Chapter 2
Fatigue Actions (Loading)

All types of fluctuating load acting on the component and the resulting stresses at potential sites for fatigue have to be considered. Stresses or stress intensity factors then have to be determined according to the fatigue assessment procedure applied.

The actions originate from live loads, dead weights, snow, wind, waves, pressure, accelerations, dynamic response etc. Actions due to transient temperature changes should also be considered. Improper knowledge of fatigue actions is one of the major sources of fatigue damage.

Tensile residual stresses due to welding and other manufacturing processes decrease the fatigue resistance. However, the influence of high tensile residual stresses is already included in the fatigue resistance data given in Chap. 3.

2.1 Basic Principles

2.1.1 Determination of Fatigue Actions (Loading)

In assessing fatigue performance, a safe estimate of fatigue loading to be endured throughout the life of the structure or component under consideration is crucial. All types of varying loading should be considered. Fluctuating loading from different sources may be significant at different phases of the life, e.g. construction, transportation, installation, in-service, and may involve different frequencies. The design load spectrum should be selected on the basis that it is an upper bound estimate of the accumulated service conditions over the full design life of the structure or component concerned. If relevant, this may be based on characteristic load data and partial safety factors $\gamma_F$ specified in the application code giving design values for the fatigue loading.

No guidance is given in this document for the establishing of design values for actions (loads), nor for partial safety factors $\gamma_F$ on actions (loads).
2.1.2 Stress Range

Fatigue assessment is usually based on stress range or stress intensity factor range. Thus, the fatigue loading (actions) needs to be expressed in these terms.

\[ \Delta \sigma = \sigma_{\text{max}} - \sigma_{\text{min}} \]  \hspace{1cm} (2.1)

\[ \Delta K = K_{\text{max}} - K_{\text{min}} \]  \hspace{1cm} (2.2)

The maximum and minimum stresses should be calculated from the superposition of all non permanent, i.e. fluctuating loads:

(a) Fluctuations in the magnitudes of loads
(b) Movement of loads on the structure
(c) Changes in loading directions
(d) Structural vibrations due to loads and dynamic response
(e) Temperature transients

Fatigue analysis is based on the cumulative effect of all stress range occurrences during the anticipated service life of the structure.

2.1.3 Types of Stress Concentrations and Notch Effects

The stress required to assess the fatigue resistance of a particular stress concentration feature depends on the type and the fatigue assessment procedure used, see Table 2.1.

Figure 2.1 shows an example of different stress definitions, such as gross nominal stress and modified or local nominal stress. Figure 2.2 shows the increase in stress in the vicinity of the notch, caused by the structural detail and the weld toe.

2.2 Determination of Stresses and Stress Intensity Factors

2.2.1 Definition of Stress Components

In the vicinity of a notch the stress distribution over the plate thickness is non-linear (Fig. 2.3).

The stress components of the notch stress \( \sigma_{\text{nl}} \) are [2]:

- \( \sigma_m \) membrane stress
- \( \sigma_b \) shell bending stress
- \( \sigma_{\text{nl}} \) non-linear stress peak
If a refined stress analysis method is used, which gives a non-linear stress distribution, the stress components can be separated by the following method:

The membrane stress $\sigma_m$ is equal to the average stress calculated through the thickness of the plate. It is constant through the thickness.

The shell bending stress $\sigma_b$ is linearly distributed through the thickness of the plate. It is found by drawing a straight line through the point $O$ in Fig. 2.3 where the membrane stress intersects the mid-plane of the plate. The gradient of the shell bending stress is chosen such that the remaining non-linearly distributed component is in equilibrium.

### Table 2.1 Stress concentrations and notch effects considered

<table>
<thead>
<tr>
<th>Type</th>
<th>Stress concentrations</th>
<th>Stress determined</th>
<th>Assessment procedure</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>None</td>
<td>Gross average stress from sectional forces, calculated using general theories, e.g. beam theory</td>
<td>Not applicable for fatigue analysis of joints, only for component testing</td>
</tr>
<tr>
<td>B</td>
<td>Macro-geometrical effects due to the design of the component, but excluding stress concentrations due to the welded joint itself.</td>
<td>Range of nominal stress (also modified or local nominal stress)</td>
<td>Nominal stress approach</td>
</tr>
<tr>
<td>C</td>
<td>B + structural discontinuities due to the structural detail of the welded joint, but excluding the notch effect of the weld toe transition</td>
<td>Range of structural hot-spot stress</td>
<td>Structural hot-spot stress approach</td>
</tr>
<tr>
<td>D</td>
<td>A + B + C + notch stress concentration due to the weld bead notches (a) actual notch stress (b) effective notch stress</td>
<td>Range of elastic notch stress (total stress)</td>
<td>(a) Fracture mechanics approach (b) Effective notch stress approach</td>
</tr>
</tbody>
</table>

**Fig. 2.1** Modified or local nominal stress
The non-linear stress peak $\sigma_{nl}$ is the remaining component of the stress.

The stress components can be separated analytically for a given through thickness stress distribution $\sigma(x)$ from $x = 0$ at the surface to $x = t$:

$$\sigma_m = \frac{1}{t} \cdot \int_{x=0}^{x=t} \sigma(x) \cdot dx$$ (2.3)

$$\sigma_b = \frac{6}{t^2} \cdot \int_{x=0}^{x=t} (\sigma(x) - \sigma_m) \cdot \left(\frac{t}{2} - x\right) \cdot dx$$ (2.4)

$$\sigma_{nl}(x) = \sigma(x) - \sigma_m - \left(1 - \frac{2x}{t}\right) \cdot \sigma_b$$ (2.5)

Note: In Fig. 2.3 and at formulae (2.3)–(2.5) a linear distribution of bending stress according to the Bernoulli theory of beams was assumed. Prior to an application of the formulae, that condition should be checked.
2.2 Determination of Stresses and Stress Intensity Factors

2.2.2 Nominal Stress

2.2.2.1 General

Nominal stress is the stress calculated in the sectional area under consideration, disregarding the local stress raising effects of the welded joint, but including the stress raising effects of the macro-geometric shape of the component in the vicinity of the joint, such as e.g. large cutouts. Overall elastic behaviour is assumed.

The nominal stress may vary over the section under consideration. For example at a beam-like component, the modified (also local) nominal stress and the variation over the section can be calculated using simple beam theory. Here, the effect of a welded on attachment is ignored (Fig. 2.4).

The effects of macro-geometric features of the component and stress fields in the vicinity of concentrated loads must be included in the nominal stress. Both may cause significant redistribution of the membrane stresses across the section. Significant shell bending stress may also be generated, as in curling of a flange, or distortion of a box section (Figs. 2.5, 2.6a, b).

The secondary bending stress caused by axial or angular misalignment (e.g. as considered to be acceptable in the fabrication specification) needs to be considered if the misalignment exceeds the amount which is already covered by the fatigue

![Fig. 2.4 Nominal stress in a beam-like component](image)

![Fig. 2.5 Examples of macrogeometric effects. Stress concentrations at a cut-outs, b curved beams, c wide plates, d curved flanges, e concentrated loads, f excentricities](images)
resistance S-N curve for the structural detail. This is done by the application of an additional stress magnification factor $k_{m,\text{eff}}$ (see Sect. 3.8.2). Either the applied stress is multiplied by $k_{m,\text{eff}}$ or the fatigue resistance (stress) is divided by it.

### 2.2.2.2 Calculation of Nominal Stress

In simple components the nominal stress can be determined using elementary theories of structural mechanics based on linear-elastic behaviour. Nominal stress is the average stress in the weld throat or in the plate at the weld toe as indicated in the tables of structural details. A possible misalignment shall be considered either in analysis or in resistance data (Fig. 2.7a)

The weld throat is determined at (Fig. 2.7b)

- **Butt welds** Wall thickness of the plates, at dissimilar wall thicknesses, the smaller wall thickness has to be taken
- **Fillet welds** The smallest distance from the root or deepest point of penetration to the surface of the fillet weld bead

The stress $\sigma_w$ or $\tau_w$ in weld throat $a$ for a weld of length $l_w$ and a force in the weld $F$ becomes

$$\sigma_w \quad \text{or} \quad \tau_w = \frac{F}{A_w} = \frac{F}{a \cdot l_w} \quad (2.6)$$

![Fig. 2.6](image) **Fig. 2.6** a Modified (local) nominal stress near concentrated loads. b Modified (local) nominal stress at hard spots
In other cases, finite element method (FEM) modelling may be used. This is primarily the case in
(a) complex statically over-determined (hyperstatic) structures
(b) structural components incorporating macro-geometric discontinuities, for which no analytical solutions are available

If the finite element method is used, meshing can be simple and coarse. Care must be taken to ensure that all stress concentration effects from the structural detail of the welded joint are excluded when calculating the modified (local) nominal stress.

If nominal stresses are calculated for fillet welds by coarse finite element meshes, nodal forces rather than element stresses should be used in a section through the weld in order to avoid stress underestimation.

When a nominal stress is intended to be calculated by finite elements, the more precise option of the structural hot spot stress determination should be considered.

2.2.2.3 Measurement of Nominal Stress

The fatigue resistance S-N curves of classified structural details are based on nominal stress, disregarding the stress concentrations due to the welded joint. Therefore the measured nominal stress must exclude the stress or strain concentration due to the corresponding discontinuity in the structural component. Thus, strain gauges must be placed outside the stress concentration field of the welded joint.

In practice, it may be necessary first to evaluate the extent and the stress gradient of the field of stress concentration (see Sect. 2.2.3.4) due to the welded joint. For further measurements, simple strain gauge application outside this field is sufficient.
When a nominal stress is intended to be measured by strain gauges, the more precise option of the structural hot spot stress measurement should be considered.

### 2.2.3 Structural Hot Spot Stress

#### 2.2.3.1 General

The structural or geometric stress $\sigma_{hs}$ at the hot spot includes all stress raising effects of a structural detail excluding that due to the local weld profile itself. So, the non-linear peak stress $\sigma_{nl}$ caused by the local notch, i.e. the weld toe, is excluded from the structural stress. The structural stress is dependent on the global dimensional and loading parameters of the component in the vicinity of the joint (type C in Sect. 2.1.3 Table 2.1). It is determined on the surface at the hot spot of the component which is to be assessed. Structural hot spot stresses $\sigma_{hs}$ are generally defined for plate, shell and tubular structures. Figure 2.8 shows examples of structural discontinuities and details together with the structural stress distribution.

The structural hot spot stress approach is typically used where there is no clearly defined nominal stress due to complex geometric effects, or where the structural discontinuity is not comparable to a classified structural detail [9, 11–13].

The structural hot-spot stress can be determined using reference points by extrapolation to the weld toe under consideration from stresses at reference points.

![Fig. 2.8](image)

**Fig. 2.8** Structural details and structural stress, e.g. at a end of longitudinal lateral attachment, b joint of plates with unequal width, c end of cover plate, d end of longitudinal attachment, e joint with unequal thickness
Strictly speaking, the method as defined here is limited to the assessment of the weld toe, i.e. cases a to e in Fig. 2.10. However, the approach may be extended to the assessment of other potential fatigue crack initiation sites including the weld root, by using the structural hot spot stress on the surface as an indication of that in the region of interest. The S-N curves or the stress concentration factors used for verification in such cases depend largely on the geometric and dimensional parameters and are only valid in the range of these parameters.

In the case of a biaxial stress state at the plate surface, it is recommended that the principal stress which acts approximately in line with the perpendicular to the weld toe, i.e. within ±60° (Fig. 2.11) is used. The other principal stress may need to be analysed, if necessary, using the fatigue class in the nominal stress approach for welds parallel to the stress.

![Diagram of structural hot-spot stress](image)

**Fig. 2.9** Definition of structural hot-spot stress

![Diagram of various locations of crack propagation](image)

**Fig. 2.10** Various locations of crack propagation in welded joints. a–e with weld toe cracks, f–j with weld root cracks
2.2.3.2 Types of Hot Spots

Besides the definitions of structural hot spot stress as given above, two types of hot spots are defined according to their location on the plate and their orientation in respect to the weld toe as defined in Fig. 2.12, Table 2.2:

The structural stress acts normal to the weld toe in each case and is determined either by a special FEA procedure or by extrapolation from measured stresses.

Fig. 2.12  Types of hot spots

<table>
<thead>
<tr>
<th>Type</th>
<th>Description</th>
<th>Determination</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>Weld toe on plate surface</td>
<td>FEA or measurement and extrapolation</td>
</tr>
<tr>
<td>b</td>
<td>Weld toe at plate edge</td>
<td>FEA or measurement and extrapolation</td>
</tr>
</tbody>
</table>
2.2.3.3 Determination of Structural Hot Spot Stress

The structural hot spot stress can be determined either by measurement or by calculation. Here the non-linear peak stress is eliminated by linearization of the stress through the plate thickness (see Sect. 2.2.1) or by extrapolation of the stress at the surface to the weld toe. The following considerations focus on surface stress extrapolation procedures of the surface stress, which are essentially the same for both measurement and calculation.

The procedure is first to establish the reference points and then to determine the structural hot spot stress by extrapolation to the weld toe from the stresses of those reference points. Depending on the method, there may be two or three reference points.

The reference point closest to the weld toe must be chosen to avoid any influence of the notch due to the weld itself (which leads to a non-linear stress peak). This is practically the case at a distance of \(0.4 \ t\) from the weld toe, where \(t\) is plate thickness. The structural hot spot stress at the weld toe is then obtained by extrapolation.

Identification of the critical points (hot spots) can be made by:

(a) Measuring several different points
(b) Analysing the results of a prior FEM analysis
(c) Experience of existing components, especially if they failed

2.2.3.4 Calculation of Structural Hot Spot Stress

In general, analysis of structural discontinuities and details to obtain the structural hot spot stress is not possible using analytical methods. Parametric formulae are rarely available. Thus, finite element analysis (FEA) is generally applied.

Usually, structural hot spot stress is calculated on the basis of an idealized, perfectly aligned welded joint. Consequently, any possible misalignment has to be taken explicitly into consideration explicitly in the FEA model or by applying an appropriate stress magnification factor \(k_m\), see also Sect. 3.8.2. This applies particularly to butt welds, cruciform joints and one-sided transverse fillet welded attachments on one side of an unsupported plate.

The extent of the finite element model has to be chosen such that constraining boundary effects of the structural detail analysed are comparable to the actual structure.

Models with either thin plate or shell elements or with solid elements may be used. It should be noted that on the one hand the arrangement and the type of the elements must allow for steep stress gradients and for the formation of plate bending, but on the other hand, only the linear stress distribution in the plate thickness direction needs to be evaluated with respect to the definition of the structural hot spot stress. The stresses should be determined at the specified reference points.
A reasonably high level of expertise is required on the part of the FEA analyst. Guidance is given in [11]. In the following, only some rough recommendations are given:

In a **plate or shell element** model (Fig. 2.13, left part), the elements are arranged in the mid-plane of the structural components. 8-noded elements are recommended particularly in regions of steep stress gradients. In simplified models, the welds are not modelled, except for cases where the results are affected by local bending, e. g. due to an offset between plates or due to a small distance between adjacent welds. Here, the welds may be included by vertical or inclined plate elements having appropriate stiffness or by introducing constraint equations or rigid links to couple node displacements. Thin-shell elements naturally provide a linear stress distribution through the shell thickness, suppressing the notch stress at weld toes. Nevertheless, the structural hot-spot stress is frequently determined by extrapolation from the reference points mentioned before, particularly at points showing an additional stress singularity such as stiffener ends.

Alternatively, particularly for complex cases, prismatic **solid elements** which have a displacement function allowing steep stress gradients as well as plate bending with linear stress distribution in the plate thickness direction may be used. An example is isoparametric 20-node elements with mid-side nodes at the edges, which allow only one element to be arranged in the plate thickness direction due to the quadratic displacement function and the linear stress distribution. By reduced integration, the linear part of the stresses can be directly evaluated at the shell surface and extrapolated to the weld toe. Modelling of welds is generally recommended as shown in Fig. 2.13 (right part). The alternative with a multi-layer arrangement of solid elements allows to linearize the stresses over the plate thickness directly at the weld toe.

**Surface Stress Extrapolation Methods:**

If the structural hot-spot stress is determined by extrapolation, the **element lengths** are determined by the reference points selected for stress evaluation. In order to avoid an influence of the stress singularity, the stress closest to the hot spot is usually evaluated at the first nodal point. Therefore, the length of the element at the hot spot corresponds to its distance from the first reference point. If finer meshes are used, the refinement should be introduced in the thickness direction as well.
Coarser meshes are also possible with higher-order elements and fixed lengths, as explained further below.

Appropriate element widths are important, particularly in cases with steep stress gradients. The width of the solid element or the two shell elements in front of the attachment should not exceed the attachment width $w$, i.e. the attachment thickness plus two weld leg lengths as indicated in Fig. 2.13.

Typical extrapolation paths for determining the structural hot spot stress components on the plate surface or edge are shown by arrows in Fig. 2.13. If the weld is not modelled, extrapolation to the structural intersection point is recommended in order to avoid stress underestimation due to the missing stiffness of the weld.

**Type “a” Hot Spots:**

The structural hot spot stress $\sigma_{hs}$ is determined using the reference points and extrapolation equations as given below (see also Fig. 2.14).

(1) Fine mesh with element length not more than $0.4 \ t$ at the hot spot: Evaluation of nodal stresses at two reference points $0.4 \ t$ and $1.0 \ t$, and linear extrapolation (Eq. 2.7).

$$\sigma_{hs} = 1.67 \cdot \sigma_{0.4,t} - 0.67 \cdot \sigma_{1.0,t} \quad (2.7)$$

(2) Fine mesh as defined in (1) above: Evaluation of nodal stresses at three reference points $0.4 \ t$, $0.9 \ t$ and $1.4 \ t$, and quadratic extrapolation (Eq. 2.8). This method is recommended for cases of pronounced non-linear structural stress increase towards the hot spot, at sharp changes of direction of the applied force or for thick-walled structures.

$$\sigma_{hs} = 2.52 \cdot \sigma_{0.4,t} - 2.24 \cdot \sigma_{0.9,t} + 0.72 \cdot \sigma_{1.4,t} \quad (2.8)$$
(3) Coarse mesh with higher-order elements having lengths equal to plate thickness at the hot spot: Evaluation of stresses at mid-side points or surface centres respectively, i.e. at two reference points $0.5 \times t$ and $1.5 \times t$, and linear extrapolation (Eq. 2.9).

$$\sigma_{hs} = 1.50 \cdot \sigma_{0.5t} - 0.50 \cdot \sigma_{1.5t}$$ (2.9)

Application of the usual wall thickness correction, as given in Sect. 3.5.2 is required when the structural hot spot stress of type “a” is obtained by surface extrapolation. For circular tubular joints, the wall thickness correction exponent of $n = 0.4$ is recommended.

**Type “b” Hot Spots:**

The stress distribution is not dependent on plate thickness. Therefore, the reference points are given at absolute distances from the weld toe, or from the weld end if the weld does not continue around the end of the attached plate.

(4) Fine mesh with element length of not more than 4 mm at the hot spot: Evaluation of nodal stresses at three reference points $4$, $8$ and $12$ mm and quadratic extrapolation (Eq. 2.10).

$$\sigma_{hs} = 3 \cdot \sigma_{4mm} - 3 \cdot \sigma_{8mm} + \sigma_{12mm}$$ (2.10)

(5) Coarse mesh with higher-order elements having length of $10$ mm at the hot spot: Evaluation of stresses at the mid-side points of the first two elements and linear extrapolation (Eq. 2.11).

$$\sigma_{hs} = 1.5 \cdot \sigma_{5mm} - 0.5 \cdot \sigma_{15mm}$$ (2.11)

**Table 2.3** Recommended meshing and extrapolation (see also Fig. 2.14)

<table>
<thead>
<tr>
<th>Type of model and weld toe</th>
<th>Relatively coarse models</th>
<th>Relatively fine models</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Type a</td>
<td>Type b</td>
</tr>
<tr>
<td></td>
<td>Type a</td>
<td>Type b</td>
</tr>
<tr>
<td><strong>Element size</strong></td>
<td>Shells</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$t \times t$</td>
<td>$10 \times 10$ mm</td>
</tr>
<tr>
<td></td>
<td>$\max t \times w/2^a$</td>
<td>$\leq 0.4 \times t$ or</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\leq 0.4 \times t$ x w/2</td>
</tr>
<tr>
<td></td>
<td>Solids</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$t \times t$</td>
<td>$10 \times 10$ mm</td>
</tr>
<tr>
<td></td>
<td>$\max t \times w$</td>
<td>$\leq 0.4 \times t$ or</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\leq 0.4 \times t$ x w/2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\leq 4 \times 4$ mm</td>
</tr>
<tr>
<td><strong>Extra-polation points</strong></td>
<td>Shells</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$0.5$ and $1.5$ t</td>
<td>$0.4$ and $1.0$ t</td>
</tr>
<tr>
<td></td>
<td>mid-side points</td>
<td>nodal points</td>
</tr>
<tr>
<td></td>
<td>$5$ and $15$ mm</td>
<td>$4$, $8$ and $12$ mm</td>
</tr>
<tr>
<td></td>
<td>mid-side points</td>
<td>nodal points</td>
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<tr>
<td></td>
<td>$0.4$ t and $1.0$ t</td>
<td>$4$, $8$ and $12$ mm</td>
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<td></td>
<td>nodal points</td>
<td>nodal points</td>
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<tr>
<td></td>
<td>Solids</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$0.5$ and $1.5$ t</td>
<td>$0.4$ t and $1.0$ t</td>
</tr>
<tr>
<td></td>
<td>surface centre</td>
<td>nodal points</td>
</tr>
<tr>
<td></td>
<td>$5$ and $15$ mm</td>
<td>$4$, $8$ and $12$ mm</td>
</tr>
<tr>
<td></td>
<td>surface centre</td>
<td>nodal points</td>
</tr>
</tbody>
</table>

$a$ $w$ = longitudinal attachment thickness +2 weld leg lengths

$b$ surface centre at transverse welds, if the weld below the plate is not modelled (see left part of Fig. 2.13)
In the case of type “b” hot spots obtained by surface stress extrapolation, the wall thickness correction (see Sect. 3.5.2) is applied with an exponent of \( n = 0.1 \) (Table 2.3).

**Alternative Methods:**

Alternative methods of estimation the structural hot spot stress may be useful in special cases. However, care is needed to ensure that they are compatible with the fatigue design resistance data recommended in this document. In the method after Haibach [15], the stress on the surface 2 mm away from the weld toe is determined. In the method after Xiao and Yamada [16], the stress 1 mm below the weld toe on the anticipated crack path is taken. Both methods are useful at sharp changes in the direction of the applied force or at thick-walled structures. In both methods no correction is required for wall thickness. The results from FEA can also be evaluated using nodal forces or through thickness integration to estimate the structural hot spot stress.

A further alternative procedure after Dong and Hong [12] uses a special stress parameter based partly on structural hot spot stress and partly on fracture mechanics analysis, with a consideration of wall thickness and stress gradient.

### 2.2.3.5 Measurement of Structural Hot Spot Stress

The recommended placement and number of strain gauges depends on the extent of shell bending stresses, the wall thickness and the type of structural stress.

The centre point of the first gauge, whose gauge length should not exceed 0.2 \( t \), is located at a distance of 0.4 \( t \) from the weld toe. If this is not possible for example due to a small plate thickness, the leading edge of the gauge should be placed at a distance of 0.3 \( t \) from the weld toe. The following extrapolation procedure and number of gauges are recommended (Fig. 2.15):

**Type “a” Hot Spots:**

(a) Two gauges at reference points 0.4 \( t \) and 1.0 \( t \) and linear extrapolation (Eq. 2.12).

![Fig. 2.15 Examples of strain gauges in plate structures](image-url)
\[ e_{hs} = 1.67 \cdot e_{0,4-t} - 0.67 \cdot e_{1,0-t} \]  \hspace{1cm} (2.12)

(b) Three gauges at reference points 0.4 t, 0.9 t and 1.4 t, and quadratic extrapolation. This method is particularly suitable for cases of pronounced non-linear structural stress increase towards the hot spot (Eq. 2.13).

\[ e_{hs} = 2.52 \cdot e_{0,4-t} - 2.24 \cdot e_{0,9-t} + 0.72 \cdot e_{1,4-t} \] \hspace{1cm} (2.13)

Precise positioning is not necessary if multi-grid strip gauges are used, since the results can be used to plot the stress distribution approaching the weld toe. The stresses at the required positions can then be read from the fitted curve.

**Type “b” Hot Spots:**
Three gauges are attached to the plate edge at reference points 4, 8 and 12 mm distant from the weld toe. The hot spot strain is determined by quadratic extrapolation to the weld toe (Eq. 2.14):

\[ e_{hs} = 3 \cdot e_{4mm} - 3 \cdot e_{8mm} + e_{12mm} \] \hspace{1cm} (2.14)

**Determination of Stress:**
If the stress state is close to uniaxial, the approximation to the structural hot spot stress is obtained approximately from Eq. (2.15).

\[ \sigma_{hs} = E \cdot e_{hs} \] \hspace{1cm} (2.15)

For biaxial stress states, the actual stress may be up to 10% higher than that obtained from Eq. (2.15). In this case, use of rosette strain gauges is recommended. If the ratio of longitudinal to transversal strains \( \varepsilon_y/\varepsilon_x \) is available, for example from FEA, the structural hot spot stress \( \sigma_{hs} \) can then be resolved from Eq. (2.16), assuming that this principal stress is approximately perpendicular to the weld toe.

\[ \sigma_{hs} = E \cdot \varepsilon_x \cdot \frac{1 + \nu}{1 - \nu^2} \] \hspace{1cm} (2.16)

The above equations also apply if strain ranges are measured, producing the range of structural hot spot stress \( \Delta\sigma_{hs} \).

### 2.2.3.6 Tubular Joints

Special recommendations exist for determining the structural hot spot stress in tubular joints [14]. In general these allow the use of linear extrapolation from the measured or calculated stresses at two reference points. The measurement of simple uni-axial stress is sufficient.
Parametric formulae have been established for the stress concentration factor $k_{hs}$ in many joints between circular and rectangular section tubes, see Ref. [14]. Hence the structural hot spot stress $\sigma_{hs}$ becomes:

$$\sigma_{hs} = k_{hs} \cdot \sigma_{nom}$$ (2.17)

where $\sigma_{nom}$ is the nominal axial membrane or bending stress in the braces, calculated by elementary stress analysis or uni-axial measurement.

### 2.2.4 Effective Notch Stress

#### 2.2.4.1 General

Effective notch stress is the total stress at the root of a notch, obtained assuming linear-elastic material behaviour. To take account of the variation of the weld shape parameters, as well as of the non-linear material behaviour at the notch root, the actual weld contour is replaced by an effective one (Fig. 2.16). For structural steels and aluminium alloys an effective notch root radius of $r = 1 \text{ mm}$ has been verified to give consistent results. For fatigue assessment, the effective notch stress is compared with a single fatigue resistance curve, although, as with other assessment methods, it is necessary to check that the fatigue resistance curve for parent metal is not exceeded in the direct vicinity of the weld [17–21].

The method is restricted to the assessment of welded joints with respect to potential fatigue failures from the weld toe or weld root (Fig. 2.17). The fatigue assessment must be additionally performed at the weld toes for the parent material using structural hot-spot stress (see Sect. 2.2.3) and the associated fatigue class (FAT) for the base material. Other modes of fatigue failure, such as crack growth from surface roughness or embedded defects, are not covered. The method is also not applicable if there is a significant stress component parallel to the weld.

The method is also restricted to assessment of naturally formed as-welded weld toes and roots (Fig. 2.18). At weld toes, an effective notch stress of at least 1.6 times the structural hot-spot stress should be assumed. This condition is usually given at welded roots. More details for practical application can be found in reference [23].
The method is well suited to the comparison of alternative weld geometries. Unless otherwise specified, it is suggested that welds should be modelled with flank angles of 30° for butt welds and 45° for fillet welds.

The method is limited to thicknesses $t \geq 5\text{ mm}$, since the method has not yet been verified for smaller wall thicknesses.

Welds toes, machined or ground to a specified profile, shall be assessed using the notch stress of the actual profile in conjunction with the nominal stress based fatigue resistance curve for a butt weld ground flush to plate.

### 2.2.4.2 Calculation of Effective Notch Stress

Effective notch stresses or stress concentration factors can be calculated by parametric formulae, taken from diagrams or calculated by finite element or boundary element models. The effective notch radius is introduced such that the tip of the
radius coincides with the root of the real notch, e.g. the end of an unwelded root gap.

For the determination of effective notch stress by FEA, element sizes of not more that 1/6 of the radius are recommended in case of linear elements, and 1/4 of the radius in case of higher order elements (Fig. 2.19 and Table 2.4). These sizes have to be observed in the curved parts as well as in the beginning of the straight part of the notch surfaces in both directions, tangential and normal to the surface, see also Ref. [22].

Possible misalignment has to be considered explicitly in the calculations.

The model may be simplified from a 3-dimensional to a 2-dimensional one under the following conditions:

(a) The loading should be mainly perpendicular to the weld, i.e. normal and shear stress in direction of the weld are not existent or small and can be neglected.
(b) The loading and the geometry of the weld should not vary in the area to be assessed.

At an occurrence of multiaxial stress, the principles of Chap. 4 should be applied. If there is a proportional loading, i.e. all stress components are in a constant phase, then the maximum principle stress may be used, provided that the minimum principle stress has the same sign. Both should be either positive or negative. In all other cases the regulations of Sect. 4.3 should be applied.

### 2.2.4.3 Measurement of Effective Notch Stress

Because the effective notch radius is an idealization, it cannot be measured directly in the welded component. In contrast, the simple definition of the effective notch can be used for photo-elastic stress measurements in resin models.
2.2.5 Stress Intensity Factors

2.2.5.1 General

Fracture was developed to assess the behaviour of cracks or crack-like imperfections in components (Fig. 2.20). The methods are well established, but require an adequate level of knowledge and experience. It is recommended to perform the assessment procedures using the recommendations given here and consulting the actual compendia of the method [43, 53]. Fracture mechanics is used for several purposes as e.g.:

(a) Assessment of fracture, especially brittle fracture, in a component containing cracks or crack-like details.
(b) Assessment of fatigue properties in a component containing cracks or crack-like imperfection as e.g. in welded joints.
(c) Predicting the fatigue properties of severely notched components with no or a relatively short crack initiation phase. Welded joints behave as being severely notched. Predictions are made assuming small initial defects.

The fatigue assessment procedure as in (b) and (c) is performed by the calculation of the growth of an initial crack $a_i$ to a final size $a_f$. Since crack initiation occupies only a small proportion of the lives of welded joints in structural metals, the method is suitable for assessment of fatigue life, inspection intervals, crack-like weld imperfections and the effect of variable amplitude loading. The final crack $a_f$ may be estimated as about one half wall thickness, since there is a rapid onset of crack propagation. Only a few and insignificant numbers of cycles are spent in that phase of fatigue.

![Finite surface crack](image1)
![Continuous surface crack](image2)
![Continuous embedded crack](image3)
![Side-crack](image4)

**Fig. 2.20** Examples for different categories of cracks
2.2.5.2 Determination of Stress Intensity Factors

The parameter which describes the fatigue action at a crack tip in terms of crack propagation is the stress intensity factor (SIF) range $\Delta K$. The starting crack configuration is the centre crack in an infinite plate. The stress intensity factor $K$ is defined by the formula $K = \sigma \cdot \sqrt{\pi \cdot a}$. Where $\sigma$ is the remote stress in the plate and $a$ is the crack parameter, here the half distance from tip to tip.

2.2.5.2.1 Standard Configurations

In existing components, there are various crack configurations and geometrical shapes. So, corrections are needed for the deviation from the centre cracked plate. The formula for the stress intensity factor has to be expanded by a correction function $Y_u(a)$. These corrections take into account the following parameters and crack locations:

(a) Free surface of a surface crack.
(b) Embedded crack located inside of a plate.
(c) Limited width or wall thickness.
(d) Shape of a crack, mostly taken as being elliptic.
(e) Distance to an edge.

For a variety of crack configurations, parametric formulae for the correction function $Y_u(a)$ have been developed (see Appendix 6.2 and references [25, 53]). These correction functions are based on different applied stress types (e.g. membrane, bending, structural hot spot stress, nominal stress). The one used must correspond to the stress type under consideration.

2.2.5.2.2 Stress Intensity Factor for Weld Toes

Fracture mechanics calculations related to welded joints are generally based on the total stress at the notch root, e.g. at the weld toe. The universal correction function $Y_u(a)$ may be separated into the correction of a standard configuration $Y(a)$ and an additional correction for the local notch of the weld toe $M_k(a)$. A further separation into membrane stress and shell bending stress was done at most of the parametric formulae for the functions $Y(a)$ and $M_k(a)$ [32, 34].

$$K = \sigma \cdot \sqrt{\pi \cdot a} \cdot Y_u(a)$$ (2.18)

In practical application, first the relevant applied stress (usually the local nominal or the structural hot spot stress) at the location of the crack is determined, assuming that no crack is present. If required, the stress should be separated into membrane and shell bending stress components. The stress intensity factor (SIF) $K$ then results as a superposition of the effects of both stress components. The effects of the crack
shape and size are covered by the correction function $Y$. The effects of the any remaining stress raising discontinuity or notch from the weld toe (non-linear peak stress) can to be covered by additional factors $M_k$, while

$$K = \sqrt{\pi \cdot a} \cdot (\sigma_m \cdot Y_m(a) \cdot M_{k, m}(a) + \sigma_b(a) \cdot Y_b(a) \cdot M_{k, b}(a))$$  \hspace{1cm} (2.18a)

where

- $K$: stress intensity factor
- $\sigma_m$: membrane stress
- $\sigma_b$: shell bending stress
- $Y_m$: correction function for membrane stress intensity factor
- $Y_b$: correction function for shell bending stress intensity factor
- $M_{k, m}$: correction for non-linear stress peak in terms of membrane action
- $M_{k, b}$: Correction for non-linear stress peak in terms of shell bending

The correction functions $Y_m$ and $Y_b$ can be found in the literature. The solutions in Ref. [25–30] are particularly recommended. For most cases, the formulae for stress intensity factors given in Appendix 6.2 are adequate. $M_k$-factors may be found in references [31, 32].

### 2.2.5.2.3 Weight Function Approach

The weight function approach is based on the idea that a given stress distribution can discretized into differential pairs of split forces which open a crack. The action of each differential force on a crack can be described by a function, the so called weight function $h(x, a)$. The determination of the stress intensity factor is thus reduced into an integration over the crack length. By this method, arbitrary stress distributions can be assessed. The basic formulation of the weight function approach is

$$K = \int_{x=0}^{x=a} \sigma(x) \cdot h(x, a) \cdot dx$$  \hspace{1cm} (2.19)

Weight functions have been developed for 2-dimensional (Fett and Munz Ref. [39]) and 3-dimensional problems (Glinka et al., see Appendix 6.2 and Ref. [40]). More weight functions may be found in literature (Ref. [43]).

The application of weight functions requires an integration process to obtain the stress intensity factor. Here it must be observed that several weight functions lead to improper integrals, i.e. integrals with infinite boundaries but finite solutions. There are two ways to overcome. Firstly to use very fine steps near the singularity, or secondly to integrate analytically, if possible, and to calculate small stripes, which are later summed up for the number of cycles.
For transverse loaded welds, parametric formulae for the stress distribution in the plate have been developed. In these cases a finite element calculation may not be necessary (Hall et al. Ref. [41])

2.2.5.2.4 Finite Element Programs

The determination of stresses and stress distributions finite element programs may be used. It must be made sure that the refinement of the meshing corresponds to method, which is used for deriving the stress intensity factors.

For the use of standard solutions and existing $M_k$ formulae, a coarse meshing may be sufficient to determine the membrane and the shell bending stress. If a weight function approach is used, a fine meshing is needed for a full information about the stress distribution at the weld toe or root, whichever is considered.

Several program systems exist which provide a direct determination of stress intensity factors. The meshing should be made according to the method used and to the recommendations of the program manual.

2.2.5.2.5 Aspect Ratio

The aspect ratio $a:c$ is a significant parameter for the stress intensity factor (Fig. 2.21). It has to be taken into consideration at fracture mechanics calculations. This consideration can be done in different ways:

(a) Direct determination and calculation of crack growth in $c$-direction, e.g. by 3-dimensional weight functions or $M_k$-formulae. These formulae give the stress intensity factor at the surface, which governs the crack propagation in $c$-direction.

(b) Application of formulae and values which have been derived from toes of fillet welds by fitting of experimental data, a possible example is given by Engesvik [44].

$$2 \cdot c = -0.27 + 6.34 \cdot a \text{ if } a > 0.1 < a < 3 \text{ mm}$$

$$a/(2 \cdot c) = 0 \text{ if } a > 3 \text{ mm}$$

Fig. 2.21 Crack parameters
(c) If only 2-dimensional Mk values are given, then the crack depth of $a = 0.15$ or $0.1$ mm may be used to calculate the effective stress intensity factor at the surface for the crack propagation in “c”-direction [53].

(d) A constant aspect ratio of $a:c = 0.1$ may be taken as a conservative approach.

2.2.5.2.6 Assessment of Welded Joints Without Detected Imperfections

Fracture mechanics may be used to assess the fatigue properties of welded joints in which no imperfections have been detected. In such cases it is necessary to assume the presence of an initial crack, for example based on prior metallurgical evidence, the detection limit of the used inspection method or fitting from fatigue data, and then to calculate the stress intensity factor as above.

In case of post-weld treatment there is a possibly larger number of cycles for crack initiation. That shall be assessed and/or considered by an appropriate calculation procedure, which might be taken from the relevant literature.

For cracks starting from a weld toe, in absences of other evidence, it is recommended that an initial crack depth of at $a = 0.1$ mm and an aspect ratio as given above might be taken considering that there might be multiple spots for crack initiation. The initial cracks have been derived from fitting the assessment procedure to experimental data, disregarding possible fracture mechanics short crack effects. If possible, the calculations should be compared or calibrated at similar joint details with known fatigue properties.

If no weld toe radius $\rho$ was specified or determined by measuring, it is recommended to assume a sharp corner i.e. a toe radius of $\rho = 0$ to $\rho = 0.2$ mm.

For root gaps in load-carrying fillet welded cruciform joints, the actual root gap should be taken as the initial crack.

It is convenient to disregard the threshold properties. Later the obtained fatigue cycles may be converted into a FAT class and to proceed using that S-N curve.

$$FAT = \Delta \sigma_{applied} \cdot \sqrt{\frac{N}{2 \cdot 10^6}}$$ (2.21)

2.3 Stress History

2.3.1 General

The fatigue design data presented in Chap. 3 were obtained from tests performed under constant amplitude loading. However, loads and the resulting fatigue actions (i.e. stresses) in real structures usually fluctuate in an irregular manner and give rise to variable amplitude loading. The stress range may vary in both magnitude and period from cycle to cycle.
The stress history is a record and/or a representation of the fluctuations of the fatigue actions in the anticipated service time of the component. It is described in terms of successive maxima and minima of the stress caused by the fatigue actions (Fig. 2.22). It should aim to cover all loading events and the corresponding induced dynamic response in a conservative way.

In most cases, the stress-time history is stationary and ergodic, which allows the definition of a mean range and its variance, a statistical histogram and distribution, an energy spectrum and a maximum values probabilistic distribution from a representation covering a limited period of operation. Therefore, the data needed to perform a fatigue analysis can be determined from service load measurements or observations conducted over a limited time, as long as it is reasonably representative of the loading to be experienced during the whole fatigue life.

A stress history may be given as

(a) a record of successive maxima and minima of stress measured in a comparable structure for comparable loading and service life, or a typical sequence of load events.

(b) a two dimensional transition matrix of the stress history derived from a).

(c) a one- or two-dimensional stress range histogram (stress range occurrences) obtained from a) by a specified counting method.

(d) a one-dimensional stress range histogram (stress range exceedences, stress range spectrum) specified by a design code.

The representations (a) and (b) may be used for component testing, while (c) and (d) are most useful for fatigue assessment by calculation.

### 2.3.2 Cycle Counting Methods

Cycle counting is the process of converting a variable amplitude stress sequence into a series of constant amplitude stress range cycles that are equivalent in terms of damage to the original sequence. Various methods are available including zero crossing counting, peak counting, range pair counting and rainflow counting. For
welded components, the ‘rainflow’ or similar ‘reservoir’ methods are recommended for counting stress ranges [58, 59].

### 2.3.3 Cumulative Frequency Diagram (Stress Spectrum)

The cumulative frequency diagram (stress spectrum) corresponds to the cumulative probability of stress range expressed in terms of stress range level exceedences versus the number of cycles. The curve is therefore continuous.

It is usually more convenient to represent the spectrum by a table of discrete blocks of cycles of constant amplitude stress range, typically up to 20 different stress levels. The assumed magnitude of the stress range in a given block would then depend on the conservatism required. Typical values would be the maximum or the mean of the stress range in the block.

Besides the representation in probabilities, a presentation of the number of occurrences or exceedances in a given number of cycles, e.g. 1 million, is used. An example showing a Gaussian normal distribution is given below (Table 2.5 and Fig. 2.23):

#### Table 2.5 Example of a stress range occurrence table (stress histogram or frequency)

<table>
<thead>
<tr>
<th>Block No.</th>
<th>Relative stress range</th>
<th>Occurrence (frequency)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.000</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>0.950</td>
<td>16</td>
</tr>
<tr>
<td>3</td>
<td>0.850</td>
<td>280</td>
</tr>
<tr>
<td>4</td>
<td>0.725</td>
<td>2720</td>
</tr>
<tr>
<td>5</td>
<td>0.575</td>
<td>20000</td>
</tr>
<tr>
<td>6</td>
<td>0.425</td>
<td>92000</td>
</tr>
<tr>
<td>7</td>
<td>0.275</td>
<td>280000</td>
</tr>
<tr>
<td>8</td>
<td>0.125</td>
<td>605000</td>
</tr>
</tbody>
</table>

#### Fig. 2.23 Example of a cumulative frequency diagram (stress spectrum)
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