Preface to the second edition

In producing this second edition, I acknowledge first and foremost the contributions of the amazing Elizabeth Loew, my editor at Springer. Without her unwavering enthusiasm and support, and, yes, some firm prodding, this project would not have made it this far.

The response I have received to the first edition has been gratifying. I thank the many readers who have sent along their questions, comments, compliments, and lists of typos. All these have been both helpful and encouraging. I have also received repeated requests for any materials I could provide to supplement the text. At first, I had very little available for public consumption. This gradually changed as I used the book myself, for courses at both the undergraduate and master's degree levels. By necessity, I developed additional exercises, created short computer-based assignments, and found new ways to explain some of the tricky concepts. So, in this second edition, I have sought to enhance the development of the main themes and ideas from the earlier edition. This includes the addition of a few new topics that, I hope, lend deeper insight into the core concepts.

Chapter 2 includes new material on the Radon transform and its properties as well as more figures, more worked examples, and a new section that offers a computer-based, hands-on guide to creating phantoms. Chapters 5 and 7 have more on the Dirac delta function and its role in X-ray imaging analysis. Chapter 8 has an expanded look at interpolation using cubic splines, more illustrations of the principal image reconstruction algorithms based on the filtered back projection, and examples of how to implement the algorithms on a computer. Chapter 9, on algebraic image reconstruction techniques, includes more thorough discussions of Kaczmarz's method and least squares approximation, a new section on regularization and spectral filtering, and more computer-based examples and exercises. A new appendix collects some useful results concerning matrices and their transposes that figure in the discussion in Chapter 9, including the eigenvalue decomposition of a (real) symmetric matrix of the form $A^T A$ and the singular value decomposition of a matrix. Additional exercises are to be found in most chapters, with about 30% more exercises overall than in the first edition.

The use of technology has been revamped throughout this second edition, with the incorporation of the open-source programming environment *R* [42]. I began to study and learn *R* not too long ago, as part of a research collaboration with a colleague of mine in statistics here at Villanova University. In the midst of running Monte Carlo simulations, I realized that the discrete, vector-based nature of *R* is well suited to many of the applications found here. Nearly all of the figures have been redesigned using *R*, and quite a few new figures have been added. More than 20 examples using *R* are included throughout the text, offering new opportunities for hands-on exploration. SpringerLink includes additional *R* scripts, including some of those used to produce the figures in the book.

I hope this new edition will stoke your enthusiasm for mathematics and the powerful impact of its applications. Enjoy!

Villanova, PA, USA Timothy G. Feeman

Preface

In 1979, the Nobel Prize for Medicine and Physiology was awarded jointly to Allan McLeod Cormack and Godfrey Newbold Hounsfield, the two pioneering scientist-engineers primarily responsible for the development, in the 1960s and early 1970s, of computerized axial tomography, popularly known as the CAT or CT scan. In his papers [14], Cormack, then a professor at Tufts University, in Massachusetts, developed certain mathematical algorithms that, he envisioned, could be used to create an image from X-ray data. Working completely independently of Cormack and at about the same time, Hounsfield, a research scientist at EMI Central Research Laboratories in the United Kingdom, designed the first operational CT scanner as well as the first commercially available model. (See [27] and [28].)

Since 1980, the number of CT scans performed each year in the United States has risen from about 3 million to over 67 million. What few people who have had CT scans probably realize is that the fundamental problem behind this procedure is essentially mathematical: If we know the values of the integral of a two- or three-dimensional function along all possible cross sections, then how can we reconstruct the function itself? This particular example of what is known as an *inverse problem* was studied by Johann Radon, an Austrian mathematician, in the early part of the twentieth century. Radon's work incorporated a sophisticated use of the theory of transforms and integral operators and, by expanding the scope of that theory, contributed to the development of the rich and vibrant mathematical field of functional analysis. Cormack essentially rediscovered Radon's ideas, but did so at a time when technological applications were actually conceivable. The practical obstacles to implementing Radon's theories are several. First, Radon's inversion methods assume knowledge of the behavior of the function along every cross section, while, in practice, only a discrete set of cross sections can feasibly be sampled. Thus, it is possible to construct only an approximation of the solution. Second, the computational power needed to process a multitude of discrete measurements and, from them, to obtain a useful approximate solution has been available for just a few decades. The response to these obstacles has been a rich and dynamic development both of theoretical approaches to approximation methods, including the use of interpolation and filters, and of computer algorithms to effectively implement the approximation and inversion strategies. Alongside these mathematical and computational advances, the machines that perform the scans have gone through several generations of improvements in both the speed of data collection and the accuracy of the images, while the range of applications has expanded well beyond the original focus on imaging of the brain. Other related processes, such as positron emission tomography (PET), have developed alongside the advances in CT.

Clearly, this subject crosses many disciplinary boundaries. Indeed, literature on technical aspects of medical imaging appears in journals published in engineering, mathematics, computer science, biomedical research, and physics. This book, which grew out of a course I gave for undergraduate mathematics majors and minors at Villanova University in 2008, addresses the mathematical fundamentals of the topic in a concise way at a relatively elementary level. The emphasis is on the mathematics of CT, though there is also a chapter on magnetic resonance imaging (MRI), another medical imaging process whose originators have earned Nobel prizes. The discussion includes not only the necessary theoretical background but also the role of approximation methods and some attention to the computer implementation of the inversion algorithms. A working knowledge of multivariable calculus and basic vector and matrix methods should serve as adequate prerequisite mathematics.

I hope you will join me, then, in this quest to comprehend one of the most significant and beneficial technological advances of our time and to experience mathematics as an inextricable part of human culture.

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