

# Prof. F.L. Litvin: Contribution to the Formation of the Russian School of the Theory of Gearing

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**Abstract** A brief overview of selected scientific works by Prof. F.L. Litvin performed up to 1980 is presented. His role in the development of the Russian school of the theory of gearing is shown.

**Keywords** Generating of gearing · Kinematic method of investigation · Envelope surfaces · Analysis and synthesis of gears

## 1 Introduction

The end of 1950s was marked by two outstanding events in the formation of the Russian school of the theory of gearing. First, in 1959, the engineer Colonel M.L. Novikov was granted an afterlife with the Lenin award for the development of gears with a new system of gearing, which caused an outbreak of interest in gears. Second, in 1960, the first edition of a monograph by Prof. F.L. Litvin was published, though at the time it was noticed only by a very limited number of experts.

The fundamentals of gearing research were laid in the first half of the 20th century through works by outstanding scientists, namely Ch.I. Gochman, Ch.F. Ketov, N.I. Kolchin, V.N. Kudryavtsev, and V.A. Gavrilenko. But publication of the monograph by Litvin [1] specifically marked the identification of the theory of gearing as a separate independent scientific subject. Beginning exactly at its inception, a “Sturm und Drang” period in the development of this theory started.

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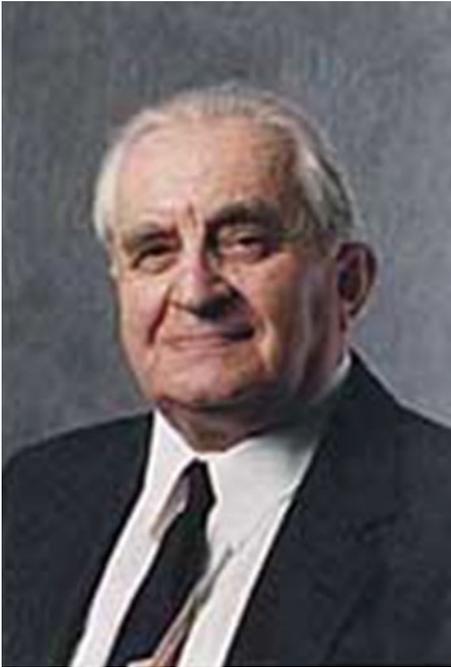
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At that time a whole constellation of bright researchers gathered around F.L. Litvin. Among them were: Ya.S. Davydov, L.V. Korostelev, N.N. Krylov, K.M. Pismanik, K.I. Gulyaev, G.I. Sheveleva, M.L. Erikhov et al. Cooperating and competing with each other, within a few years they had created a marvelous building of a new science at the junction of differential geometry and machine mechanics.

While we can talk about each of these brainstorming participants as contributing some part to this new science, F.L. Litvin laid the essential primary foundation most completely represented in monograph [2].

An attempt is made below to enumerate, even if not in great detail, the separate, closely related blocks of this foundation.



## 2 A Little Prehistory. Principles of Generation of Spatial Gearing

Fundamentals of the theory of spatial gearing were laid out by a French geome-trician Theodore Olivier [3]. He was the first to consider the generation of conju-gated surfaces as the enveloping process and introduce the concept of an auxiliary generating surface. Considering tooth flanks as enveloping to the auxiliary surface, he found the condition of their linear and point contact. Olivier suggested then that the theory of gearing is a particular subject of projective geometry.

Restrictions imposed by projective geometry were withdrawn by Gochman [4]. Having developed analytic investigation methods, he projected the theory of gearing into the area of analytical and differential geometry.



Olivier, Theodore  
(1793-1853)



Gochman, Chaim I.  
(1851-1916)

Principles of Olivier-Gochman, generalized by Ya.S. Davydov for the case of applying non-rigid and non-congruent pairs of generating surfaces [5], have been and remain now the foundation of any method for the generating of gears cut by enveloping. While developing a system of gearing named after him, M.L. Novikov proposed a principle of generating of gearing that differed from the principles of Olivier-Gochman [6]; however, industrial implementation of Novikov gears became possible only on the basis of applying the basic rack profile of a pair of non-congruent generating racks. Even the recently outlined trend of cutting gears by pin tools with NC machines will not save the design engineers from the necessity of virtual tooth cutting by Olivier-Gochman principles.

### 3 Kinematic Method for Investigation of Spatial Gearing

Principles of generating of gearing require an adequate mathematical apparatus for their investigation. While developing the analytical methods of Ch.I. Gochman, N.I. Kolchin brought them into a state for solving the important practical tasks, such as: investigation of spur bevel gears, non-orthogonal worm gears and globoid gears of Cone type, profiling the cutting tool and analysis of errors in gearing [7].



Kolchin, Nikolai I.  
(1894-1975)



Davydov, Yakov S.  
(1914-2003)

However, the methods of analysis by Gochman and Kolchin were very bulky. The next step in the development of the theory of gearing was creation of a kinematic method of investigation, based on the simple and obvious fact that at any contact point of mutually enveloping surfaces their common contact-normal  $\mathbf{n}$  should be perpendicular to the vector of their relative velocity  $V^{(1,2)}$ . The kinematic approach to gearing investigation was proposed at the beginning of the 1950s by three researchers independent of each other. But while Shishkov [8] described such an approach verbally, and Davydov [9] expressed it through projections of the above-mentioned vectors, it was F.L. Litvin who presented the main law of gearing in a perfect and concise way [10]:

$$nV^{(1,2)} = 0.$$

This law allows to describe the enveloping surface and to investigate the conditions of surface contact analytically. For 60 years already this exact law has been the basis of a majority of investigations both in the area of analysis and the synthesis of gearing.

However, to be fair, in a number of cases, a non-differential approach to solving the enveloping problem, i.e., unrelated to finding normal vectors to surfaces, is sometimes of greater advantage. Fundamentals of such an approach were proposed in [11, 12].

## 4 Matrix Method of Coordinate Transformation in the Theory of Gearing

In any case, the investigation of gearing is related to a repeated transformation of equations of lines and surfaces from one coordinate system to another: from the system related to a tool to a fixed coordinate system, from a fixed one to a system related to the gearwheel to be cut, etc. In order to describe these transformations, F.L. Litvin proposed applying a  $4 \times 4$  matrix, where the  $3 \times 3$  determinant is for cosines of angles between axes of the old and new coordinate systems and the 4th column is for coordinates of the vector connecting the origins of these systems. This method is so vital that Litvin describes it in the first chapters of all his monographs up to the last edition [13], and all his further investigations were performed based on application of this method.

The versatility of this method becomes even more important when one considers that, according to the reasonable note of M.L. Erikhov, having mastered this mathematical apparatus, any average post-graduate is promptly capable of solving the task of investigating any type of new gearing. This statement is proved by the fact that 48 Ph.D. theses were presented under the direct supervision of F.L. Litvin in Leningrad. And the number of Litvin's absentee followers, from both Russia and abroad, who applied his methods, including the method of matrix transformation, is uncountable.

## 5 General Problems of the Theory of Spatial Gearing

Among the most important theoretical tasks successfully solved by F.L. Litvin in his works and described in the monograph [2], the following should be mentioned:

- (i) Applying the kinematic method of investigation, he obtained equations of a surface enveloping a family of generating surfaces both for one and, what is more complicated, two parameters of enveloping. He revealed the conditions sufficient for the existence of the envelope to the family of generating surfaces represented by equations both in parametrical and implicit forms.
- (ii) Investigating curvatures in the gearing, he determined the main and normal curvatures of the enveloping surface and its torsion. He found direct relations between curvatures of the enveloped and enveloping surfaces in one- and two-parametrical enveloping processes. He determined the dimensions and orientation of the Dupin indicatrix as the prototype of the instant contact ellipse.
- (iii) He developed the method for finding singular points on the enveloping surface and methods of their prevention at its active segment or, in engineering language, methods of preventing tooth undercutting.

- (iv) The concept (well-known for gears with parallel and intersected axes) of the axis of meshing as the straight line intersected by all contact normals of the gear was extended by him to gears with crossed axes. He proved the existence of a pair of such equivalent axes in the case in which one of engaged surfaces is helical with a constant pitch, and he showed how application of these axes allows for simplifying the investigation of gearing, in particular, worm gears.
- (v) He considered and described mathematically seven different ways of synthesis of conjugated gearing, including by means of a rigid or non-rigid non-congruent pair of generating surfaces as well as by that of generating lines.
- (vi) Litvin proposed methods of synthesis of approximated gearing with the optimal conditions of gearing in the vicinity of the predesigned contact point.
- (vii) He investigated the effect of errors in gear manufacturing and assembly on the function of transmission errors in gearing with linear and point contact.

## 6 Geometry of Gearing for Specific Types of Gears

Statement of the above enumerated general problems of the theory of gearing takes up about one third of the monograph [2]. The remaining two thirds are devoted to investigation of gearing geometry for specific types of gears. Among them are:

- (i) Non-circular gears with closed centrodes, intended to transmit the rotation with variable gear ratio. The first monograph by the author [14] was devoted to investigation of these gears.
- (ii) Helical gears with parallel axes, both involute and Novikov-Wildhaber gears cut by a non-congruent pair of generating racks.
- (iii) Helical gears with crossed axes, including involute ones and a new type of crossed gears cut by a congruent pair of concave and concave generating racks, proposed together with V.V. Shultz.
- (iv) Worm gear drives, orthogonal and non-orthogonal, with cylindrical worms, both with ruled surfaces ZA (Archimedes), ZN (convolute), and ZI (involute), and non-ruled surfaces, such as surface ZK, reproduced by a grinding cone and, finally, with two new versions of ground worms with a concave profile, later included into Russian standards as types ZT1 and ZT2.
- (v) Face-gear drives with orthogonal and non-orthogonal, intersecting and crossed axes, for which engineering techniques of geometrical design were developed taking into account conditions preventing tooth undercut and sharpening and providing contact localization.
- (vi) Face-milled spiral bevel gears with tapered and uniform teeth, Helixform bevel gears with helicoid tooth surface and cyclo-palloid bevel gears with cycloid tooth profile of the generating wheel. In all cases, investigation is brought up to obtaining the equations of conjugate surfaces and the design of the set-up parameters of cutter heads to provide contact localization in gears.

## 7 Investigations Made in the 1970s

During this time, F.L. Litvin's main attention was focused on scientific supervision of his numerous post-graduates. For instance, at the 6th All-Union Conference on ToMM in 1973, he presented his survey paper with seven young co-authors [15]. The following issues were discussed in this paper, amongst others: analysis of pitch surfaces of hypoid and spiroid gears, synthesis of involute spiroid gearing, synthesis of double-enveloping worm gears with a grounded worm, synthesis of double-enveloping worm gears with reduced vibration activity, synthesis of bevel gears with circular teeth based on uniting the local synthesis through solving the so-called reverse task of the theory of gearing.

Among other practically important works of that period, the authors should mention the work [16], in which fundamentals of the synthesis of cylindrical worm gear drives with localized contact have been presented.

Simultaneous to working through the issues of specific gear geometry, F.L. Litvin continued investigating general issues of the theory of gearing.

In particular, he generalized the formula of Euler-Savary for determining the relation between the main curvatures of tooth flanks in spatial gearing [17].

Together with M.L. Erikhov, he created the vector field of normals for ordinary nodes of contact of the enveloped surface [18].

Working through the concept of the node line of contact as the envelope to the family of instant contact lines, Litvin revealed the necessary and sufficient conditions of such a line existence on the generating surface, having thus determined the boundary of this surface beyond which the area not participating in the enveloping process is located [19].

He revealed the sufficient conditions of existence for the similar node line of contact and the edge of regression on the enveloping surface, i.e., determined its theoretical boundaries [20].

The full list of scientific publications and patents that Doctor of Technical Sciences Prof. Litvin published in Russia within the period from 1939 until 1978 comprises 121 papers and 22 inventor's certificates. This list was made by F.L. Litvin himself and is presented in the Appendix.

The 1970s were also the beginning of Litvin's publication abroad. The first of these papers was devoted to investigation of the relation between tooth curvatures in spatial gearing [21]. Another notable publication of this period is the fundamental generalization of methods of gearing synthesis by two-parameter enveloping, written together with N.N. Krylov and M.L. Erikhov [22]. It is difficult for a modern internet user to understand the psychological effect these breakthrough Russian articles, coming from behind the Iron Curtain to the international podium, had on his contemporaries.

In 1979, Faydor L'vovich immigrated to the USA with his family. There, he continued his active scientific work. He organized the Gear Research Centre at the University of Illinois in Chicago, and his credibility lends this Centre the support of

the leading mechanical engineering enterprises in the country. This period of activity is better described by his post-graduate students in Chicago.

We note only that, working in the United States, F.L. Litvin didn't interrupt his personal and scientific relations with his Russian colleagues. In particular, in 2003, he visited Moscow, and at the ensuing workshop, which brought together all contemporary Russian experts, delivered a brilliant lecture on the latest scientific achievements of his school.

## 8 Conclusion

Thankfully, Faydor L'vovich is still with us, healthy and continuing his activity. He works 364 days a year. On the 365th, Judgment Day, he reports to God and begins a new work. Thus, he does not allow us, his students and followers, to overestimate our small successes and rest on our laurels.

He does not let us forget that, if we want to live in a civilized world, someone has to design and cut those gears without which we can drive neither car wheels, nor power plant turbines, nor gear pumping units for oil extraction, nor mill rolls.

Let us hope that supreme forces will maintain his health and buoyancy until the age of 120, as given to a man in one old and wise book.

## Appendix

### **Full List of Scientific Publications and Inventions of Professor, Doctor of Technical Sciences F.L. Litvin for the Period of 1939–1978**

#### *Publications*

1. Basic statements of shaving process. Proc. of Committee on tooth-cutting engineering. VNITOMASH-LONITOMASH, N1, 1939 (in Russian).
2. To calculation of rectilinear guidance mechanism of the type of Evans mechanism. "Journal of Engineers and Technicians", N9, 1939 (in Russian).
3. Influence of eccentricity of spur pinions of involute profile on reading accuracy gears. Jnl. "Optical and Mechanical Industry", N4, 1940 (in Russian).
4. Control of spur pinions by means of wires. Jnl. "Optical and Mechanical Industry", N6, 1941 (in Russian).
5. Spur and helical involute pinions with crossed axes; influence of errors on accuracy of operation. PhD thesis, maintained in Tomsk polytechnic institute in 1944 (in Russian).
6. Crossed helical gears. Proc. "Gearing", N1, Mashgiz, L., 1946. (Awarded by LONITOMASH as the best scientific research work) (in Russian).

7. Main problems of the theory of non-circular gearwheels. In Proc. of LONITOMASH, N6 "Theory and Analysis of Gearwheels", Mashgiz, L., 1949 (in Russian).
8. Some problems of the theory of gearing for non-circular spur and helical gearwheels. "Proceedings of Seminar on Machine and Mechanism Theory", N29, USSR AS, 1949 (in Russian).
9. Non-circular gearwheels (monograph, the first edition). Mashgiz, 1950 (in Russian).
10. Basic theorems of planar gearing and their application to profiling the flat gearing. "Proceedings of LPI", N7, Mashgiz, 1950 (in Russian)
11. Non-circular gearwheels for fluid meter. "Journal of Engineers and Technicians", N3, 1950 (in Russian).
12. To the question of noncircular gearwheel tooth cutting by generating method "Proceedings of Seminar on Machine and Mechanism Theory". N 34, USSR AS, 1950 (in Russian).
13. Determination of tooth thickness of worms and helical gearwheels by rollers and balls. "Proceedings of Seminar on Machine and Mechanism Theory", N39, USSR AS, 1951 (in Russian).
14. Application of bevel gearing analytical theory to determination of tooth thickness of straight bevel gearwheels by balls. "Proceedings of Seminar on Machine and Mechanism Theory", N44, USSR AS, 1952 (in Russian).
15. Calculation methods in production and control of gear parts (coauthor: N.I. Kolchin), Mashgiz, 1952 (in Russian).
16. Cutting non-circular gearwheels by division method. "Report of technical information" N1, Glavpribor, 1952 (in Russian).
17. Toward the Investigation of Spatial Gearing with Line Contact of Surfaces. "Proceedings of Seminar on Machine and Mechanism Theory", Vol. 49, USSR AS, 1953 (in Russian)
18. Tool profiling for helical surface machining according to pitch. "Proceedings of LPI", N4, 1953 (in Russian).
19. Improvement of gear hobbing machine for cutting gearwheels (coauthor: G.G. Pavlov). "Machines and tools", N4, 1953 (in Russian).
20. Theory of gearing, design and production of non-circular gearwheels. "Gears in Mechanical Engineering. Proceedings of the first Moscow conference on gears in mechanical engineering", USSR AS, 1953 (in Russian).
21. Adjustment of stroke, force calculation and balancing of a piston machine (coauthors: N.I. Kolchin, A.A. Valkovskiy). Study guide. LPI Publ., 1954 (in Russian).
22. Production of multi-thread worm gears with new geometry (coauthors: S.G. Printsental, G.F. Shigorin). Mashgiz, 1953 (in Russian).
23. Theory and methods of production of gears with variable gear ratio (Doctor of technical sciences thesis, maintained in Leningrad polytechnic institute in 1954), (in Russian).
24. Device for cutting non-circular gearwheels (coauthor: G.G. Pavlov). "Machines and tools", N2, 1954 (in Russian).

25. Analytical method of investigating the worm gearing with variable gear ratio and variable center distance. "Proceedings of LPI", N3, 1954 (in Russian).
26. Analysis of cylindrical screw manifolds (coauthor: D.L. Faynshtein). "Machines and tools", N7, 1955 (in Russian).
27. Designing non-circular gearwheels, applied in mechanical engineering. "Proceedings of IMASH. Seminar on TMM", N55, USSR AS, 1954 (in Russian).
28. Application of matrices and theory of screws to investigation of spatial gearing. "Proceedings of LPI", N182, 1955 (in Russian).
29. Reproducing the function of one independent variable by a simple row of non-circular gearwheels. "Proceedings of LPI", N182, 1955 (in Russian).
30. Non-circular gears: design, theory of gearing and manufacture, 2nd ed, Mashgiz, 1956. 320p. (in Russian).
31. Non-circular gears. "Machines and tools", N9, 1956 (in Russian).
32. Addendum correction of spur gears and drawing of their meshing (coauthor: N.I. Kolchin). Study guide. LPI Publ., 1957 (in Russian).
33. On design and cutting teeth of multithread non-circular gearwheels (coauthor: N.S. Yablonskiy). "Priborostroeniye", N6, 1957 (in Russian).
34. To investigation of steady motion of machine aggregate with elastic driving shaft for forces dependent on velocity and position of links. "Proceedings of IMASH. Seminar on TMM", N68, USSR AS, 1958 (in Russian).
35. Simplified method of determining the contact lines and contact area of a worm gearing with a cylindrical worm. "Vestnik mashinostroyeniya", N2, 1959 (in Russian).
36. To calculation of a functional shunted potentiometer (coauthor: S.G. Kislitsin). "Automatics and telemechanics", N11, 1959 (in Russian).
37. Simplified method of determining the position of a ball or roller in the root of a screw or worm. "Izvestiya Vuzov. Mashinostroyeniye", N1, 1960 (in Russian).
38. Simplified method of investigating the worm gearing with a cylindrical worm. "Proceedings of the 2nd All-Union Conference on main problems of TMM", USSR AS, 1960 (in Russian).
39. Problems of geometry and manufacture of worm gears with the increased load-carrying capacity (coauthor: I.K. Koryshev). "Proceedings of the conference on technique of gears", Mashgiz, 1960 (in Russian).
40. Analytical methods of spatial gearing investigation. "Proceedings of IMASH. Seminar on TMM", N84, USSR AS, 1961 (in Russian).
41. Engagement of bevel gears with epi- hypotrochoid longitudinal profile (coauthor: N.B. Firun). "Proceedings of IMASH. Seminar on TMM", N86, USSR AS, 1961 (in Russian).
42. Theory of gearing (monograph, the 1st ed.) FIZMATGIZ, 1960 (in Russian).
43. Grinding method and geometry of thread surfaces of cylindrical worms with concave profile. "Izvestiya OTN USSR AS. Mechanics and mechanical engineering", N4, 1961 (in Russian).

44. Dynamic balancing of rotors (coauthors: S.G. Smirnov, N.F. Utekhin). LPI Publ., 1961 (in Russian).
45. Geometry of worm gear drives with cylindrical worms with concave profile, ground by a toroidal wheel. "Proceedings of Odessa conference", Mashgiz, 1961 (in Russian).
46. Geometry of surfaces and efficiency of screw pairs with rolling friction (coauthor: B.A. Konstantinov). "Machines and tools", N7, 1962 (in Russian).
47. Geometrical investigation of a variety of Novikov gearing. "Proceedings of mechanical institute", N24, 1962 (in Russian).
48. Undercutting of bevel gearwheels with circular teeth (coauthor: Guo-Kai). "Machines and tools", N11, 1962 (in Russian).
49. New types of worm gear drives with cylindrical worms (monograph). Mashgiz, 1962 (in Russian).
50. Investigation of meshing of bevel gears with tapered teeth. "Theory of machines and mechanisms" (coauthor: Guo-Kai). "Theory of machines and mechanisms", N92-93, USSR AS, 1962 (in Russian).
51. Method of determination of contact lines and limiting points of tooth flanks of a spatial gearing mechanism. "Izvestiya OTN USSR AS. Mechanics and mechanical engineering", N6, 1962 (in Russian).
52. Matrix method of determination of relation between curvatures of tooth flanks in a gearing (coauthor: Chzhan Tsy-syan). "Izvestiya OTN USSR AS. Mechanics and mechanical engineering", N3, 1963 (in Russian).
53. Tooth undercutting of planar and spatial gearing. "Proceedings of the 3rd all-Russian meeting on TMM", Vol. 4, Mashgiz, 1963 (in Russian).
54. Improvement of engagement conditions of bevel gearwheels with arch teeth (coauthor: Guo-Kai). "Proceedings of Seminar on Machine and Mechanism Theory", N98-99, USSR AS, 1963 (in Russian).
55. Investigation of engagement conditions in vicinity of the tooth flanks contact point. "Proceedings of Seminar on Machine and Mechanism Theory", N100, USSR AS, 1963 (in Russian).
56. Position function and transmission ratio of the driving arm mechanism. "Analysis and Synthesis of Mechanisms and Gear Theory", Nauka, 1965 (in Russian).
57. Application of the kinematic method to determine the relation between curvatures of mutually enveloping surfaces, and conditions for the absence of teeth undercutting.. "Proceedings of Seminar on Machine and Mechanism Theory", N103, USSR AS, 1964 (in Russian).
58. Main problems of the theory of spatial gearing and methods of their solution. Proceedings of Miskolc-Universitat, Hungary, 1964 (in Hungarian).
59. Geometry and methods of manufacture of worm gear drives with concave worm profile. Proceedings of Miskolc-Universitat, Hungary, 1964 (in Hungarian).
60. Reference book of designer of precision instrument industry. (Chief editing and author of several sections). "Mashinostroyeniye", 1964 (in Russian).

61. Functional cam mechanisms (monograph). Publ. house of Leningrad institute of precise mechanics and optics, 1965 (in Russian).
62. Synthesis of approximated bevel and hypoid gearing with improved parameters. "Analysis and synthesis of mechanisms and theory of gearing", M. Nauka, 1965 (in Russian).
63. Determination of initial parameters, their correction when cutting bevel gearwheels (coauthor: A.A. Zaostrovskiy). "Theory of gears in machines", Mashinostroyeniye, 1966 (in Russian).
64. Helical gearwheels with new geometry, providing the increased load-carrying capacity (coauthor: V.V. Schultz). "Theory of gears in machines", Mashinostroenie, 1966 (in Russian).
65. Improving the engagement conditions in Helixform hypoid gears (coauthor: Yu.I. Shurygin). "Theory of gears in machines", Mashinostroenie, 1966 (in Russian).
66. Determination of the error of the dividing circuit for tooth hobbing machines by electro-magnetic method and error influence on the accuracy of gearwheels to be cut (coauthor: N.G. Lindtrop). "Proc. of technical economic conference of Neva plant n.a. Lenin", Mashinostroyeniye, 1966 (in Russian).
67. Determination of main parameters of gearwheels of Helixform bevel gears with helicoid tooth surface (coauthor: B.D. Zilberman). "Mechanics of machines", N1-2, Nauka, 1966 (in Russian).
68. Application of kinematic method to determine curvatures and conditions of undercutting elimination for the envelope of two-parameter set of tool surfaces (coauthor: M.L. Erikhov). "Mechanics of machines", N3-4, Nauka, 1966 (in Russian).
69. Engagement and cutting the bevel gearwheels with cycloidal tooth lines (coauthors: V.N. Kedrinskiy, K.I. Gulyaev, N.B. Firun). "Machines and tools", N2, 1967 (in Russian).
70. Synthesis of approximated gearing according to local conditions (Yu.A. Sinichkin, V.S. Smirnov). "Mashinovedeniye", N2, 1967 (in Russian).
71. Worm gear drives with the concave profile worm (coauthors: V.N. Komkov, I. P. Bernatskiy). "Vestnik mashinostroeniya", N9, 1967 (in Russian).
72. To generation of non-circular bevel gearwheels (coauthor: G.A. Livshits). "Mashinovedeniye", N1, 1968 (in Russian).
73. Theory of gearing (monograph, 2nd ed.). "Nauka" Publ., 1968 (in Russian; it was translated into Hungarian and published in Hungary in 1973).
74. Transmission of motion by eccentrically fixed helical gearwheels (coauthor: N.G. Lindtrop). "Gears and worm gear drives", Mashinostroenie, 1968 (in Russian).
75. Contact conditions and radii of curvatures in the design point for straight bevel gears cut by continuous circular drawing (coauthors: Yu.A. Sinichkin, V.S. Smirnov). "Theory of mechanisms and machines", N5, Kharkov, 1969 (in Russian).

76. Determination of main curvatures of tooth flanks and parameters of cutting heads for spiral bevel gears with tapered teeth (coauthor: B.P. Timofeyev). "Mashinovedeniye", N1, 1969 (in Russian).
77. Curvature determination for tooth flanks of straight bevel gearwheels at synthesis according to local conditions (coauthors: L.Ya. Liburkin, B.D. Zilberman, Yu.A. Sinichkin). "Mashinovedeniye", N5, 1969 (in Russian).
78. Cutting of non-circular bevel gearwheels by intermittent face milling (coauthor: R.Sh. Varsimashvili). "Machines and tools", N5, 1970 (in Russian).
79. Generalization of the Euler-Savary formula and determination of relations between main curvatures of tooth flanks in spatial gearing. "Mashinovedeniye", N6, 1970 (in Russian).
80. Determination of function of the position in a spatial rod mechanism by a conditional open circuit. "Mashinovedeniye" N3, 1970 (in Russian).
81. Finding the line of boundary points on the tool surface (coauthors: V.D. Britskiy, B.P. Timofeyev). "Izvestiya VUZov. Mashinostroyeniye, N3, 1970 (in Russian).
82. "Theory of gears in machines" (Chief editing). Mashinostroyeniye, 1970. (in Russian).
83. Development of geometry and theory of gearing. "Theory of gears in machines", Mashinostroyeniye, 1970 (in Russian).
84. Vector field of normals in ordinary node contact points of the enveloped surface (coauthor: M.L. Erikhov). "Theory of gears in machines". Mashinostroyeniye, 1970 (in Russian).
85. Analysis of settings to cut Helixform bevel gears with tapered teeth (coauthor V.N. Rubtsov). "Machines and tools", N12, 1970 (in Russian).
86. Worm link stopper of rotation (coauthors: V.D. Britskiy, I.P. Tyrsa). "Izvestiya VUZov. Priborostroyeniye", N8, 1970 (in Russian).
87. Peculiarities of generating the contact pattern of bevel gears with circular teeth (coauthors: V.N. Rubtsov, Kh.I. Marinov, B.P. Timofeyev). "Machines and tools", N6, 1971 (in Russian).
88. Mathematical modeling of approximated gearing and their optimization. "Proceedings of international congress on theory of mechanisms and machines", 1971, Belgrad (in Russian).
89. Synthesis of face-milled and Helixform bevel gearwheels with uniform teeth (coauthors: V.N. Rubtsov, Kh.I. Marinov). "Izvestiya Vuzov. Mashinostroyeniye", N1, 1971 (in Russian).
90. Kinematic way of determining the curvature of centrodes (coauthor: E.E. Peisakh). "Izvestiya Vuzov. Mashinostroyeniye", N1, 1971 (in Russian).
91. Elimination of node contact points at synthesis of involute spiroid gears (coauthor: V.A. Ganshin). "Theory of gears in machines", Nauka, 1971 (in Russian).
92. Probabilistic estimation of kinematic error of eccentric gearwheels (coauthors: Y.I. Gutman, V.A. Kutsokon). "Izvestiya VUZov. Priborostroyeniye", N10, 1972 (in Russian).

93. Synthesis of straight bevel gears cut by circular broaching (coauthors: L.Ya. Liburkin, B.D. Zilberman, Yu. A. Sinichkin). "Mechanics of machines", N 31–32, 1972 (in Russian).
94. Transmission of forces and friction losses in spur involute gears with spring mechanism to adjust the dead stroke (coauthors: Yu.Ya. Grossmann, M.A. Nozdrin. "Izvestiya VUZov. Priborostroeniye", N5, 1972 (in Russian).
95. Transmission of forces in zoom-lens (coauthors: E.E. Peisakh, Ya.M. Kruger). "Izvestiya VUZov. Priborostroeniye", N12, 1972 (in Russian).
96. Synthesis of face-milled and Helixform bevel gearwheels according to local conditions (coauthors: B.P. Timofeyev, V.N. Rubtsov). "Mechanics of machines", N31-32, 1972 (in Russian).
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*Inventor's Certificates (Granted in Russia)*

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2. Method of checking the centrodes of non-circular gearwheels. Certificate of authorship N 88033 priority dated April 25, 1949 (coauthors: Ch.F. Ketov and B.S. Kogan).
3. Device to gear-hobbing machine to cut convex-concave non-circular gearwheels. Certificate of authorship N 91505 priority dated January 24, 1950.
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