Chapter 2
Basic Definitions

The basic tool for understanding wind turbine aerodynamics is the momentum theory in which the flow is assumed to be steady, inviscid, incompressible and axisymmetric. The momentum theory basically consists of control volume integrals for conservation of mass, axial and angular momentum balances, and energy conservation

$$\oint_{\mathrm{CV}} \rho \mathbf{V} \cdot \mathrm{d}\mathbf{A} = 0 \tag{2.1}$$

$$\oint_{\mathrm{CV}} u\rho \mathbf{V} \cdot \mathrm{d}\mathbf{A} = T - \oint_{\mathrm{CV}} p\mathrm{d}\mathbf{A} \cdot \mathbf{e}_x \tag{2.2}$$

$$\oint_{\mathrm{CV}} ru_\theta \rho \mathbf{V} \cdot \mathrm{d}\mathbf{A} = Q \tag{2.3}$$

$$\oint_{\mathrm{CV}} \left[p/\rho + 1/2\|\mathbf{V}\|^2\right] \rho \mathbf{V} \cdot \mathrm{d}\mathbf{A} = P \tag{2.4}$$

where $\mathbf{V} = (u, v, u_\theta)$ is the velocity vector in axial, radial and azimuthal direction, respectively, ρ is the density of air, $\mathbf{A}$ denotes the outward pointing area vector of the control volume, p is the pressure, T is the axial force (thrust) acting on the rotor, Q is the torque, and P is the power extracted from the rotor.

The main dimensionless parameters to characterize the aerodynamic operation of a wind turbine are the following;

$$\text{Tip speed ratio:} \quad \lambda = \frac{\Omega R}{U_0} \tag{2.5}$$

$$\text{Thrust coefficient:} \quad C_T = \frac{T}{1/2\rho A U_0^2} \tag{2.6}$$

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$$\text{Power coefficient:} \quad C_P = \frac{P}{1/2\rho A U_0^3} \tag{2.7}$$

where Ω is the angular velocity of the rotor, A is the rotor area, R is the radius of the rotor, and U_0 is the wind speed.

Essentially a wind turbine is a rotating flow machinery that extracts the kinetic energy in the wind to useful mechanical power in the rotor and drive train and from this into electrical power in the generator. The first part of this process, i.e. the extraction of energy from the wind to the rotor can be modelled using the actuator disc concept. An actuator disc is an idealized rotor representation, in which a sudden pressure difference is created over the rotor without having any discontinuity of the velocity. Thus, in front of the disc a high pressure, p^+, appears, whereas a lower pressure, p^-, acts behind the rotor disc. The pressure jumps over the rotor disc, $(p^+ - p^-)$, then corresponds to the local thrust force acting over the rotor, $\Delta T = (p^+ - p^-) \cdot \Delta A$, where $\Delta A = 2\pi r \Delta r$ corresponds to the local area of an annular segment of the rotor. The extracted power can then be determined by multiplying the thrust by the local velocity in the rotor plane. The above-presented equations form the background for the analysis presented in the rest of the thesis.

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