

Contents

Part I Basic toolbox	1
1 Introduction	3
1.1 Formal definitions	12
2 Kernelization	17
2.1 Formal definitions	18
2.2 Some simple kernels	20
2.2.1 VERTEX COVER	21
2.2.2 FEEDBACK ARC SET IN TOURNAMENTS	22
2.2.3 EDGE CLIQUE COVER	25
2.3 Crown decomposition	26
2.3.1 VERTEX COVER	29
2.3.2 MAXIMUM SATISFIABILITY	29
2.4 Expansion lemma	30
2.5 Kernels based on linear programming	33
2.6 Sunflower lemma	38
2.6.1 d -HITTING SET	39
3 Bounded search trees	51
3.1 VERTEX COVER	53
3.2 How to solve recursive relations	55
3.3 FEEDBACK VERTEX SET	57
3.4 VERTEX COVER ABOVE LP	60
3.5 CLOSEST STRING	67
4 Iterative compression	77
4.1 Illustration of the basic technique	78
4.1.1 A few generic steps	80
4.2 FEEDBACK VERTEX SET IN TOURNAMENTS	81
4.2.1 Solving DISJOINT FEEDBACK VERTEX SET IN TOURNAMENTS in polynomial time	83
4.3 FEEDBACK VERTEX SET	86
4.3.1 First algorithm for DISJOINT FEEDBACK VERTEX SET	87
*4.3.2 Faster algorithm for DISJOINT FEEDBACK VERTEX SET	88
4.4 ODD CYCLE TRANSVERSAL	91

5	Randomized methods in parameterized algorithms	99
5.1	A simple randomized algorithm for FEEDBACK VERTEX SET	101
5.2	Color coding	103
5.2.1	A color coding algorithm for LONGEST PATH	104
5.3	Random separation	106
*5.4	A divide and color algorithm for LONGEST PATH	108
5.5	A chromatic coding algorithm for d -CLUSTERING	113
5.6	Derandomization	117
5.6.1	Basic pseudorandom objects	118
5.6.2	Derandomization of algorithms based on variants of color coding	120
6	Miscellaneous	129
6.1	Dynamic programming over subsets	130
6.1.1	SET COVER	130
6.1.2	STEINER TREE	131
6.2	INTEGER LINEAR PROGRAMMING	135
6.2.1	The example of IMBALANCE	136
6.3	Graph minors and the Robertson-Seymour theorem	140
7	Treewidth	151
7.1	Trees, narrow grids, and dynamic programming	153
7.2	Path and tree decompositions	157
7.3	Dynamic programming on graphs of bounded treewidth	162
7.3.1	WEIGHTED INDEPENDENT SET	162
7.3.2	DOMINATING SET	168
7.3.3	STEINER TREE	172
7.4	Treewidth and monadic second-order logic	177
7.4.1	Monadic second-order logic on graphs	178
7.4.2	Courcelle's theorem	183
7.5	Graph searching, interval and chordal graphs	185
7.6	Computing treewidth	190
7.6.1	Balanced separators and separations	192
7.6.2	An FPT approximation algorithm for treewidth	195
7.7	Win/win approaches and planar problems	199
7.7.1	Grid theorems	200
7.7.2	Bidimensionality	203
7.7.3	Shifting technique	211
*7.8	Irrelevant vertex technique	216
7.9	Beyond treewidth	228
	Part II Advanced algorithmic techniques	245
8	Finding cuts and separators	247
8.1	Minimum cuts	249

8.2	Important cuts	254
8.3	EDGE MULTIWAY CUT	261
8.4	(p, q) -clustering	264
8.5	Directed graphs	272
8.6	DIRECTED FEEDBACK VERTEX SET	274
8.7	Vertex-deletion problems	278
9	Advanced kernelization algorithms	285
9.1	A quadratic kernel for FEEDBACK VERTEX SET	287
9.1.1	Proof of Gallai’s theorem	290
9.1.2	Detecting flowers with Gallai’s theorem	295
9.1.3	Exploiting the blocker	296
*9.2	Moments and MAX- E_r -SAT	299
9.2.1	Algebraic representation	301
9.2.2	Tools from probability theory	302
9.2.3	Analyzing moments of $X(\psi)$	304
9.3	CONNECTED VERTEX COVER in planar graphs	307
9.3.1	Plane graphs and Euler’s formula	308
9.3.2	A lemma on planar bipartite graphs	309
9.3.3	The case of CONNECTED VERTEX COVER	310
9.4	Turing kernelization	313
9.4.1	A polynomial Turing kernel for MAX LEAF SUBTREE	315
10	Algebraic techniques: sieves, convolutions, and polynomials	321
10.1	Inclusion–exclusion principle	322
10.1.1	HAMILTONIAN CYCLE	323
10.1.2	STEINER TREE	324
10.1.3	CHROMATIC NUMBER	326
10.2	Fast zeta and Möbius transforms	328
10.3	Fast subset convolution and cover product	331
10.3.1	Counting colorings via fast subset convolution	334
10.3.2	Convolutions and cover products in min-sum semirings	334
10.4	Multivariate polynomials	337
10.4.1	LONGEST PATH in time $2^k n^{\mathcal{O}(1)}$	340
10.4.2	LONGEST PATH in time $2^{k/2} n^{\mathcal{O}(1)}$ for undirected bipartite graphs	346
*10.4.3	LONGEST PATH in time $2^{3k/4} n^{\mathcal{O}(1)}$ for undirected graphs	349
11	Improving dynamic programming on tree decompositions	357
11.1	Applying fast subset convolution	358
11.1.1	Counting perfect matchings	358
11.1.2	DOMINATING SET	359
11.2	Connectivity problems	361
11.2.1	Cut & Count	361
*11.2.2	Deterministic algorithms by Gaussian elimination	365

12 Matroids	377
12.1 Classes of matroids	379
12.1.1 Linear matroids and matroid representation	379
12.1.2 Representation of uniform matroids	380
12.1.3 Graphic matroids	381
12.1.4 Transversal matroids	382
12.1.5 Direct sum and partition matroids	383
12.2 Algorithms for matroid problems	383
12.2.1 Matroid intersection and matroid parity	386
12.2.2 FEEDBACK VERTEX SET in subcubic graphs	389
12.3 Representative sets	392
12.3.1 Playing on a matroid	394
12.3.2 Kernel for d -HITTING SET	398
12.3.3 Kernel for d -SET PACKING	399
12.3.4 ℓ -MATROID INTERSECTION	401
12.3.5 LONG DIRECTED CYCLE	403
12.4 Representative families for uniform matroids	409
12.5 Faster LONG DIRECTED CYCLE	410
12.6 LONGEST PATH	413

Part III Lower bounds 419

13 Fixed-parameter intractability	421
13.1 Parameterized reductions	424
13.2 Problems at least as hard as CLIQUE	426
13.3 The W-hierarchy	435
*13.4 Turing machines	439
*13.5 Problems complete for $W[1]$ and $W[2]$	443
13.6 Parameterized reductions: further examples	448
13.6.1 Reductions keeping the structure of the graph	448
13.6.2 Reductions with vertex representation	451
13.6.3 Reductions with vertex and edge representation	453
14 Lower bounds based on the Exponential-Time Hypothesis	467
14.1 The Exponential-Time Hypothesis: motivation and basic results	468
14.2 ETH and classical complexity	473
14.3 ETH and fixed-parameter tractable problems	475
14.3.1 Immediate consequences for parameterized complexity	476
*14.3.2 Slightly super-exponential parameterized complexity	477
*14.3.3 Double exponential parameterized complexity	484
14.4 ETH and $W[1]$ -hard problems	485
14.4.1 Planar and geometric problems	489
*14.5 Lower bounds based on the Strong Exponential-Time Hypothesis	502

14.5.1	HITTING SET parameterized by the size of the universe	503
14.5.2	Dynamic programming on treewidth	508
14.5.3	A refined lower bound for DOMINATING SET	514
15	Lower bounds for kernelization	523
15.1	Compositionality	524
15.1.1	Distillation	525
15.1.2	Composition	529
15.1.3	AND-distillations and AND-compositions	533
15.2	Examples	534
15.2.1	Instance selector: SET SPLITTING	534
15.2.2	Polynomial parameter transformations: COLORFUL GRAPH MOTIF and STEINER TREE	537
15.2.3	A more involved one: SET COVER	540
15.2.4	Structural parameters: CLIQUE parameterized by the vertex cover number	544
15.3	Weak compositions	547
	References	556
	Appendix	577
	Notation	577
	Problem definitions	581
	Index	599
	Author index	609



<http://www.springer.com/978-3-319-21274-6>

Parameterized Algorithms

Cygan, M.; Fomin, F.V.; Kowalik, Ł.; Lokshtanov, D.; Marx, D.; Pilipczuk, M.; Pilipczuk, M.; Saurabh, S.

2015, XVII, 613 p. 84 illus., 25 illus. in color., Hardcover

ISBN: 978-3-319-21274-6