \leq n} x_i$;
- The κ-*voting* function ($0 < \kappa < 1$): $f^\kappa(x_1, \ldots, x_n) = 0$ if $Zero(\{x_1, \ldots, x_n\}) = n$; $f^\kappa(x_1, \ldots, x_n) = \frac{1}{2}$ if $\lceil \kappa \cdot n \rceil \leq Zero(\{x_1, \ldots, x_n\}) < n$ and $f^\kappa(x_1, \ldots, x_n) = 1$ otherwise, where $Zero(\{x_1, \ldots, x_m\})$ is the number of zeros in $\{x_1, \ldots, x_n\}$.

Definition 2. *Let Σ be a signature, τ a type and $\Pi = \{\Xi_1, \ldots, \Xi_n\}$ a type group. Given a distance function d and an aggregation function f, $\lambda_{d,f}$ between τ and Π is defined as $\lambda_{d,f}(\tau, \Pi) = f(\{d(\tau, \Xi_1), \ldots, d(\tau, \Xi_n)\})$. Furthermore, τ is called df*-minimal *(for short,* minimal*) w.r.t. Π if for any type $\tau' \in T_\Sigma$, $\lambda_{d,f}(\tau, \Pi) \leq \lambda_{d,f}(\tau', \Pi)$.*

We use $\Lambda_{d,f}(\Pi, \Xi)$ to denote a set of all *df*-minimal types w.r.t. Π in Ξ.

Proposition 1. *Let Σ be a finite signature and $\Pi = \{\Xi_1, \ldots, \Xi_n\}$ ($n \geq 1$) a type group over Σ. If Ξ_i For any distance function d and any aggregation function f, we have (1) $\Lambda_{d,f}(\Pi, T_\Sigma) \neq \emptyset$ and (2) If $\cap\Pi \neq \emptyset$ then $\Lambda_{d,f}(\Pi, T_\Sigma) = \cap\Pi$.*

The first statement guarantees that every type group has always at least minimal type if this type group contains a non-empty type set and the second shows that each type belong to all members of a type group is exactly a minimal type.

Let Σ be a signature and $\mathcal{T} = \{\psi_1, \ldots, \psi_n\}$ a TBox over Σ. Each axiom ψ_i is of the form $C_i \sqsubseteq D_i$ $(1 \leq i \leq n)$ where C_i, D_i $(1 \leq i \leq n)$ are concepts. We simply write $\Pi_\Sigma(\mathcal{T})$ as $\Pi(\mathcal{T})$ if $\Sigma = Sig(\mathcal{T})$.

Corollary 1. *Let Σ be a finite signature and $\mathcal{T}$ a TBox over Σ. For any distance function d and any aggregation function f, we have (1) $\Lambda_{d,f}(\Pi_\Sigma(\mathcal{T}), T_\Sigma) \neq \emptyset$; and (2) If $\mathcal{T}$ is consistent then $\Lambda_{d,f}(\Pi_\Sigma(\mathcal{T}), T_\Sigma) = \cap\Pi_\Sigma(\mathcal{T})$.*

The above second item is no longer true if a TBox $\mathcal{T}$ is not consistent.

Example 1. Let $\mathcal{T} = \{\top \sqsubseteq A, A \sqsubseteq \exists P, \exists P \sqsubseteq \bot\}$ and $\Sigma = Sig(\mathcal{T})$. So $\Sigma = \{A, P\}$ and $\mathcal{T}$ is inconsistent. $\mathcal{T}$ has eight possible types: $\tau_{11} = \{\}$, $\tau_{12} = \{\exists P^-\}$, $\tau_{21} = \{\exists P\}$, $\tau_{22} = \{\exists P, \exists P^-\}$, $\tau_{31} = \{A\}$, $\tau_{32} = \{A, \exists P^-\}$, $\tau_{41} = \{A, \exists P\}$ and $\tau_{41} = \{A, \exists P, \exists P^-\}$. Thus, we have $\Lambda_{d^H,f^s}(\Pi(\mathcal{T}), T_\Sigma) = \{\tau_{11}, \tau_{12}, \tau_{31}, \tau_{32}, \tau_{41}, \tau_{42}\}$ while $\cap\Pi(\mathcal{T}) = \emptyset$.

Unfortunately, $\Lambda_{d,f}(\Pi(\mathcal{T}), T_\Sigma)$ does not always satisfy the role coherence as the following example shows.

Example 2. Let $\mathcal{T} = \{\top \sqsubseteq A \sqcap \exists P, \exists P^- \sqsubseteq \bot\}$ and $\Sigma = Sig(\mathcal{T})$. If d is the Hamming distance function and f is the summation function, then $\Lambda_{d^H,f^s}(\Pi(\mathcal{T}), T_\Sigma) = \{\{A, \exists P\}\}$. Note that $\exists P^- \notin \cup\Lambda_{d^H,f^s}(\Pi(\mathcal{T}), T_\Sigma)$.

The reason that the role coherence might be absent in $\Lambda_{d,f}(\Pi(\mathcal{T}), T_\Sigma)$ is that $\exists P$ and $\exists P^-$ are taken as two independent concepts so that the relation of satisfiability between $\exists P$ and $\exists P^-$ cannot be captured when minimal types are computed [24]. To construct a model type set from a random type set Ξ, we introduce an iterative operator $\mu_{d,f}(\Xi)$ and its fixpoint.

Given an arbitrary type set Ξ, if it is not a model type set of any TBox, there are two possible options to recovery the role coherence: removing and adding. For instance, if $\tau \in \Xi$ such that $\exists R \in \tau$ and $\exists R^- \notin \cup\Xi$ for some role R, then we can either remove τ from Ξ or add a new type τ such that $\exists R^- \in \tau$ in to

Ξ. In Example 2, if we remove the type $\{A, \exists P\}$, then $\Lambda_{d^H,f^s}(\Pi(\mathcal{T}), T_\Sigma)$ will be empty, which is not desirable. In other words, the removing approach could cause the empty type set where the reasoning becomes trivial. So we will extend the type violating the role coherence. Consider Example 2 again, there are three possible types $\tau_1 = \{\exists P^-\}, \tau_2 = \{A, \exists P^-\}$ and $\tau_3 = \{A, \exists P, \exists P^-\}$ such that $\exists P^- \in \tau_i$ $(i = 1, 2, 3)$ where $\lambda_{d^H,f^s}(\tau_1, \Pi(\mathcal{T})) = 3$, $\lambda_{d^H,f^s}(\tau_2, \Pi(\mathcal{T})) = 2$ and $\lambda_{d^H,f^s}(\tau_3, \Pi(\mathcal{T})) = 1$. So we can pick τ_3 as the desired minimal type. Furthermore, this extension is an iterative process since newly added types possibly contains new role names and role incoherence is not yet satisfied at every step. To construct a model type set from a random type set Ξ, we introduce an iterative operator $\mu_{d,f}(\Xi)$ and its fixpoint.

Formally, let Σ be a finite signature and Π a type group over Σ. Given a type set Ξ over Σ, let $\mu_{d,f}(\Xi) = \Xi \cup \Xi'$, where $\Xi' \subseteq T_\Sigma$ and $\Xi' = \{\tau \mid$ for some role R, $\exists R \in \cup\Xi$ and $\exists R^- \notin \cup\Xi$, $\exists R^- \in \tau$ and for any type $\tau' \in T_\Sigma$, $\exists R^- \in \tau'$ implies $\lambda_{d,f}(\tau, \Pi) \leq \lambda_{d,f}(\tau', \Pi)\}$. We use Ξ^+ to denote the *fixpoint* of $\mu_{d,f}$, i.e., $\Xi^+ = FP(\mu_{d,f})(\Xi)$. For any distance function d, any aggregation function f, and any type set Ξ, we can conclude that Ξ^+ always exists since $\mu_{d,f}$ is inflationary (i.e., $\Xi \subseteq \mu_{d,f}(\Xi)$) and Σ is finite.

Given a signature Σ and a TBox $\mathcal{T}$ over Σ, we say $\Lambda^+_{d,f}(\Pi(\mathcal{T}), T_\Sigma)$ is the *minimal model type set* of $\mathcal{T}$. Intuitively, a minimal model type set is a set of minimal types with maintaining role coherence. In Example 2, $\Lambda^+_{d^H,f^s}(\Pi(\mathcal{T}), T_\Sigma)$ $= \Lambda_{d^H,f^s}(\Pi(\mathcal{T}), T_\Sigma) \cup \{\tau_3\} = \{\{A, \exists P\}, \{A, \exists P, \exists P^-\}\}$.

We show that minimal model type sets meet our motivation.

Proposition 2. *Let Σ be a signature and $\mathcal{T}$ a TBox over Σ. For any distance function d and aggregation function f, we have*

- $\Lambda^+_{d,f}(\Pi_\Sigma(\mathcal{T}), T_\Sigma) \neq \emptyset$;
- $\Lambda^+_{d,f}(\Pi_\Sigma(\mathcal{T}), T_\Sigma) = \cap\Pi_\Sigma(\mathcal{T})$, *if $\mathcal{T}$ is coherent;*
- $\exists P \in \cup\Lambda^+_{d,f}(\Pi_\Sigma(\mathcal{T}), T_\Sigma)$ *iff* $\exists P^- \in \cup\Lambda^+_{d,f}(\Pi_\Sigma(\mathcal{T}), T_\Sigma)$ *for any* $P \in \Sigma_R$.

In Proposition 2, the first item states that there always exist minimal model types for any non-empty TBox; the second shows that when a TBox is consistent, each minimal model type is exactly a model type; and the third ensures that minimal model type sets always satisfy the role coherence.

Definition 3. *Let Σ be a signature, $\mathcal{T}$ a TBox, and, ϕ an inclusion over Σ. Given a distance function d and an aggregation function f, $\mathcal{T}$ distance-based entails (d-entails) ϕ, denoted by $\mathcal{T} \models_{d,f} \psi$, if $\Lambda^+_{d,f}(\Pi_\Sigma(\mathcal{T}), T_\Sigma) \subseteq Mod^T(\{\phi\})$.*

In Example 2, $\mathcal{T} \models_{d^H,f^s} \top \sqsubseteq A$.

4 Distance-Based Semantics for Knowledge Bases

Compared with inconsistency of TBoxes, inconsistency occurring in KBs is much more complex. For instance, *Example 3.* Let $\mathcal{K} = (\{\exists P^- \sqsubseteq \bot\}, \{\exists P(a)\})$ be a KB and $\Sigma = \{P, a, 1\}$. $\mathcal{K}$ is inconsistent and thus has no model feature.

We first introduce *concept profiles* and then use type distance function to describe how far apart features are. Let Σ be a signature and $\mathcal{A}$ an ABox over Σ. Assume that N_A is a set of all named individuals in $\mathcal{A}$. Let $\mathcal{A}_R = \{P(a,b) \mid P(a,b) \text{ or } P^-(b,a) \in \mathcal{A}\}$. A *concept profile* of a in $\mathcal{A}$, denoted by $\Sigma_C(a)$, is defined as follows:

$$\Sigma_C(a) = \bigcup_{D(a)\in\mathcal{A}} \{D\} \cup \bigcup_{P(a,b_1),\ldots,P(a,b_n)\in\mathcal{A}_R} \{\geq m\, P \mid m \in \Sigma_N, m \leq n\}$$
$$\cup \bigcup_{P(b_1,a),\ldots,P(b_n,a)\in\mathcal{A}_R} \{\geq m\, P^- \mid m \in \Sigma_N, m \leq n\}.$$

Intuitively, a set of concept profiles is a partition of concepts that are realized in that ABox w.r.t. individuals. For instance, let $\Sigma = \{C, D, P, a, b_1, b_2, 1, 2\}$ and $\mathcal{A} = \{C \sqcap D(a), P(a, b_1), P(a, b_2), D(b_1)\}$. Thus $\Sigma_C(a) = \{C \sqcap D, \exists P, \geq 2P\}$, $\Sigma_C(b_1) = \{D, \exists P^-\}$, and $\Sigma_C(b_2) = \{\exists P^-\}$.

Let $\mathcal{K} = (\mathcal{T}, \mathcal{A})$ be a KB. We extend the signature $Sig(K)$ of $\mathcal{K}$ as $Sig^*(\mathcal{K}) = Sig(\mathcal{T}) \cup Sig(\Sigma_C(\mathcal{A}))$ where $\Sigma_C(\mathcal{A}) = \bigcup_{a\in N_A} \Sigma_C(a)$. Indeed, $Sig^*(\mathcal{K})$ is obtained from $Sig(\mathcal{K})$ by adding all possible natural numbers occurring in all concept profiles but not occurring in $\mathcal{K}$. In other words, $Sig^*(\mathcal{K})$ and $Sig(\mathcal{K})$ are no different except Σ_N. In the above example, $Sig(\mathcal{A}) = \{C, D, P, a, b_1, b_2\}$ while $Sig^*(\mathcal{A}) = \{C, D, P, a, b_1, b_2, 1, 2\}$.

Next, we will define the notion of minimal model features.

Definition 4. *Let Σ be a signature and $\mathcal{K} = (\mathcal{T}, \mathcal{A})$ a KB over Σ. Denote $\Pi_\Sigma(a) = \{T_\Sigma(D) \mid D \in \Sigma_C(a)\}$. Given a distance function d and an aggregation function f, a df-*minimal model feature *of $\mathcal{K}$ is a feature $\mathcal{F} = \langle \Xi, \mathcal{H}\rangle$ satisfying the following four conditions:*

- *$\Xi \subseteq \Lambda^+_{d,f}(\Pi_\Sigma(\mathcal{T}), T_\Sigma)$;*
- *for each $P \in \Sigma_R$, $\exists P \in \cup\Xi$ iff $\exists P^- \in \cup\Xi$;*
- *$\tau \in \Lambda^+_{d,f}(\Pi_\Sigma(a), \Lambda^+_{d,f}(\Pi_\Sigma(\mathcal{T}), T_\Sigma)) \cap \Xi$ for each $a \in \Sigma_I$ and $\tau(a) \in \mathcal{H}$;*
- *for any role assertion $P(a,b) \in \mathcal{A}_R - \mathcal{H}$, either $\geq n+1\, P(a) \notin \mathcal{H}$ and $P(a,b_1)$, $\ldots$, $P(a,b_n) \in \mathcal{H}$, or $\geq n+1\, P^-(b) \notin \mathcal{H}$ and $P(a_1,b), \ldots, P(a_n,b) \in \mathcal{H}$.*

Let $Mod^F_{d,f}(\mathcal{K})$ denote the set of df-minimal model features of $\mathcal{K}$.

In Definition 4, a minimal model feature is a feature $\mathcal{F}$ which contains two parts, namely, a type set Ξ and a Herbrand set $\mathcal{H}$. The first condition requires that all types of Ξ are minimal; the second says that Ξ should be a model type set, i.e., it satisfies the property of role coherence; the third guarantees that each type of Ξ satisfying each concept assertion in $\mathcal{H}$ has the minimal distance function to its corresponding concept profile, that is, if a concept assertion $D(a)$ is satisfied by $\mathcal{H}$ then types satisfying D are minimal w.r.t. type group $\Pi_\Sigma(a)$ of concept profile $\Sigma_C(a)$; and the last ensures that $\mathcal{F}$ is consistent by those role assertions conflicting with concept assertions.

Example 4. In *Penguin* KB, we abbreviate *Penguin* to P, *Swallow* to S, *Bird* to B, *Fly* to F, *tweety* to t and *fred* to r. Let $\Sigma = \{P, S, B, F, t, r\}$, $\Sigma_C(t) = \{P, \neg F\}$ and $\Sigma_C(r) = \{S\}$. We have $\Lambda^{+}_{d^H, f^s}(\Pi_\Sigma(\mathcal{T}), T_\Sigma) = \{\tau_1, \tau_2, \tau_4, \tau_8, \tau_{12}, \tau_{16}\}$. Here $\tau_1 = \{\}$, $\tau_2 = \{F\}$, $\tau_4 = \{B, F\}$,$\tau_8 = \{S, B, F\}$, $\tau_{12} = \{P, B, F\}$, and $\tau_{16} = \{P, S, B, F\}$. All of whose distance is 0. We have $\Lambda^{+}_{d^H, f^s}(\Pi_\Sigma(t), \Lambda^{+}_{d^H, f^s}(\Pi_\Sigma(\mathcal{T}), T_\Sigma)) = \{\tau_1, \tau_{12}, \tau_{16}\}$ and $\Lambda^{+}_{d^H, f^s}(\Pi_\Sigma(r), \Lambda^{+}_{d^H, f^s}(\Pi_\Sigma(\mathcal{T}), T_\Sigma)) = \{\tau_8, \tau_{16}\}$. All types in $\Lambda^{+}_{d^H, f^s}(\Pi_\Sigma(t), T_\Sigma)$ have distance equal to 1 while all types in $\Lambda^{+}_{d^H, f^s}(\Pi_\Sigma(r), T_\Sigma)$ have distance equal to 0. Thus, $Mod^F_{d,f}(\mathcal{K}) = \{\langle \Xi, \tau(t) \cup \tau'(r)\rangle \mid \tau \in \{\tau_1, \tau_{12}, \tau_{16}\}$, $\tau' \in \{\tau_8, \tau_{16}\}$, $\{\tau, \tau'\} \subseteq \Xi$ and $\Xi \subseteq \{\tau_1, \tau_8, \tau_{12}, \tau_{16}\}\}$.

We find that minimal model features can reach our aim.

Proposition 3. *Let Σ be a signature and $\mathcal{K}$ a KB over Σ. For any distance function d and any aggregation function f, we have*

– $Mod^F_{d,f}(\mathcal{K}) \neq \emptyset$;
– $Mod^F_{d,f}(\mathcal{K}) = Mod^F(\mathcal{K})$, *if $\mathcal{K}$ is consistent.*

An expected result is that the second statement of Proposition 3 does not necessarily hold if $\mathcal{K}$ is inconsistent. For instance, in Example 3, $Mod^F_{d,f}(\mathcal{K}) = \{\mathcal{F}_1, \mathcal{F}_2\}$ where $\mathcal{F}_1 = \langle\{\exists P\}, \{\exists P(a)\}\rangle$ and $\mathcal{F}_2 = \langle\{\exists P, \exists P^-\}, \{\exists P(a), \exists P^-(a)\}\rangle$ while $Mod^F(\mathcal{K}) = \emptyset$.

Now, based on minimal model features, we are ready to define the *distance-based entailment* for KBs, written $\models_{d,f}$, under which meaningful information can be entailed from an inconsistent KB.

Definition 5. *Let Σ be a signature, $\mathcal{K}$ a KB, and, ϕ an axiom over Σ. Given a distance function d and an aggregation function f, $\mathcal{K}$ distance-based entails (d-entails) ϕ, still denoted by $\mathcal{K} \models_{d,f} \phi$, if $Mod^F_{d,f}(\mathcal{K}) \subseteq Mod^F(\{\phi\})$.*

Distance-based entailment brings a new semantics (called *distance-based semantics*) for inconsistent KBs by weakening classical entailment. It is not hard to see that no contradiction can be entailed in this semantics. For instance, in *Penguin* KB, $\neg$*Fly* $\sqcap$ *Fly*(*tweety*) cannot be entailed but $\neg$*Fly* $\sqcup$ *Fly*(*tweety*) can under our new semantics.

In the rest of this section, we exemplify that the distance-based semantics is suitable for reasoning with inconsistent KBs.

Consequences are intuitive and reasonable under the distance-based semantics. In *Penguin* KB, $\mathcal{K} \models_{d^H, f^s}$ *Fly*(*fred*) while $\mathcal{K} \not\models_{d^H, f^s}$ *Penguin*(*tweety*) and $\mathcal{K} \not\models_{d^H, f^s}$ *Fly*(*tweety*). We further analyze those conclusions under distance-based semantics. The inconsistency of $\mathcal{K}$ is caused by statement about *tweety*. On the one hand, *tweety* is a penguin which cannot fly, i.e., $\neg$*Fly*(*tweety*). On the other hand, a penguin is a bird which can fly, i.e., *Fly*(*tweety*). Moreover, there exists no more argument for either *Penguin*(*tweety*) or *Fly*(*tweety*). In this sense, neither *Penguin*(*tweety*) nor *Fly*(*tweety*) can be entailed under distance-based semantics. However, the statement about *fred* in $\mathcal{K}$ contains no conflict. Thus *Fly*(*fred*) can be entailed under distance-based semantics. Additionally, let us consider a simple example: let $\mathcal{A} = \{A(a), \neg A(a), B(b)\}$. We can conclude that $\mathcal{A} \models_{d^H,f^s} B(b)$ while neither $\mathcal{A} \not\models_{d^H,f^s} A(a)$ nor $\mathcal{A} \not\models_{d^H,f^s} \neg A(a)$.

In general, different result for a KB would be brought by selecting different distance function and different aggregation.

Example 5. Let $\mathcal{A} = \{A(a), \neg A \sqcap \exists P(a), \neg\exists P(a)\}$ be an inconsistent ABox and $\Sigma = \{A, P, a\}$. Thus $\mathcal{A} \models_{d^H,f^m} \neg A \sqcup \exists P(a)$ while $\mathcal{A} \not\models_{d^D,f^m} \neg A \sqcup \exists P(a)$.

For instance, in Example 3, $\mathcal{A} \models_{d^H,f^{\frac{1}{2}}} A(a)$ while $\mathcal{A} \not\models_{d^H,f^s} A(a)$.

5 Properties of Distance-Based Semantics

In this section, we present some useful properties of the distance-based semantics.

If $\mathcal{K}$ is inconsistent and there exists an axiom ϕ such that $\mathcal{K} \not\models_p \phi$ where $\models_p$ is an entailment relation, then we say $\models_p$ is *paraconsistent*. It is well known that the classical entailment $\models$ is not paraconsistent. We reconsider Example 3 and we have $\mathcal{K} \models_{d^H,f^s} \exists P^- \sqsubseteq \bot$ while $\mathcal{K} \not\models_{d^H,f^s} \exists P(a)$.

The following result shows that the distance-based entailment is paraconsistent.

Proposition 4. *For any distance function d and any aggregation function f, $\models_{d,f}$ is paraconsistent.*

Most existing semantics for paraconsistent reasoning in DLs are much weaker than the classical semantics in this sense that there exists a consistent KB $\mathcal{K}$ and an axiom ϕ such that $\mathcal{K} \models \phi$ (also called *consistency preservation*) but ϕ is not entailed by $\mathcal{K}$ under the paraconsistent semantics. The following result shows that the distance-based semantics does not have such shortcoming.

We can conclude a result directly following Proposition 3.

Proposition 5. *Let Σ be a signature, $\mathcal{K}$ a KB, and, ϕ an axiom over Σ. For any distance function d and any aggregation function f, if $\mathcal{K}$ is consistent then we can conclude that $\mathcal{K} \models_{d,f} \phi$ iff $\mathcal{K} \models \phi$.*

Under the classical semantics, a property that $\mathcal{K} \models \psi$ iff $\mathcal{T} \models \psi$ for any inclusion ψ is called *TBox-preservation* where the problem of subsumption checking is irrelevant to ABoxes. Our distance-based semantics satisfies such a property.

Proposition 6. *Let Σ be a signature, $\mathcal{K} = (\mathcal{T}, \mathcal{A})$ a KB, and, ψ an inclusion over Σ. For any distance function d and any aggregation function f, $\mathcal{K} \models_{d,f} \psi$ iff $\mathcal{T} \models_{d,f} \psi$.*

By Proposition 6, TBox preservation property means that if the TBox by itself is consistent, then it will be entailed (and hence preference is given to preserving TBox statements over ABox statements), such as the same treatment in [13]. This is different from some other approaches to inconsistency-handling in DLs, where the TBox and ABox are equally treated, or the ABox is given preference such as [15,21,23].

The closure w.r.t. $\models_{d,f}$ of an arbitrary KB is always consistent.

Proposition 7. *Let Σ be a signature and $\mathcal{K} = (\mathcal{T}, \mathcal{A})$ a KB over Σ. For any distance function d and any aggregation function f, let $Cn_{d,f}(\mathcal{T}) = \{\psi$ is an inclusion $\mid \mathcal{T} \models_{d,f} \psi\}$ and $Cn^{\mathcal{T}}_{d,f}(\mathcal{A}) = \{\varphi$ is an assertion $\mid (\mathcal{T}, \mathcal{A}) \models_{d,f} \varphi\}$. We conclude that both $Cn_{d,f}(\mathcal{T})$ and $Cn^{\mathcal{T}}_{d,f}(\mathcal{A})$ are consistent.*

Proposition 7 provides a theoretical foundation of applying our approach to *inconsistency-tolerant conjunctive query answering* [3].

Let Σ be a signature. A distance function d is *Σ-unbiased*, if for any Σ-concept C and any two Σ-types τ_1, τ_2, $B \in \tau_1$ iff $B \in \tau_2$ for any basic concept B occurring in C implies $d(\tau_1, T_\Sigma(C)) = d(\tau_2, T_\Sigma(C))$. The Hamming distance function and the drastic distance function are unbiased.

Let us consider a distance function $d^\cup$ defined as follows: for any two sets S_1, S_2, $d^\cup(S_1, S_2) = 0$ if $S_1 = S_2$; and $d^\cup(S_1, S_2) = 1 + |S_1 \cup S_2|$. It clearly concludes that $d^\cup(S_1, S_2) = 0$ iff $S_1 = S_2$ and $d^\cup(S_1, S_2) = d^\cup(S_2, S_1)$. Thus $d^\cup$ is a distance function. Let $\Sigma = \{A_1, A_2, A_3, A_4\}$ and $C = A_1 \sqcap A_2$. For each type $\tau \in T_\Sigma(C)$, $\{A_1, A_2\} \subseteq \tau$. Let $\tau_1 = \{A_1, A_2\}$ and $\tau_2 = \{A_1, A_2, A_3, A_4\}$. Thus $d(\tau_1, T_\Sigma(C)) = 5$ and $d(\tau_2, T_\Sigma(C)) = 7$. Then $d^\cup$ is not unbiased.

Unbiasedness will bring a good property of relevance in reasoning since the unbiased distance is not sensitive to those irrelevant basic concepts.

Proposition 8. *Let Σ be a signature, $\mathcal{K}$ a KB, and, ϕ a non-tautology over Σ. If d is an unbiased distance function and $Sig(\mathcal{K}) \cap Sig(\{\phi\}) = \emptyset$ then for any aggregation function f, $\mathcal{K} \not\models_{d,f} \phi$.*

An entailment relation $\models_m$ is *monotonic* if $\mathcal{K}' \models_m \phi$ implies $\mathcal{K} \models_m \phi$ for any KB $\mathcal{K}' \subseteq \mathcal{K}$; and *nonmonotonic* otherwise. Another characteristic property of $\models_{d,f}$ is its non-monotonic nature.

Proposition 9. *For any distance function d and any aggregation function f, $\models_{d,f}$ is non-monotonic.*

A relation $|\!\approx$ is *cautious* if it satisfies:

- (*cautious reflexivity*) If $\mathcal{K} = \mathcal{K}' \oplus \mathcal{K}''$ and $\mathcal{K}'$ is consistent, then $\mathcal{K} |\!\approx \varphi$ for all axiom $\varphi \in \mathcal{K}'$;
- (*cautious monotonicity*) If $\mathcal{K} |\!\approx \varphi$ and $\mathcal{K} |\!\approx \psi$, then $\mathcal{K} \cup \{\varphi\} |\!\approx \psi$;
- (*cautious cut*) If $\mathcal{K} |\!\approx \varphi$ and $\mathcal{K} \cup \{\varphi\} |\!\approx \psi$ then $\mathcal{K} |\!\approx \psi$.

Proposition 10. *For any distance function d and any monotonic hereditary aggregation function f, $\models_{d,f}$ is cautious.*

Example 6. Consider an ABox $\mathcal{A} = \{HasWife(Mike, Rose), HasWife(Mike, Mary), \neg(\geq 2\, HasWife)(Mike)\}$. Let $\Sigma = \{HasWife, Mike, Mary, Rose, 1, 2\}$. The first statement claims that *Mike* has at most one wife. Moreover, we are informed that *Mike* has two wives *Rose* and *Mary*. We conclude that $\mathcal{A}$ is inconsistent and $\mathcal{A} \models_{d^H, f^s} \geq 1 HasWife(Mike)$. Moreover, we can also conclude that $\mathcal{A} \not\models_{d^H, f^s} HasWife(Mike, Rose)$, and $\mathcal{A} \not\models_{d^H, f^s} HasWife(Mike, Mary)$. Intuitively, *Mike* has a wife while we don't know whether his wife is *Rose* or *Mary* under our distance-based semantics.

6 Discussions

Existing model-centered approaches for inconsistency handling are usually based on various forms of inconsistency-tolerant semantics, such as four-valued description logics [14,15], quasi-classical description logics [23], the argumentation-based semantics for description logics [10,21], and the MKNF-based semantics for description logics [7]. Compared to them, our distance-based semantics works on classical interpretations but still can draw more useful and reasonable logical consequences. Different from [7] which introduces a weak negation **not** to tolerate inconsistency, our approach does not change the syntax of DLs. Different from syntax-based paraconsistent approaches taking some consistent subsets as substitutes of KBs in reasoning [19,8,9,16,6,18], our approach can satisfy the closure consistency.

There are some model-based approaches presented in [17,13]. Compared with it directly working on models, our approach works on types and features which take advantage of finiteness. Moreover, we construct those models which are closer to a KB according to some distance functions and aggregation functions when there exists no model in an inconsistent KB. A distance-based approach is proposed to measure inconsistency of TBoxes [15]. However, this approach might be difficult to do so because of infinite number of models of DL KBs since it is based on the distance between models. As a future work, we employ our distance-based technique to measure inconsistency of KBs. A simpler semantic characterisation called type semantics has been developed for DL-Lite in [24]. We plan to study the issue of handling DL paraconsistency using the type semantics.

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