Preface

renaissance [...]

> enthusiastic and vigorous activity along literary, artistic, and cultural lines distinguished by a revival of interest in the past, by an increasing pursuit of learning, and by an imaginative response to broader horizons generally [...]
> a return of youthful vigor, freshness, zest, or productivity a renewal of life or interest in some aspect of it [...]
>
> Webster's Third New International Dictionary

Sampling theory has played a central role in mathematics, science, and engineering for over 75 years now. The original quest of identifying a continuous function on Euclidean space from discrete data is addressed in the classical sampling theorem, commonly attributed to Cauchy, Kotelnikov, Ogura, Raabe, Shannon, and/or Whittaker. It states that a bandlimited function can be recovered in full from values measured on a regular sampling grid whenever the bandlimitation is described by an interval whose length does not exceed the density of the sampling grid. A multitude of variants and extensions of this result have cemented the extensive role of sampling theory in engineering and science during the second half of the 20th century.

Today, the original emphasis on recovery from samples is complemented by the need for efficient digital representations of signals and images by various kinds of available, but at first sight insufficient, measurements. In addition, fast and noise resistant algorithms aimed at recovering from such measurements are of increasing importance. The assumption that a signal is bandlimited in the classical setting is commonly replaced by possibly nonlinear constraints on the objects at hand; and the need to efficiently obtain reliable nonredundant representations of such objects may involve a nonlinear measurement procedure as well.

Such and related considerations have lately reenergized the area of sampling theory and inspired the rapid growth of new interdisciplinary research areas such as compressive sensing and phase retrieval.

Compressive sensing is based on the observation that many practical signals like images, speech, music, radar signals, ultrasound signals, and man-made communication signals are well characterized by a relatively small number of relevant parameters when compared to the dimension of the ambient space. That is, we assume that the signal is contained in – or is well approximated by a signal in – the union of low-dimensional subspaces of a high dimensional space; the signal depends on a *sparse* set of parameters and the difficulty lies in realizing which parameters are active and which ones can be ignored. For example, if a high dimensional signal is known to have few nonzero Fourier coefficients of unknown locations, then compressive sensing algorithms exploit this sparsity assumption and recover the signal from samples far below the Nyquist rate.

In compressive sensing, the nonlinearity of the signal space leads to challenging mathematical problems when attempting to prove performance guarantees for realistic recovery algorithms such as Basis Pursuit or Orthogonal Matching Pursuit. State-of-the-art results control the recovery probability of sparse signals when the number of required measurements grows only linearly in the number of nonzero parameters and logarithmically in the ambient dimension.

The second example of a flourishing research area in sampling theory is motivated by X-ray crystallography where, in essence, only magnitudes of Fourier coefficients of an image are measured. In order to reconstruct the image, some additional insights on the image need to be utilized to recover the phase of each Fourier coefficient and thereby the original image. To achieve this in a provably numerically stable manner remains an open problem to date. This being said, the described problem spearheaded the novel research area of phase retrieval. The question addressed herein is the following: in which settings and for what kind of measurements can we design algorithms that recover images or other signals from magnitudes of those measurements?

Compressed sensing and phase retrieval are just two examples that illustrate the influx of new ideas and paradigms in sampling theory; they form the foundation of the sampling theory renaissance that we enjoy today in mathematics, science, and engineering.

The contributed chapters in this volume are authored by invited speakers and session organizers of the *10th International Conference on Sampling Theory and Applications* (SampTA) which took place on July 1st to 5th, 2013, in Bremen, Germany. The authors' contributions are organized into five parts, "Random Measurements of High Dimensional Data," "Finite and Structured Frames," "Band-limitation and Generalizations," "Sampling and Parametric Partial Differential Equations," and "Data Acquisition," thereby representing a good portion of research areas discussed at SampTA 2013.

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