Chapter 2
Multiple Flare-Angle Smooth-Walled Horn

Overview: In this chapter, we shall discuss the employment of multiple flare-angle smooth-walled horns optimised to operate at 700 GHz. We will first describe the design of the feed horn, then the fabrication technique we developed to machine these horns, followed by the experimental setup we built to measure the far field radiation patterns. The measured results will be analysed and compared with the theoretical predictions produced using our own software package. In addition, we shall discuss the use of Ansys HFSS for tolerance analysis.

2.1 Introduction

Feed horns with a good radiation characteristic are an essential component in the design of a sub-mm wave heterodyne receivers. They offer high beam efficiency and low sidelobes level required by sensitive astronomical observations. Most of modern sub-mm receivers use corrugated horns (Clarricoats and Olver 1984) to efficiently couple the RF signal from the sky to the detector. Corrugated horns however require azimuthal corrugations in their interior wall to create isotropic surface boundary conditions to the propagating E- and H-field wavefronts. This suppresses unwanted waveguide modes, and allows only the propagation of the hybrid HE_{11} mode (Olver et al. 1994), to produce uniform and polarized illumination at the horn aperture. The result is a far-field radiation pattern that has high directivity, low sidelobes and low cross-polarization levels across a large operational bandwidth (around 50%).

Corrugated horns are fabricated using either direct machining or electroforming. Direct machining needs a high precision milling machine to achieve the required machining tolerances and repeatability, in particular near the horn’s throat where the diameter is too small for a robust tool. Electroforming, on the other hand, requires complicated mandrel fabrication, electroplating and chemically dissolving stages, albeit the machining of the mandrel is easier. Both processes are costly (approximately $1,000–2,000 per horn) and time consuming (several weeks per horn).
The new generation of sub-mm astronomical telescopes would require a larger number of feed horns for their large multi-pixel focal plane array receiver systems (∼10–1000 pixels). The cost for producing these large number of horns has become considerably high and the time required for their construction has become a major concern.

In recent years therefore, there has been considerable interest to find alternatives to corrugated horns without compromising the radiation pattern quality (e.g. Granet et al. 2004; Yassin et al. 2007; Britton et al. 2010). Smooth-walled horns that have comparable radiation characteristics to a corrugated horn, but without the large number of high density corrugation grooves (∼5 corrugations per wavelength) is an attractive option. The simplest form of a smooth-walled horn is a Pickett-Potter horn (Pickett et al. 1984; Potter 1963), which has only a single step discontinuity near the throat followed by a smooth conical flaring section. The step discontinuity excites the TM$_{11}$ mode at about 16% of the total incident power of the incoming TE$_{11}$ mode. Both fields propagate along the conical flaring section until they arrive in phase at the horn aperture. Figure 2.1a shows the resulting far field pattern of the TE$_{11}$ and TM$_{11}$ modes, and it can be seen clearly that the sidelobes level and position of both modes are now coincide with each other. The combination of both modes therefore results in cancellation of sidelobes, and a main beam with high circularity. This generates a highly uniform field that give low sidelobes level, and low cross-polarization in the far-field radiation pattern.

One disadvantage of a Pickett-Potter horn is that it has a narrow operating bandwidth. The performance however can be substantially improved by increasing the number of discontinuities near the throat of the horn (Yassin et al. 2007), that generate a carefully chosen combination of the higher order modes which could widen the operating bandwidth of the horn (Kittara 2002; Kittara et al. 2000, 2004). The optimised depth of these discontinuities, the horn length and flare angles can be predicted using modal matching in conjunction with optimisation algorithms (Granet et al. 2004; Zeng et al. 2010). For designing the multiple flare-angle smooth-walled horn described in this chapter, we employed modal matching for design
analysis, and Genetic Algorithm (GA) for optimisation (Haupt and Haupt 1998; Kittara et al. 2007).

The two algorithms were incorporated into a self-contained C++ package written by Kittara et al. (2007), HornSynth. Using this package, they have successfully produced a variety of high performance feed horns, with two, three and four flare-angle sections, giving respectively 15, 20 and 25% operating bandwidth (Kittara et al. 2008; Leech et al. 2012). As shall be seen later, one major advantage of the multiple flare-angle horn is that they only require a simple fabrication process due to their simple interior geometry (see Fig. 2.1b). A series of 230 GHz multiple flare-angle horns and horn arrays have been fabricated using this technique and tested extensively by Leech et al. (2009, 2010, 2011, 2012), and the results showed low level of sidelobes, good beam circularity, low cross-coupling level and low cross-polarization over a 20% bandwidth, in agreement with the HornSynth’s predictions.

2.2 Feed Horn Design

As we have explained before, HornSynth comprises two main components, the modal matching analysis and the Genetic Algorithm. Modal matching is used to predict the fields propagating along the feed horn, and to calculate the horn aperture field by calculating the scattering matrix, whilst GA is employed for reliable optimisation.

Modal-matching is a powerful technique that had been used extensively by many authors (e.g. James 1981; Olver and Xiang 1988) to accurately predict the feed horns aperture distribution. In the discussion that follows, we shall briefly summarise the basic principles used in modal matching employing a conical horn as an example. The reader is referred to Kittara et al. (2004) and Olver et al. (1994) for a more detailed description.

In modal matching analysis, the feed horn’s interior is first divided into a series of cylindrical sections, with the tangential field components propagating within each section described as the superposition of waveguide base functions (modes). Using an appropriate coordinate system (i.e., cylindrical coordinates for conical horns), these modes can be obtained by solving Helmholtz equations. Assuming the wave is propagating in the $z$-direction, the solutions are normally found by calculating the $z$ components of the $E$- and $H$-field within the horn that satisfy the wave equations and the boundary condition. For a conical horn, this can be written as

\[ \psi_z(\rho, \phi, z, t) = J_n(k\rho) \cos(n\phi + \phi_0)e^{-\gamma z}e^{j\omega t} \]  

(2.1)

where $\psi_z$ is either the $E_z$ or $H_z$ component, $\rho$ and $\phi$ are the radial distance from the $z$ axis and the azimuth angle respectively, $J_n$ is Bessel function of the 1st kind of order $n$, and $\phi_0$ is a phase constant. The wave number is defined as $k^2 = \gamma^2 + \omega^2 \mu \epsilon$ where $\gamma$ is the propagation constant and $\omega, \mu$ and $\epsilon$ representing the radial frequency, the permeability and electric permittivity, respectively. The remaining transverse
components of the E- and H-field propagation wave can then be expressed in terms of $E_z$ and $H_z$ as

\begin{align}
E_\rho &= -\frac{1}{k^2} \left( \frac{\gamma}{\rho} \frac{\partial}{\partial \rho} E_z + \frac{j \omega \mu}{\rho} \frac{\partial}{\partial \phi} H_z \right), \\
E_\phi &= \frac{1}{k^2} \left( -\frac{\gamma}{\rho} \frac{\partial}{\partial \phi} E_z + j \omega \mu \frac{\partial}{\partial \rho} H_z \right), \\
H_\rho &= \frac{1}{k^2} \left( \frac{j \omega \mu}{\rho} \frac{\partial}{\partial \phi} E_z - \frac{\gamma}{\rho} \frac{\partial}{\partial \rho} H_z \right), \\
H_\phi &= -\frac{1}{k^2} \left( j \omega \mu \frac{\partial}{\partial \rho} E_z + \frac{\gamma}{\rho} \frac{\partial}{\partial \phi} H_z \right).
\end{align}

A conical horn with a perfectly conducting wall supports two orthogonal mode sets, the transverse magnetic $\text{TM}_{nm}$ modes and the transverse electric $\text{TE}_{nm}$ modes, where the subscript $n$ and $m$ are integers representing the cyclic variation with $\phi$ and the $m$th root of the Bessel function of order $n$, respectively. TM modes have only the tangential magnetic fields with $H_z = 0$ while TE modes have only the tangential electric fields with $E_z = 0$. The boundary conditions imposed on the perfect conductor wall require that $E_\phi = 0$ for TM modes, and $E_\phi = E_z = 0$ for TE modes, at $\rho = a$ where $a$ is the radius of the cylindrical section. Applying these boundary conditions to Eqs. 2.2 gives the expressions of the specific transverse modes of TE and TM fields within that particular cylindrical section. The total modal fields at the interface between two adjacent cylindrical sections are matched with boundary conditions that are dictated by the conservation of power. As shown in Fig. 2.2, this interface junction is represented by a scattering matrix, and the power coupling across the junction is related using the scattering matrix defined by

\begin{equation}
\begin{pmatrix}
B \\
D
\end{pmatrix} = \begin{pmatrix}
S_{11} & S_{12} \\
S_{21} & S_{22}
\end{pmatrix} \begin{pmatrix}
A \\
C
\end{pmatrix},
\end{equation}

where $A$ and $B$ are the input and reflected mode coefficient matrices from the input side (left), while $C$ and $D$ are the similar matrices from the output side (right).
The transverse electric fields in each uniform cylindrical section are represented as a spectrum of $N$ nodes. From Fig. 2.2, the electric and magnetic fields on the left hand side are given as

\[
E_L = \sum_{n=1}^{N} [A_n e^{-\gamma_n z} + B_n e^{\gamma_n z}] e_{nL}, \quad (2.4a)
\]

\[
H_L = \sum_{n=1}^{N} [A_n e^{-\gamma_n z} - B_n e^{\gamma_n z}] h_{nL}, \quad (2.4b)
\]

where $A_n$ and $B_n$ are the forward and reflected amplitude coefficients of mode $n$ from the input side, and $e$ and $h$ represent the transverse electric and magnetic modal functions respectively. On the right hand side of the junction, the fields are

\[
E_R = \sum_{m=1}^{M} [C_m e^{-\gamma_m z} + D_m e^{\gamma_m z}] e_{mR}, \quad (2.5a)
\]

\[
H_R = \sum_{m=1}^{M} [C_m e^{-\gamma_m z} - D_m e^{\gamma_m z}] h_{mR}, \quad (2.5b)
\]

where $C_m$ and $D_m$ are the forward and reflected amplitude coefficients of mode $m$ at the output side. Matching these transverse fields across the junction i.e., $E_L = E_R$ and $H_L = H_R$ at boundary $z = 0$, we have

\[
\sum_{n=1}^{N} (A_n + B_n) e_{nL} = \sum_{m=1}^{M} (C_m + D_m) e_{mR}, \quad (2.6a)
\]

\[
\sum_{n=1}^{N} (A_n - B_n) h_{nL} = \sum_{m=1}^{M} (C_m - D_m) h_{mR}. \quad (2.6b)
\]

Multiplying Eqs. 2.6a and 2.6b with $h_R$ and $e_L$ respectively, and integrating over the cross sectional area of the junction, produce a pair of simultaneous matrix equations:

\[
P(A + B) = Q(C + D), \quad (2.7a)
\]

\[
P^T (D - C) = R(A - B), \quad (2.7b)
\]

where the matrix $P$ representing the mutual coupled power between mode $i$ on the left hand side and the mode $j$ on the right hand side. $Q$ and $R$ are the self-coupled power between modes on the right hand and left hand side of the junction. These matrices are given by
\[ P_{ij} = \int_{S_L} (e_{iL} \times h_{jR}) \cdot ds, \quad (2.8a) \]
\[ Q_{jj} = \int_{S_R} (e_{jR} \times h_{jR}) \cdot ds, \quad (2.8b) \]
\[ R_{ii} = \int_{S_L} (e_{iL} \times h_{iL}) \cdot ds. \quad (2.8c) \]

Rearranging Eqs. 2.7 into the matrix form gives the elements in the scattering matrix \( S \) as

\[ S_{11} = (R + P^T Q^{-1} P)^{-1} \times (R - P^T Q^{-1} P) \quad (2.9a) \]
\[ S_{12} = 2 \times (R + P^T Q^{-1} P)^{-1} P^T \quad (2.9b) \]
\[ S_{21} = 2 \times (Q + P R^{-1} P^T)^{-1} P \quad (2.9c) \]
\[ S_{22} = - (Q + P R^{-1} P^T)^{-1} \times (Q - P R^{-1} P^T) \quad (2.9d) \]

All these scattering matrices describing each junction are then cascaded together to form the complete scattering matrix that represents the overall feed horn. The horn aperture field can then be directly computed from the output transmission coefficient of the complete scattering matrix, which is then Fourier-transformed to find the far-field radiation patterns of the horn (Kittara et al. 2004).

### 2.2.1 Genetic Algorithm

Genetic algorithm is an optimisation technique that imitates the natural selection process of biological evolution. It is based on the genetic evolution of a population of a certain size to find the fittest individual i.e., the optimised solution. One advantage of GA is that it is well suited for parallel computation. Another important advantage of GA is that it can simultaneously search a wide sampling parameter space and have the ability to escape local minima.

GA optimisation begins with encoding each key parameter or variable to be optimised into a binary string called a chromosome. These chromosomes are converted to a Gray code to ensure that the numerical value of the chromosomes do not change by a large fractional amount when bits are flipped through mutation and crossover (see below).

A set of chromosomes, containing each parameter, is an individual. An initial population of individuals is formed where the chromosomes of each individual are chosen stochastically within sensible constraints. The fitness of each individual is evaluated via a cost function evaluated via a and the higher fitness half of the population is selected to becomes parents that produce offspring via mating. The lower fitness half of the population is eliminated and replaced by the new offspring individuals. Together, they form the next generation of population.
In the mating stage, the parent’s chromosomes are spliced at random length. Each ends are then swapped between the parents and recombined to produce two new offspring chromosomes. This process is called crossover. The Gray code makes sure that the chromosomes of the offsprings are not too numerically different from their parents, hence preserving the overall fitness quality of the population. This also prevents the algorithm from suddenly straying away from a converging solution.

After crossover, a single bit of the offspring chromosome is flipped with a small probability, mimicking the mutation process in the genetic evolution. A combination of crossover and mutation ensures the diversity of the offspring, and it is these processes that give the searching algorithm the ability to explore a wider parameter space. After a new population is formed, the fitness of each offspring is re-evaluated and the mating process is repeated for a few generations. In general, the average fitness of the population increases from one generation to the other. The optimisation process is stopped after it reaches a user specified limit of the generation (typically 1,000–2,000 generations). The fittest individual is then further optimised using the Simplex minimisation (Nelder and Mead 1965) in the HornSynth case.

2.2.2 700 GHz Multiple Flare-Angle Horn

For the operation at the 700 GHz waveband, we have chosen a multiple flare-angle smooth-walled horn with 3 flared sections. The 5 parameters to be optimised are $R_1$, $R_2$, $L_1$, $L_2$ and $L_3$ (see Fig. 2.1). $R_0$ is set by the cutoff frequency of the input circular waveguide, and the horn aperture $R_3$ is determined by the required beam width of the feed horn, which in this case is about 14.6° Full-Width Half-Maximum (FWHM). The initial chromosomes in each individual in the first generation are randomly assigned within a range that ensure a long phasing section, and the flare discontinuities are located nearer to the throat of the horn. The rationale behind this is that the resulting feed horn design is not too dissimilar with the Pickett-Potter horn to retain its general good features.

The cost function which measures the fitness of each individual horn is evaluated from the far-field beam pattern calculated using the modal matching technique (Yassin et al. 2007). At a specific frequency $f$, the cost function is given by

$$\delta_f^2 = w_X \left[ \sum_{P=-30}^{P=30} \left( \frac{\sigma_P}{\sigma_P^{av}} \right)^2 w_P \right],$$

(2.10)

where $w_X$ is the peak cross-polar power relative to main beam peak power, $P$ is the power response in dB, $\sigma_P$ and $\sigma_P^{av}$ are the difference between the E- and the H-plane beam widths and the average beam width of both planes at the power response $P$ dB, and $w_P = 10^{P/15}$ is the weighting function for the beam circularity (Kittara et al. 2007). This cost function is evaluated at several frequencies across the entire bandwidth. The final cost function for a particular horn design centred at $f_0$ is calculated by
Table 2.1 Geometrical parameters for the 3-section multiple flare-angle horn design

<table>
<thead>
<tr>
<th>Centre frequency</th>
<th>Parameter</th>
<th>( R_0 )</th>
<th>( R_1 )</th>
<th>( R_2 )</th>
<th>( R_3 )</th>
<th>( L_1 )</th>
<th>( L_2 )</th>
<th>( L_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>700 GHz</td>
<td>Length (mm)</td>
<td>0.203</td>
<td>0.488</td>
<td>0.595</td>
<td>1.200</td>
<td>0.487</td>
<td>0.398</td>
<td>7.886</td>
</tr>
</tbody>
</table>

\[
\delta^2 = \sum_f \delta_f^2 \exp\left(-\frac{(f - f_0)^2}{2\sigma_f^2}\right),
\]  

(2.11)

where the exponential term is the frequency dependent weighting factor, and \( \sigma_f \) is the bandwidth \((f_{\text{upper}} - f_{\text{lower}})\).

These cost function was chosen to specify the fitness criterion for high beam circularity and low cross-polarization level. The combination of these two criteria produced a horn that exhibits low sidelobes level and since Pickett-Potter type horns generally have low return loss (Pickett et al. 1984), this requirement was not included in the cost function. Other criteria can clearly be chosen, depending on the horn applications.

The most computationally intensive step in the entire design procedure is the beam pattern calculation via the modal matching for each individual in the population across a wide bandwidth. Since GA optimisation is well suited for parallelisation, the beam patterns and the cost function of each individual within a certain generation can be distributed to separate central processing units (CPUs) as a slave tasks for simultaneous calculations. The parallel computing technique vastly reduces the time required to optimised a multiple flare-angle horn. For example, using a 28 cores machine, we managed to optimise the 3-section horn within 1,000 generation of 27 individuals, taking only about 3 hours of computing time.

Table 2.1 shows the result from the GA optimisation for a 3-section multiple flare-angle horn centred at 700 GHz. As shown in Fig. 2.3, the predicted beam patterns have high circularity and low sidelobes levels across \( \sim 140 \) GHz bandwidth.

Fig. 2.3 Theoretical E-, H-plane and cross-polarization beam pattern calculated using modal matching.
2.3 Horn Fabrication

The major advantage of the multiple flare-angle horn is that its simple interior profile lends itself to easy and fast fabrication process. These horns can be made via the simple direct milling technique using a standard 5-axis computer-numerical-control (CNC) machine. Figure 2.4a shows a high-speed steel machine tool manufactured (by a standard machine tool company) to have its cutting edge shaped according to the horn profile. This tool is used to mill the desired feed horn directly into a block of aluminium. The circular waveguide that feeds the horn is formed by simply milling through the apex of the feed horn profile from the other end of the aluminium plate. Figure 2.4b shows an example of the fabricated horn made using this technique.

In order to demonstrate that the direct milling method is repeatable, the sharpness of the discontinuities of the shaped cutting tool was inspected after using it to fabricate several horns. As shown in Fig. 2.5a, the discontinuities of the machine tool remained reasonably sharp. The apparent damage of the cutting edge away from the first discontinuity (enlarged in Fig. 2.5b) is mainly caused by the aluminium waste not being properly cleared out of the milling area, hence adhering to the cutting tool.

Using the direct milling method described above, a large number of horns can be fabricated very rapidly. Once the machine tool and the working metal plate are properly aligned within the 5-axis CNC machine, the feed horns can be fabricated

Fig. 2.4 a The high speed steel machine tool used to fabricate the multiple flare-angle horns. b A prototype of 700 GHz smooth-walled horn made by direct milling into an aluminium block

Fig. 2.5 Sharpness of the discontinuities of the machine bit examined under a microscope
simply by repeating the milling process along the programmed coordinates, forming a horn array. Leech et al. (2012) have successfully fabricated and measured high quality beam patterns across a 37 horn array operating at 230 GHz, demonstrating that this fabrication technique is feasible for constructing large format focal plane arrays.

### 2.4 Experimental Setup

Our far-field test facility consisted of a rotary table, a local oscillator (LO), a lock-in amplifier, a data logging system, and 4 K cryogenic bolometric detector, all housed in an anechoic chamber. The anechoic chamber walls were lined with high loss microwave absorbers (Eccosorb® AN-72 from Emersion & Cuming Microwave Products), to reduce electromagnetic interference and reflection.

Figure 2.6 shows a schematic diagram of the test range, without the surrounding Eccosorb®. The horn under test was used as a transmitter, fed by the LO via a circular-to-rectangular waveguide transition. The whole arrangement was aligned to the cooled receiver horn feeding the bolometer inside the cryostat. This assembly was affixed firmly onto a rotary table, with the aperture of the horn aligned parallel to the rotating axis of the rotary table. Extra Eccosorb® RF absorbers were placed at key positions around the experimental setup to eliminate stray power pickup and reduce the effect of standing waves.

The separation distance between the horn under test and the window of the cryogenic Dewar housing the detector is set to be further than \( \frac{D^2}{\lambda} \) to ensure that the

![Fig. 2.6 Schematic diagram showing the setup of the far-field beam pattern test range](image-url)
horn is in the far-field region. The radiation patterns were measured by logging the signal received from the lock-in amplifier while rotating the transmitter. This method gave a high dynamic range of \(\sim 60\) dB.

For measuring co-polar and cross-polarization patterns, a terahertz polarizer grid was placed in front of the detector cryostat window, with the polarization axis tilted at 45° to the plane of rotation (Ludwig 1973). The input polarization of the horn under test was then aligned parallel to the grid polarization axis for measuring a co-polar pattern, or perpendicular to the grid axis for measuring a cross-polar pattern.

2.5 Measurement Results and Analyses

We had fabricated six multiple flare-angle horns in two fabrication runs, with three prototypes at each run. At 700 GHz, the tolerance required to achieve good performance is stringent since an error similar to that of the fabrication for a 230 GHz horn would be a significant fraction of a wavelength at this higher frequency. The cutting tool at this frequency is also much smaller, with a maximum radius of 1.2 mm.

Along with the feed horns, we had also fabricated a 13° semi-flare angle conical circular-to-rectangular waveguide transition as a separate piece to fit the rectangular output waveguide of the 700 GHz LO. This transition was made by direct milling as well, with a 13° conical cutting tool milled into the aperture of the rectangular waveguide formed in a split-block beforehand. A standard UGC 387/U (modified) flanges was then machined to provide an interface for attaching the transition to the horn and the LO source.

Figure 2.7 shows the beam patterns measured for one of the best prototype horn (Horn No. 1) among the three fabricated in the first run. The results were reasonably

![Figure 2.7](image)

**Fig. 2.7** Beam pattern measurements of the first generation smooth-walled horn No. 1 at the central frequency 700 GHz. The measured beam patterns match reasonably well with theory, given the tight tolerances required at this wavelength. Grid No. 1 and grid No. 2 represent two different polarizers used during the measurements for comparison purpose. The beam patterns measured at various other frequencies across the wavebands are presented in Appendix C.
Fig. 2.8 Measurements of co-polar and cross-polar pattern for the first generation of horns at 660 GHz, compared with modal matching predictions. The solid curves are the measured data while the dotted curves are computed patterns. The black curves represent the co-polar patterns, while the red curves correspond to the cross-polar patterns.

Fig. 2.9 Microscope photos show the lateral misalignment between the input circular waveguide and the horn aperture of the first generation horns. a Horn 1. b Horn 2. c Horn 3

good, but there were some notable differences between the calculated E- and H-plane and the measured beam patterns. The sidelobe positions were almost coincide with the theoretical predictions and the sidelobe level was only a few dB higher. The measured cross-polarization was higher than expected though, with a slight asymmetry in all the measured beam patterns. It can however be clearly seen that the pattern quality is quite acceptable at this frequency.

Figure 2.8 shows the measured co-polar and cross-polar beam pattern of the three prototypes at 660 GHz. There was a clear asymmetry in the main beam of the co-polarization patterns for 2 out of the 3 prototypes. The measured cross-polar level was significantly higher than that predicted by modal matching. We believe that these effects were caused by a lateral translational offset between the input circular waveguide and the horn section.

We have inspected the interior of the horns under microscope to look for any obvious lateral misalignment, and as shown in Fig. 2.9, there was evidence that the input circular waveguide was indeed offset with respect to the horn. This showed up as asymmetry in the circularity of the first dark rim around the first smooth-walled section of the horn profile.
With the experience gained from the first fabrication run, a new batch of the horns were fabricated to minimise the translational offset error of the input waveguide. We realised that the major contributor to this translational offset is the inability to accurately place the cylinder aluminium block back to its original position after removing it for the manufacturing of the circular waveguide. Hence, in the new batch of feed horns, we used a longer aluminium cylinder block to ensure that the CNC machine clamp can hold it firmly in place. The circular waveguide was then milled along with all the holes of the flanges to ensure that the circular waveguide was aligned to the centre of the flange with minimum offset.

Figure 2.10 shows that the asymmetry in the measured beam patterns for all three horns was substantially reduced now, except for prototype No. 1. The cross-polarization level for all three horns was also lowered by several dB compared to the first batch of prototypes. Figure 2.11 shows the top-down view into the interior of all the three new horns. As can be seen, the first dark rim is now much more circular.

Figure 2.12 shows the E-, H-, co-polar and cross-polar radiation patterns measured for Horn No. 3 at the design central frequency of 700 GHz, and at the edges of the LO
Fig. 2.12 Beam pattern measurements of horn No. 3 from the second generation smooth-walled horn in the 700 GHz frequency band. The solid lines are the measured beam patterns, while the dash lines are the calculated patterns using modal matching. In the right column, the red colour lines represent the cross-polar patterns, and the black lines are the co-polar patterns. a 600 GHz. b 700 GHz. c 740 GHz.

It can clearly be seen that the measured first sidelobe level is below –30 dB at the central frequency. The main beam has excellent beam circularity down to the –30 dB frequency range 600–740 GHz. The measured patterns at other frequencies across the band are shown in the Appendix C.
level and is symmetrical and clear of standing wave ripples. The co-polar patterns agreed very well with the computed prediction and the peak cross-polarization level was below −22 dB, limited by the polarizer’s efficiency to filter out non-polarized component (typically about 99% at high frequencies). This limitation can be seen by noting that the level of cross-polarization remains approximately fixed across a bandwidth of 140 GHz. In particular, at the 600 GHz plot, the cross-polar level remains approximately at −20 dB, showing no trend of worsening performance as predicted by theory.

2.6 Study of Multiple Flare-Angle Horn Using Ansys HFSS

Most horn design software packages, including *HornSynth*, can only model horns with axial symmetry. They are particularly useful in predicting the behaviour of a certain feed horn design with circular or rectangular symmetry, but cannot be employed to incorporate non-axisymmetric features. In this section, we describe the use of HFSS to model the feed horn in 3-D, including the non-axisymmetric errors.

3-D electromagnetic simulators are not commonly employed for predicting the performance of horns because the electrically-large dimension of the model (∼20λ) often presents a huge challenge for the computational memory requirement, particularly for corrugated horns with large numbers of corrugations per wavelength. Multiple flare-angle horn however, consist of only a few flare-angle discontinuities and long phasing sections, hence the volume of meshing needed to accurately describe the surface of the horn model is greatly reduced. This relaxes the heavy requirement on computer memory, and makes possible the use of HFSS to study critical features of our horn design, such as tolerances on fabrication errors and cross coupling between adjacent horns.

2.6.1 Comparing Simulation Results with Modal Matching

Constructing a 3-D model of a multiple flare-angle horn using HFSS is rather straightforward. First, the feed horn model is set to be aluminium or a perfect conductor, within a free space environment (boundary at least λ/4 away from the feed horn). This free space structure is assigned with the *Radiation Boundary Condition*, which presents an infinite boundary by presenting a perfect match to the electromagnetic field at the boundary. An E-field polarized wave port is then launched at the base of the input waveguide, shown as a small circle at the bottom of Fig. 2.13 with a coordinate triad. An infinite far-field radiation sphere is setup by assigning the origin of this *infinite sphere* at the centre of the horn aperture plane.

For generating the E- and H-plane far-field radiation patterns, we plot the *total gain* at the x-plane (φ = 0°) and the y-plane (φ = 90°), assuming the polarized
E-vector is aligned to the x-axis). The co-polar and cross-polar measurements are obtained from the gain X and gain Y at $\phi = 45^\circ$, respectively.

In some cases, to further reduce the computational loading and time, we could take advantage of the symmetry of the E- and H-field configuration of the horn (hereinafter simplified model). This is done by drawing only a quarter portion of the original model, including the free space environment, and assigning a perfect E-boundary condition at the x-plane, and a perfect H-boundary condition at the y-plane, as shown in Fig. 2.13b. This virtually creates a mirror image of the model on the other side of the x- or y- symmetric plane, hence resembling a full feed horn model.

As shown in Fig. 2.14, the far-field patterns simulated by HFSS using both simplified and full model match very well the results produced using modal matching. The sidelobe positions were almost identical, but at the power level below $-30$ dB, results from HFSS often show slightly lower power level and slight asymmetry. This is because the HFSS finite tetrahedron meshing imitating the horn model might not necessarily be entirely axial symmetric (see Fig. 2.13b).

Note that in the cross-polarization plots, the matching with modal matching was remarkably similar at the edge of the waveband. However, there were significantly different in the central waveband where the cross-polar level is substantially lower.
Fig. 2.14 Far-field patterns of a 3-section smooth-walled horn across the 230 GHz design wavebands, generated by HornSynth, a full-horn HFSS model and a simplified quarter-wedged horn HFSS model. a E- and H-plane. b Co- and cross-polarization
This again, shows that the HFSS has the limitation in accuracy for prediction of power levels below $-40$ dB, hence care should be taken when interpreting predictions at lower power levels.

Increasing the number of modes in the simulated wave port improves the accuracy of the HFSS model. However, this also increases the meshing frequency, hence effectively enlarges the electrical-size of the model. Figure 2.15 shows the comparison of simulating a 3-section smooth-walled horn with 2, 8 and 20 modes in the exciting port. As can be seen, the results in general were reasonable even when only using the $\text{TE}_{11}$ and $\text{TM}_{11}$ mode in the simulation. The main lobes match each other very well, but increasing the number of modes does help to improve the accuracy of the sidelobes. Nevertheless, increasing the number of modes more than 8 does not seem to show much improvement.

### 2.6.2 Tolerance Analysis

The use of HFSS to study the performance of a multiple flare-angle horn was largely motivated by the need for tolerance analysis to investigate the effect of lateral misalignment between the input waveguide and the horn. In Fig. 2.16, we have simulated this effect by setting the circular waveguide to be offset by 50–100 $\mu$m, and the simulated results were able to reproduce the asymmetry and the magnitude of the cross-polar patterns of the measured pattern with these settings. The smoothing of the dip in the cross-polarization pattern is also predicted correctly, especially in the case of Horn No. 2 from the first fabrication batch, where the offset is more severe. These results, together with the inspection of the horn interior, further confirmed that the lateral misalignment is the cause of the asymmetry and high cross-polar level measured previously in the first generation of the prototype horns.
2.6 Study of Multiple Flare-Angle Horn Using Ansys HFSS

Figure 2.16 Measurements of co- and cross-polarization for all three first generation prototype horns, comparing with HFSS simulation. The lateral misalignment used in the HFSS model are labelled on the top right of the panels, with E and H indicating the plane where the misalignment shifted, and the number indicating the amount of shifting. The solid curves are the measured data while the dotted curves are the computed patterns. The black curves represent the co-polarization patterns, while the red curves correspond to the cross-polarization patterns.

2.6.3 Cross-Coupling Between Two Adjacent Horns

Cross-coupling between two adjacent horns within a focal plane array can greatly degrade the quality of astronomical data. In some cases such as CMB cosmology instruments, the level of cross-coupling must be well below –40 dB. In principle, the expected cross-coupling level can be estimated analytically (Olver et al. 1994) if the aperture fields of both feeds were known. However, this is complicated in general, as the aperture field of the horn is normally hard to describe accurately analytically, but this calculation can be easily done numerically using HFSS, utilising its full 3-D capability. Here, the cross-coupling level is defined as the ratio of the power received by one horn to the input power injected into the input of the adjacent one.

Figure 2.17 shows the cross-coupling of power between two adjacent horns, with 0.7 mm separation between the aperture edges of the two horns. HFSS predicts that the cross-coupling level is less than –50 dB across the required frequency band. The far-field pattern of the horn is not at all affected by this low level of coupling.

The simulated results were compared with the measured cross-coupling level between a two-horn array designed to operate from 210–250 GHz. Figure 2.18 shows the two adjacent horns that were milled directly into a single aluminium block with a 0.7 mm separation between the nearest edges of two horn apertures, a distance appropriate for a realistic packed array.

The cross-coupling measurement was performed using the ABmm® Vector Network Analyser (VNA) at Rutherford Appleton Laboratory, by measuring the power coupled between the transmitter and the receiver ports (see Fig. 2.18a). A carbon loaded epoxy cone was placed on top of the two-horn array to prevent the RF signal being reflected back to the receiver horn. The power measured by the VNA was then
Fig. 2.17 The HFSS predicted far-field patterns of the radiating horn in the two-horn array, compared with the same patterns of a single horn. The panel on the right shows the comparison of HFSS predicted cross-coupling with the measured data. The red triangle indicates the measured response, while the black thick line represents the result predicted by HFSS.

Fig. 2.18 a Schematic diagram showing the setup of Abmm® VNA to measure the cross-coupling between two horns. b Photo showing the two-horn array prototype.

simply the cross-coupled RF power from the transmitter horn to the receiver horn caused by the fringing of the aperture fields.

Figure 2.17c, shows that the measured cross-coupling level agreed with the level predicted by HFSS, and was lower than –67 dB across the operating bandwidth. The measured far-field beam patterns of both horns showed high similarity as well (Leech et al. 2012), with the E- and H-plane measurements showing no deviation from the computed beam patterns of the individual horns. The very low level of cross coupling...
between the two adjacent horn supported by the agreement with computed results demonstrates that smooth-walled horns fabricated by direct milling are suitable for employment in close packed focal plane arrays.

2.7 Summary

In this chapter, we have presented the design of a high performance multiple flare-angle horn that have radiation pattern similar to that of the conventional corrugated horn, but is much simpler to fabricate. The horn is suitable for producing a large number of feed horn for a closely packed focal plane arrays.

We have demonstrated that the multiple flare-angle design is feasible at frequencies as high as 700 GHz. The measured radiation patterns showed good performance and agreed well with modal matching across a bandwidth of 140 GHz, covering the entire astronomical window centred at 650 GHz. These results also demonstrate that the fabrication technology described above is effective in the sub-mm wavelength range.

We have also used HFSS to estimate and correctly predict the tolerances in fabrication that give rise to the asymmetry in the cross-polar patterns. This is important since conventional software packages cannot simulate non-axisymmetric errors. HFSS is also useful for estimating the cross-coupling level between two adjacent horns. We have measured the cross-coupling between two smooth-walled horns in an array and the results show a very low level of power coupling between the two horns, and good agreement with HFSS prediction.

References


Development of Coherent Detector Technologies for Sub-Millimetre Wave Astronomy Observations
Tan, B.-K.
2016, XXIV, 219 p., Hardcover
ISBN: 978-3-319-19362-5