

# Preface

The present book originates both from the talks delivered by the first author at several international conferences and from a mini-course given by the second author at BCAM in November 2014. The scope is narrower compared with its companion reference,<sup>1</sup> as most of the aspects related to linear (or weakly nonlinear) kinetic equations have been omitted in order to focus on the rigorous derivation of global error estimates for particular types of (systems of) balance laws in one space dimension.

The monograph presents, in a hopefully attractive and self-contained form, some techniques based on the  $L^1$  stability theory derived at the end of the 1990s by A. Bressan, T.-P. Liu, and T. Yang, which yield original error estimates for so-called well-balanced numerical schemes solving one-dimensional hyperbolic systems of balance laws.<sup>2</sup> Efforts have been focused on a practical assessment of these error bounds, too, either by a wave-front tracking technique or by a simpler Godunov process.

Well-balanced schemes, as they are studied hereafter, mostly rely on a reformulation of the original balance laws as a homogeneous, nonconservative system involving one supplementary steady “fake variable” often denoted  $a(x)$ . In a strictly hyperbolic regime, a scattering state emerges from the time decay of an extended interaction potential, including the “standing waves” associated with  $a$ . Such an asymptotic picture motivates a treatment of source terms, originally suggested by James Glimm,<sup>3</sup> such as “local scattering centers”, which we shall apply extensively.

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<sup>1</sup>*Computing Qualitatively Correct Approximations of Balance Laws*, Springer (2013).

<sup>2</sup>*cf.* Marc Laforest, *SIAM J. Math. Anal.* **35** (2004), 1347–1370.

<sup>3</sup>*cf.* J. Glimm and D.H. Sharp, *Found. Phys.* **16** (1986), 125–141.



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Balance Laws

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