## Contents

### 1 Mathematical Problem-Solving with Technology:
#### An Overview of the Problem@Web Project ........................................... 1

- **1.1 Introduction** .................................................................................... 1
- **1.2 Young People with Technology ......................................................... 2
- **1.3 Young People’s Mathematical Problem-Solving with Technology** ........................................... 4
- **1.4 The Research Focus ........................................................................ 6
- **1.5 The SUB12 and SUB14 Mathematics Competitions** ......................... 8
- **1.6 Methodological Issues .................................................................... 15
- **1.7 Concluding Comments.................................................................... 17

References ................................................................................................... 18

### 2 Youngsters Solving Mathematical Problems with Technology:
#### Their Experiences and Productions .................................................... 21

- **2.1 Introduction** .................................................................................... 21
- **2.2 The Participants in the Mathematical Competitions SUB12 and SUB14** .................................................................................... 22
- **2.3 The Participants and the Use of Digital Technologies** ................. 26
- **2.4 The Participants’ Productions with Digital Technologies** ........... 29
  - **2.4.1 From the Use of Paper and Pencil to Writing with Word and Excel** ........................................................... 29
  - **2.4.2 The Use of Tables .................................................................. 35
  - **2.4.3 The Use of Images and Diagrams .......................................... 39
  - **2.4.4 The Use of Numerical Software ............................................ 44
  - **2.4.5 The Use of Geometrical Software .......................................... 45
- **2.5 Concluding Comments.................................................................... 51

References................................................................................................... 53
3 Perspectives of Teachers on Youngsters Solving Mathematical Problems with Technology ........................................ 55
   3.1 Introduction ........................................................................................................ 55
   3.2 The Role of the Teachers in the Mathematical Competitions ......................... 57
      3.2.1 The Support of the Teachers: From the First Round to the Final .................. 59
      3.2.2 The Social Part of the Competitions: The Meeting at the Final .................... 62
   3.3 Perspectives of Teachers About the Mathematical Competitions SUB 12 and SUB14 ........................................ 63
   3.4 Mathematical Communication: An Additional Challenge ................................. 72
   3.5 The Use of Technology: The Sharing of Experiences Between Teachers and Students ........................................ 74
   3.6 Overview and Conclusion ................................................................ 79
References .............................................................................................................. 80

4 Theoretical Perspectives on Youngsters Solving Mathematical Problems with Technology ........................................ 83
   4.1 The Theoretical Stance ...................................................................................... 83
   4.2 Problem-Solving as Mathematisation ............................................................... 85
   4.3 Problem-Solving as Expressing Thinking ....................................................... 88
      4.3.1 Expository Discourse in Problem-Solving .................................................. 90
      4.3.2 Technology Used for Expressing Thinking in Problem-Solving .................. 93
   4.4 Multiple External Representations ................................................................ 96
   4.5 Humans-with-Media and Co-action with Digital Tools .................................... 98
   4.6 An Outlook .................................................................................................... 106
References .............................................................................................................. 108

5 Digitally Expressing Conceptual Models of Geometrical Invariance ............................................................... 113
   5.1 Main Theoretical Ideas .................................................................................. 113
      5.1.1 Perceiving Affordances of Digital Tools .................................................... 114
      5.1.2 The Indivisibility Between the Subject and the Context ......................... 117
      5.1.3 Humans-with-Media Mathematising ......................................................... 119
      5.1.4 Mathematisation with Dynamic Geometry Software ................................. 120
   5.2 Context and Method ...................................................................................... 122
   5.3 Data Analysis .................................................................................................. 124
      5.3.1 The Problem: Building a Flowerbed .......................................................... 124
      5.3.2 Zooming in: The Participants’ Productions ................................................. 125
      5.3.3 Zooming Out: Comparing and Contrasting ............................................... 135
   5.4 Discussion and Conclusion ........................................................................... 138
References .............................................................................................................. 139
6 Digitally Expressing Algebraic Thinking in Quantity Variation .......................... 141
   6.1 Main Theoretical Ideas .................................................................................. 141
   6.1.1 Digital Representations in the Spreadsheet ............................................. 142
   6.1.2 Algebraic Thinking .................................................................................... 144
   6.1.3 Problem-Solving with the Spreadsheet and the Development of Algebraic Thinking .......................................................... 145
   6.1.4 Expressing Algebraic Thinking and Co-action with the Spreadsheet .............. 146
   6.2 Context and Method ...................................................................................... 147
   6.3 Data Analysis ................................................................................................. 149
   6.3.1 The First Problem: The Treasure of King Edgar ........................................... 149
   6.3.2 The Second Problem: The Opening of the Restaurant “Sombrero Style” .............. 160
   6.4 Discussion and Conclusion .......................................................................... 168
References ............................................................................................................ 171

7 Digitally Expressing Co-variation in a Motion Problem ........................................ 173
   7.1 Main Theoretical Ideas .................................................................................... 173
   7.1.1 Co-variation and Modelling Motion ............................................................. 175
   7.1.2 Visualisation in Motion Problems .................................................................. 178
   7.2 Context and Method ...................................................................................... 180
   7.3 Data Analysis .................................................................................................. 182
   7.3.1 The Experts’ Solutions to the Problem ....................................................... 182
   7.3.2 Definition of Categories .............................................................................. 186
   7.4 Analysis of the Students’ Solutions to the Problem ......................................... 187
   7.4.1 Conceptual Models Involved in the Participants’ Problem-Solving and Expressing ................................................................. 188
   7.4.2 Forms of Representation in Students’ Digital Productions ......................................... 191
   7.5 Discussion and Conclusion .......................................................................... 204
References ............................................................................................................ 207

8 Youngsters Solving Mathematical Problems with Technology: Summary and Implications ................................................................. 209
   8.1 Introduction .................................................................................................... 209
   8.2 The Problem@Web Project .............................................................................. 210
   8.3 The Youngsters Solving Mathematical Problems with Technology .............................. 214
   8.4 The Perspectives of the Youngsters’ Teachers ................................................. 218
   8.5 Theoretical Framework .................................................................................. 223
   8.6 Digitally Expressing Mathematical Problem-Solving ......................................... 225
   8.6.1 Digitally Expressing Conceptual Models of Geometrical Invariance .................... 226