Preface

Watts and Strogatz, with the publication in 1998 of their seminal paper on small-world networks, opened the golden era of complex networks studies and showed in particular how statistical physics could contribute to the understanding of these objects. The first studies that followed considered the characterization of large graphs, their degree distribution, clustering coefficient, or their average shortest path. New models of random graphs, beyond the well-known Erdos–Renyi archetype, were then proposed in order to understand some of the empirically observed features. However, many complex networks encountered in the real-world are embedded in space: nodes and links correspond to physical objects and there is usually a cost associated with the formation of edges. This aspect turns out to be crucial as it determines many features of the structure of these networks that we can call "spatial". It is difficult to consider that spatial networks actually form a subclass of complex networks, but rather constitute their own family specified by a set of properties that differ from the "usual" complex networks. In particular, one of the most salient properties in complex network is a broad degree distribution with the existence of hubs. This feature has a dramatic impact on dynamical processes occurring on these networks and is at the heart of studies on scale-free networks. In contrast, the physical constraints in spatial networks prohibit in general the formation of hubs and their most interesting properties lie in their spatial organization and in the relation between space and topology.

Spatial networks—even if this was not the standard name at that time—were the subject of numerous studies in the 70s in regional science followed by quantitative geographers who were interested in characterizing the structure of transportation networks, from roads to subways and railways, and produced a number of important results about these networks and their evolution. The recent revival of the interest in this subject, combined with an always larger amount of data, allowed to make some progress in our understanding of these objects. The recent advances obtained in the understanding of spatial networks have generated an increased attention toward the potential implication of new theoretical models in agreement with data. Questions such as the structure and resilience of infrastructures and the impact of space on the formation of biological networks are fundamental questions that we hope to solve in a near future.

Most of these spatial networks are—to a good approximation—planar graphs for which edge crossing is not allowed. Planar networks were for a long time the subject of numerous studies in graph theory, but we are still lacking models and tools for their characterization.

In this book, we will discuss different aspects of spatial networks, focusing essentially on the characterization of their structure and on their modeling. Each chapter is as much as possible self-contained and for the sake of clarity and readability, we tried to be as modular as possible in order to allow the reader interested in just one specific model or tool to focus essentially on the corresponding chapter.

The first chapter introduces the subject with some definitions and basic results about planar graphs together with less trivial results about the crossing number of a graph. We will insist on the distinction between topological non-planarity and non-planarity of the physical embedding. As discussed above, many measures that were extensively used for complex networks are in fact irrelevant for spatial networks, due to constraints that make the degree bounded, and the clustering and assortativity trivial. We review both the irrelevant and the simplest measures in Chap. 2, and also a discussion on the more advanced tool that is community detection.

In Chaps. 3–7, we discuss various tools and measures for spatial networks. An important object in spatial networks, and in particular in planar graphs, is the face (or cell, block depending on the context). We discuss in Chap. 3 the statistics of the area and shape of these faces and the possibility of a mapping of a planar graph to a tree. We discuss here both an approximate mapping introduced for weighted graphs and an exact bijection obtained in mathematics for (rooted) planar graphs. In Chap. 4, we discuss the important quantity which is the betweenness centrality. It was introduced in the 70s for quantifying the importance of a node in a network and this particular "centrality" seems to be very interesting for characterizing the organization of spatial networks. We first expose general properties of the betweenness centrality such as the scaling of the maximum value or the effect of adding or removing edges. We then present empirical results about the spatial patterns of the betweenness centrality in various networks and theoretical aspects as well, such as the centrality of loops in random graphs. In Chap. 5, we also consider other path-related quantities that were used in spatial networks. The simplicity compares shortest paths and simplest paths—the paths with the smallest number of turns—and the entropy quantifies the complexity of paths in these networks.

In Chap. 6, we address a subject whose importance might grow in the future and which concerns spatial networks with attributes. In these systems, nodes have a certain attribute (a real number such as the population of a city for example) and we have to characterize the interplay between the value of the attribute and the spatial location of a node. We discuss for these objects a measure of spatial "dominance" that was developed by Okabe and his collaborators. We end this chapter with a discussion on community detection whose results depend strongly on the existence of correlations between space and attribute, and on the choice of a null model.

In Chap. 7, we address the important problem of time-evolving spatial networks and their characterization. We focus in this more empirically oriented part on the evolution of the street network and the growth of subways. The large number of parameters and possible measures is, maybe surprisingly, not very helpful and we will see how to identify the most relevant tools for the characterization of the evolution of these systems. This is a very timely subject and we can expect many development and progress about this problem in the coming years.

In Chaps. 8–14, we discuss modeling aspects of spatial networks. We start in Chap. 8 with a description of tessellations which are good "null" models for planar graphs and which also allow to characterize the statistics of a distribution of points. We will naturally discuss the Voronoi tessellation and its properties (in particular in the case of a Poisson distribution of points), but also other models such as cracks and STIT tessellations.

In Chap. 9, we discuss the random geometric graph, probably the simplest model of spatial network and some of its variants such as the soft random geometric graph, the Bluetooth graph, and the k-nearest neighbor model. We also discuss a dynamical version of the random geometric graph where agents are mobile in a plane and create a network of connections.

In Chap. 10, we present generalizations of the Erdos–Renyi random graph to the spatial case. In particular, we will discuss the Waxman model that is considered as a simple model for the structure of the Internet. We will also present spatial generalizations of the Watts–Strogatz model and its properties. In particular, after having discussed some models, we will focus on the navigability on these networks as it has important practical applications.

In Chap. 11, we discuss a particular class of spatial networks that are made of branches radiating from a node and a loop (or ring) connecting these different branches. We will see the conditions under which the loop can have a larger betweenness centrality than the origin and we will also discuss the impact of congestion at the center on the overall pattern of shortest paths.

In Chap. 12, we present optimal networks and their properties, and discuss the most important illustrations of this class of graphs such as the minimum spanning tree that minimizes the total length of the network. We will discuss the statistical properties of this tree and we will present a more general class of optimal trees that minimize a combination of length and betweenness centrality, allowing to interpolate between the minimum spanning tree and the star graph (that minimizes the average shortest path). We end this chapter with a discussion of the conditions for the appearance of loops or a hub-and-spoke structure in this optimization framework.

In Chaps. 13 and 14, we present models of network growth where a new node is added at each time step and connects to the existing network according to certain rules. In Chap. 13, we first consider spatial variants of preferential attachment where the new node will preferentially connect to well-connected nodes, up to a distance-dependent factor. We will also consider the "potential" approach where the addition of a new node is governed by a potential that gives the probability to choose a specific location and depends in general on the state of the network at this time. We describe in this chapter the general philosophy of this approach and detail the example of the growth of road networks. In Chap. 14, we consider the case of "local" optimization where each node (added sequentially) optimizes a given function. The minimization is therefore local and the resulting network at large time does not in general minimize a simple quantity. An important example in this class of greedy models is the cost–benefit model which we will discuss thoroughly here. This framework will allow us to understand some of the properties of transportation networks such as subways or railways and how they are affected by the substrate where their evolution take place.

We end this book with a (subjective) discussion in Chap. 15 about what seems to be interesting and important research directions in the study of spatial networks.

As can be seen in this short outline of the book, several disciplines are concerned. Scientists from statistical physics, random geometry, probability, and computer sciences produced a wealth of interesting results and this book cannot cover all new studies about spatial networks. Owing to personal biases, space limitations, and lack of knowledge, important topics might have been omitted, and I apologize in advance for omissions or errors and to those colleagues who feel that their work is not well represented here. Incomplete and imperfect as it is, I hope, however, that this book will be helpful to scientists interested in the formation and evolution of spatial networks, a fascinating subject at the crossroad of so many disciplines.

Paris, France
June 2017

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http://www.springer.com/978-3-319-20564-9

Morphogenesis of Spatial Networks
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2018, XIX, 331 p. 187 illus., Hardcover
ISBN: 978-3-319-20564-9