Chapter 2
Main Equations and Relations of Magnetoelasticity of Thin Plates and Shells

The hypotheses of magnetoelasticity of thin bodies is addressed in this section. On its basis, using the results from the previous section, two-dimensional equations and appropriate conditions are obtained that describe the behavior of conducting plates and shells in a magnetic field.

2.1 Two-Dimensional Equations of Magnetoelasticity of Thin Conducting Plates

2.1.1 Finitely Conducting Plates

Let us consider an isotropic elastic plate of constant thickness $2h$. The plate is made of a material with the finite conductivity σ and placed in an external nonstationary magnetic field $\mathbf{H_0}$. The plate is immersed in the Cartesian coordinate system x_1, x_2, x_3 (Fig. 2.1), and the middle plane of nondeformed plate coincides with the coordinate plane x_1, x_2. Let the problem of magnetostatics for a ondeformed body [see Eqs. (1.6.7)–(1.6.9)] be solve, i.e., the vectors of magnetic field intensity for external $\mathbf{H}_\mathbf{0}^{(\mathbf{e})}\left(H_{01}^{(e)}, H_{02}^{(e)}, H_{03}^{(e)}\right)$ and internal $\mathbf{H_0}(H_{01}, H_{02}, H_{03})$ areas are known. Let us assume that products of the type $e_{0i}h_j, e_ih_{0j}$ and $h_{0j}u_i$ are quantities of second order smallness and can be neglected in the corresponding combinations.

In the Cartesian coordinate system covariant and contravariant components of vectors and tensors, characterizing the magnetoelastic state of the plate, coincide with each other; covariant derivatives also coincide with the ordinary derivatives because in this case the metric tensor is a constant one; hence, the Christoffel symbols are equal to zero.

Taking into account the above-noted facts from relation (1.6.15), one can obtain that within the plate perturbations of magnetoelastic quantities are satisfied the following equations:

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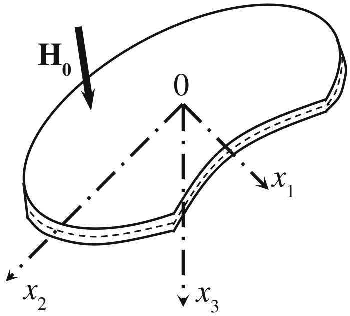

Fig. 2.1 Geometrical interpretation of the problem. Plate in a magnetic field

$$\frac{\partial}{\partial x_i}\left(s_{ik} + s_H^{im}\frac{\partial u_k}{\partial x_m}\right) + \frac{\sigma}{c}\left[\left(\mathbf{e} + \frac{1}{c}\frac{\partial \mathbf{u}}{\partial t} \times \mathbf{H_0}\right) \times \mathbf{H_0}\right]_k = \rho\frac{\partial^2 u_k}{\partial t^2}, \tag{2.1.1}$$

$$\varepsilon_{ijk}\frac{\partial h_j}{\partial x_i} = \frac{4\pi\sigma}{c}\left(\mathbf{e} + \frac{1}{c}\frac{\partial \mathbf{u}}{\partial t} \times \mathbf{H_0}\right)_k, \tag{2.1.2}$$

$$\begin{gathered}\varepsilon_{ijk}\frac{\partial e_j}{\partial x_i} = -\frac{1}{c}\frac{\partial h_k}{\partial t}, \quad \frac{\partial h_k}{\partial x_k} = 0,\\ s_{ij} = \frac{E}{1+\nu}\left(\varepsilon_{ij} + \frac{\nu}{1-2\nu}\varepsilon_{kk}\delta_i^j\right),\\ \varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right).\end{gathered} \tag{2.1.3}$$

Herein

$$E = G\frac{3\lambda + 2G}{\lambda + G}, \quad \nu = \frac{\lambda}{2(\lambda + G)} \tag{2.1.4}$$

are elasticity modulus and Poisson's ratio, respectively, and the stresses of the nondeformed state s_H^{im} are determined having solved the problem (1.6.10)–(1.6.13) for the appropriate initial conditions and conditions at infinity.

Issues of exact solution of three-dimensional equations of magnetoelasticity (2.1.1) and (2.1.2) for bodies with finite dimensions is associated with almost insurmountable mathematical difficulties. That is why great interest has formed around the development of approximate methods of solution of three-dimensional equations of magnetoelasticity for thin bodies such as shells and plates. The solution of problem of magnetoelasticity is not relieved essentially even if Kirchhoff hypothesis on nondeformable normal with respect to the mechanical quantities are used.

In such formulation, only a number of particular problems are able to be solved [5, 19, 31, 32]. The asymptotic method was more acceptable for these problems because it uses the thinness of the examined elastic body, i.e., plates or shells. In the work [5], the asymptotic method of integration of three-dimensional equations of magnetoelasticity has been reported in detail for thin plates and shells of finite conductivity. Some general regularities of change of certain magnetoelastic quantities along the thickness of a thin body were explored. As a result of analylyzing these solutions and determining exact solutions to some particular problems of magnetoelasticity of plates and shells [5, 19, 32], a hypothesis of the magnetoelasticity of these bodies was proposed in [5] with respect to the character of the change of some components of the electromagnetic field as well as elastic displacements along the thickness of the plate or shell.

This hypotheses is addressed in the following way: (1) the normal rectilinear element to the middle plane of the plate stays rectilinear and normal to the deformed middle plane of the plate and keeps its length; and (2) the tangential components of the intensity vector induced electromagnetic field in the plate's electric field, as well as the normal component of the intensity vector induced in the plate's magnetic field, remain unchanged along the thickness of the plate.

Due to the first assumption, it is also accepted that in the equation of the generalized Hooke law, the quantity s_{33} can be neglected compared with the quantities s_{11} and s_{22}. The accepted assumption has the following form:

$$u_1 = u - x_3 \frac{\partial w}{\partial x_1}, \quad u_2 = v - x_3 \frac{\partial w}{\partial x_2}, \quad u_3 = w(x_1, x_2, t); \tag{2.1.5}$$

$$e_1 = \varphi(x_1, x_2, t), \quad e_2 = \psi(x_1, x_2, t), \quad h_3 = f(x_1, x_2, t), \tag{2.1.6}$$

where $u(x_1, x_2, t), v(x_1, x_2, t), w(x_1, x_2, t)$ are the displacements of points of the middle plane of the plate, and φ, ψ, f are unknown functions of the induced magnetic field in the plate's electromagnetic field.

The range of acceptance of the hypotheses of magnetoelasticity of thin bodies was discussed in works [5, 32, 109–111]. It was shown that the error of the noted hypotheses is estimated having neglected the quantity $\left|k^2 + 4\pi\sigma\omega c^{-2}\right| h^2$ compared with the unit (here k is the wave number, and ω is the complex frequency of magnetoelastic vibrations).

The electrodynamic part (2.1.6) of the hypothesis of magnetoelasticity of thin bodies was obtained by performing asymptotic integration with respect to equation

$$\operatorname{rot} \mathbf{e} = -\frac{1}{c}\frac{\partial \mathbf{h}}{\partial t}, \quad \operatorname{div} \mathbf{h} = 0 \tag{2.1.7}$$

in the area $G\,(|x_3| \le h, (x_1, x_2) \in Q)$ occupying the plate [5]. These equations take place not only in the internal area G but, according to Eq. (1.6.16), at the whole infinite strip $|x_3| < h$. Hence, asymptotic integration of Eq. (2.1.7) in the area $|x_3| < h$, taking into account the continuity of e_1, e_2 and h_3 at the lateral surface of the plate [the condition (2.1.18)], brings us to the same result as in the case of the area G. Thus, the relation (2.1.6), up to the third approximation of the asymptotic integration, take place at the whole infinite strip $|x_3| < h$.

In the work [10], the generalized hypotheses of magnetoelasticity of thin plates are proposed according to which relation (2.1.6) take place at the whole infinite strip $|x_3| < h$, i.e., together with Eqs. (2.1.5) and (2.1.6), and the following relations are accepted [10]:

$$\left.\begin{aligned} u_1 &= u - x_3 \tfrac{\partial w}{\partial x_1}, \\ u_2 &= v - x_3 \tfrac{\partial w}{\partial x_2}, \\ u_3 &= w(x_1, x_2, t) \end{aligned}\right\} \quad \text{for} \quad |x_3| < h, (x_1, x_2) \in Q \tag{2.1.8}$$

and

$$\left.\begin{aligned} e_1 &= \varphi(x_1, x_2, t), \\ e_2 &= \psi(x_1, x_2, t), \\ h_3 &= f(x_1, x_2, t) \end{aligned}\right\} \quad \text{for} \quad |x_3| < h, -\infty < x_1, x_2 < \infty. \tag{2.1.9}$$

Using (2.1.8) and the accepted assumptions from Hooke's law (2.1.3), we have the following expression for s_{11}, s_{12} and s_{22} [3, 60, 94]:

$$\begin{aligned} s_{11} &= \frac{E}{1-\nu^2}\left[\frac{\partial u}{\partial x_1} + \nu\frac{\partial v}{\partial x_2} - x_3\left(\frac{\partial^2 w}{\partial x_1^2} + \nu\frac{\partial^2 w}{\partial x_2^2}\right)\right], \\ s_{22} &= \frac{E}{1-\nu^2}\left[\frac{\partial v}{\partial x_2} + \nu\frac{\partial u}{\partial x_1} - x_3\left(\frac{\partial^2 w}{\partial x_2^2} + \nu\frac{\partial^2 w}{\partial x_1^2}\right)\right], \\ s_{12} &= \frac{E}{2(1+\nu)}\left[\frac{\partial u}{\partial x_2} + \frac{\partial v}{\partial x_1} - 2x_3\frac{\partial^2 w}{\partial x_1 \partial x_2}\right]. \end{aligned} \tag{2.1.10}$$

Substituting (2.1.8) and (2.1.9) into Eq. (2.1.2), the following equation is obtained to define the components h_1, h_2 and e_3 of the induced electromagnetic field:

$$\begin{aligned} \frac{\partial h_1}{\partial x_3} &= \frac{\partial f}{\partial x_1} + \frac{4\pi\sigma}{c}\psi + \frac{4\pi\bar{\sigma}}{c^2}\left[H_{01}\frac{\partial w}{\partial t} - H_{03}\left(\frac{\partial u}{\partial t} - x_3\frac{\partial^2 w}{\partial x_1 \partial t}\right)\right], \\ \frac{\partial h_2}{\partial x_3} &= \frac{\partial f}{\partial x_2} - \frac{4\pi\sigma}{c}\varphi + \frac{4\pi\bar{\sigma}}{c^2}\left[H_{02}\frac{\partial w}{\partial t} - H_{03}\left(\frac{\partial v}{\partial t} - x_3\frac{\partial^2 w}{\partial x_2 \partial t}\right)\right], \\ \frac{4\pi\bar{\sigma}}{c}e_3 &= \frac{\partial h_2}{\partial x_1} - \frac{\partial h_1}{\partial x_2} - \frac{4\pi\bar{\sigma}}{c^2}\left[H_{02}\frac{\partial u}{\partial t} - H_{01}\frac{\partial v}{\partial t} - x_3\frac{\partial}{\partial t}\left(H_{02}\frac{\partial w}{\partial x_1} - H_{01}\frac{\partial w}{\partial x_2}\right)\right]. \end{aligned} \tag{2.1.11}$$

Having integrated the first two equations of system (2.1.11) with respect to x_3, and taking into account the continuity of h_1 and h_2 at the surfaces $x_3 = \pm h$, we obtain

$$\begin{aligned} h_1 &= \frac{h_1^+ + h_1^-}{2} + x_3\left(\frac{\partial f}{\partial x_1} + \frac{4\pi\bar{\sigma}}{c}\psi\right) \\ &\quad + \frac{4\pi\bar{\sigma}}{c^2}\left[a_1\frac{\partial w}{\partial t} + a_3\frac{\partial u}{\partial t} + d_3\frac{\partial^2 w}{\partial x_1 \partial t}\right], \\ h_2 &= \frac{h_2^+ + h_2^-}{2} + x_3\left(\frac{\partial f}{\partial x_2} - \frac{4\pi\bar{\sigma}}{c}\varphi\right) \\ &\quad + \frac{4\pi\bar{\sigma}}{c^2}\left[a_2\frac{\partial w}{\partial t} - a_3\frac{\partial v}{\partial t} + d_3\frac{\partial^2 w}{\partial x_2 \partial t}\right]. \end{aligned} \tag{2.1.12}$$

where

$$\bar{\sigma} = \begin{cases} \sigma & \text{for} \quad (x_1, x_2) \in Q \\ 0 & \text{for} \quad (x_1, x_2) \notin Q. \end{cases}$$

Herein, and furthermore by way of the indexes "+" and "−", the corresponding quantities at $x_3 = h$ and $x_3 = -h$ are denoted

$$a_i = A(H_{0i}), \quad d_i = A(x_3 H_{0i}), \quad i = 1, 2, 3,$$

Moreover, an integral operator $A(\alpha)$ has the form

$$A(\alpha) = \int_0^{x_3} \alpha \mathrm{d}x_3 - \frac{1}{2}\left(\int_0^{h} \alpha \mathrm{d}x_3 + \int_0^{-h} \alpha \mathrm{d}x_3\right).$$

Having performed the above-mentioned operations with respect to Eq. (2.1.11), besides the expression (2.1.12), the following differential equations, with respect to the unknown functions u, v, w, φ, ψ, and f are obtained

$$\begin{aligned} &\frac{\partial \psi}{\partial x_1} - \frac{\partial \varphi}{\partial x_2} + \frac{1}{c}\frac{\partial f}{\partial t} = 0, \\ &\frac{\partial f}{\partial x_1} + \frac{4\pi\bar{\sigma}}{c}\left[\psi + \frac{1}{2hc}\left(b_1\frac{\partial w}{\partial t} - b_3\frac{\partial u}{\partial t} + c_3\frac{\partial^2 w}{\partial x_1 \partial t}\right)\right] = \frac{h_1^+ - h_1^-}{2h}, \\ &\frac{\partial f}{\partial x_2} - \frac{4\pi\bar{\sigma}}{c}\left[\varphi - \frac{1}{2hc}\left(b_2\frac{\partial w}{\partial t} - b_3\frac{\partial v}{\partial t} + c_3\frac{\partial^2 w}{\partial x_2 \partial t}\right)\right] = \frac{h_2^+ - h_2^-}{2h}, \end{aligned} \tag{2.1.13}$$

where the following notations are performed:

$$b_i = \int_{-h}^{h} H_{0i} \mathrm{d}x_3, \quad c_i = \int_{-h}^{h} x_3 H_{0i} \mathrm{d}x_3.$$

To obtain the normal component of electric field e_3 condition (1.6.19) is used

$$\left(\mathbf{e} + \frac{1}{c}\frac{\partial \mathbf{u}}{\partial t} \times \mathbf{H_0}\right) \cdot \mathbf{N_0} = 0, \tag{2.1.14}$$

which defines the sign of the quantity e_3 at the surfaces $x_3 = \pm h$ and expresses by way of the displacements of the points of plate's middle plane:

$$e_3^{\pm} = \frac{1}{c}\left[H_{01}^{\pm}\frac{\partial v}{\partial t} - H_{02}^{\pm}\frac{\partial u}{\partial t} \pm h \frac{\partial}{\partial t}\left(H_{02}^{\pm}\frac{\partial w}{\partial x_1} - H_{01}^{\pm}\frac{\partial w}{\partial x_2}\right)\right]. \tag{2.1.15}$$

The condition (2.1.14) and, hence, the representation (2.1.15) in general is true with the accuracy of neglecting of surface charges.

Substituting (2.1.12) into the third equation of system (2.1.11) and taking into account (2.1.15), we find

$$\begin{aligned} e_3 = {} & -x_3\left(\frac{\partial \varphi}{\partial x_1} + \frac{\partial \psi}{\partial x_2}\right) \\ & + \frac{1}{c}\frac{\partial}{\partial t}\left[H_{01}v - H_{02}u + \left(a_2 - \frac{\partial d_3}{\partial x_2} + x_3 H_{02}\right)\frac{\partial w}{\partial x_1}\right. \\ & \left. - \left(a_1 - \frac{\partial d_3}{\partial x_1} + x_3 H_{01}\right)\frac{\partial w}{\partial x_2}\right]. \end{aligned} \tag{2.1.16}$$

Like the usual theory elastic stability of thin plates, here we will also assume that elongation and shear deformations are small enough compared with the corresponding angle of rotation $2\omega = \operatorname{rot} \mathbf{u}$ and these last quantities are small enough and can be neglected compared with the unit. In addition, all quantities characterizing the influence of rotation ω_3 around the axis x_3 will be neglected. Accordingly, from the first two equations of system (2.1.1) on the basis of (2.1.8) and (2.1.10), we can calculate the stresses s_{13} and s_{23}. Satisfying the appropriate conditions from (1.6.17) at the surfaces $x_3 = \pm h$, let us calculate the functions of integration included in the expressions for s_{13} and s_{23}. As a result, the following expressions for the stresses s_{13} and s_{23} are

$$
\begin{aligned}
s_{13} = {} & x_3\left[\rho\frac{\partial^2 u}{\partial t^2} - \frac{E}{1-\nu^2}\left(\frac{\partial^2 u}{\partial x_1^2} + \frac{1-\nu}{2}\frac{\partial^2 u}{\partial x_2^2} + \frac{1+\nu}{2}\frac{\partial^2 v}{\partial x_1 \partial x_2}\right)\right] \\
& + \frac{h^2 - x_3^2}{2}\left[\rho\frac{\partial^3 w}{\partial x_1 \partial t^2} - \frac{E}{1-\nu^2}\frac{\partial \Delta w}{\partial x_1}\right] \\
& - \frac{\sigma}{c}\left[a_3\psi + d_2\left(\frac{\partial \varphi}{\partial x_1} + \frac{\partial \psi}{\partial x_2}\right) - \frac{1}{c}\left(\beta_{22}\frac{\partial u}{\partial t} - \beta_{12}\frac{\partial v}{\partial t}\right)\right. \\
& \left. + \frac{\gamma_{32}}{c}\frac{\partial}{\partial t}\left(\frac{\partial v}{\partial x_1} - \frac{\partial u}{\partial x_2}\right) + \frac{\alpha_{13}}{c}\frac{\partial w}{\partial t} - \frac{1}{c}\frac{\partial}{\partial t}\left(\chi_{22}\frac{\partial w}{\partial x_1} - \chi_{12}\frac{\partial w}{\partial x_2}\right)\right], \\
s_{23} = {} & x_3\left[\rho\frac{\partial^2 v}{\partial t^2} - \frac{E}{1-\nu^2}\left(\frac{\partial^2 v}{\partial x_2^2} + \frac{1-\nu}{2}\frac{\partial^2 v}{\partial x_1^2} + \frac{1+\nu}{2}\frac{\partial^2 u}{\partial x_1 \partial x_2}\right)\right] \\
& + \frac{h^2 - x_3^2}{2}\left[\rho\frac{\partial^3 w}{\partial x_2 \partial t^2} - \frac{E}{1-\nu^2}\frac{\partial \Delta w}{\partial x_2}\right] \\
& + \frac{\sigma}{c}\left[a_3\varphi + d_1\left(\frac{\partial \varphi}{\partial x_1} + \frac{\partial \psi}{\partial x_2}\right) + \frac{1}{c}\left(\beta_{11}\frac{\partial v}{\partial t} - \beta_{21}\frac{\partial u}{\partial t}\right)\right. \\
& \left. + \frac{\gamma_{31}}{c}\frac{\partial}{\partial t}\left(\frac{\partial v}{\partial x_1} - \frac{\partial u}{\partial x_2}\right) - \frac{\alpha_{23}}{c}\frac{\partial w}{\partial t} + \frac{1}{c}\frac{\partial}{\partial t}\left(\chi_{11}\frac{\partial w}{\partial x_2} - \chi_{21}\frac{\partial w}{\partial x_1}\right)\right]
\end{aligned}
\tag{2.1.17}
$$

and the following equations, with respect to the unknown functions u, v, w, φ, ψ and f, are obtained:

$$
\begin{aligned}
& \frac{\partial^2 u}{\partial x_1^2} + \frac{1-\nu}{2}\frac{\partial^2 u}{\partial x_2^2} + \frac{1+\nu}{2}\frac{\partial^2 v}{\partial x_1 \partial x_2} \\
& \quad + \frac{1-\nu^2}{2Eh}\frac{\sigma}{c}\left[b_3\psi + c_2\left(\frac{\partial \varphi}{\partial x_1} + \frac{\partial \psi}{\partial x_2}\right)\right. \\
& \quad - \frac{1}{c}\left(F_{22}\frac{\partial u}{\partial t} - F_{12}\frac{\partial v}{\partial t}\right) + \frac{d_{32}}{c}\frac{\partial}{\partial t}\left(\frac{\partial v}{\partial x_1} - \frac{\partial u}{\partial x_2}\right) \\
& \quad \left. + \frac{b_{13}}{c}\frac{\partial w}{\partial t} - \frac{1}{c}\frac{\partial}{\partial t}\left(G_{22}\frac{\partial w}{\partial x_1} - G_{12}\frac{\partial w}{\partial x_2}\right)\right] = \frac{\rho(1-\nu^2)}{E}\frac{\partial^2 u}{\partial t^2}, \\
& \frac{\partial^2 v}{\partial x_2^2} + \frac{1-\nu}{2}\frac{\partial^2 v}{\partial x_1^2} + \frac{1+\nu}{2}\frac{\partial^2 u}{\partial x_1 \partial x_2} \\
& \quad + \frac{1-\nu^2}{2Eh}\frac{\sigma}{c}\left[-b_3\varphi - c_1\left(\frac{\partial \varphi}{\partial x_1} + \frac{\partial \psi}{\partial x_2}\right)\right. \\
& \quad - \frac{1}{c}\left(F_{11}\frac{\partial v}{\partial t} - F_{21}\frac{\partial u}{\partial t}\right) + \frac{d_{31}}{c}\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x_2} - \frac{\partial v}{\partial x_1}\right) \\
& \quad \left. + \frac{b_{23}}{c}\frac{\partial w}{\partial t} - \frac{1}{c}\frac{\partial}{\partial t}\left(G_{11}\frac{\partial w}{\partial x_2} - G_{21}\frac{\partial w}{\partial x_1}\right)\right] = \frac{\rho(1-\nu^2)}{E}\frac{\partial^2 v}{\partial t^2}.
\end{aligned}
\tag{2.1.18}
$$

In (2.1.17) and (2.1.18), the following notations are performed:

$$b_{ij} = \int_{-h}^{h} H_{0i}H_{0j}\mathrm{d}x_3, \quad d_{ij} = \int_{-h}^{h} a_i H_{0j}\mathrm{d}x_3,$$

$$\alpha_{ij} = A\left(H_{0i}H_{0j}\right), \quad \beta_{ij} = \alpha_{33}\delta_i^j + A\left(H_{0j}\frac{\partial a_3}{\partial x_i}\right),$$

$$\gamma_{ij} = A\left(a_i H_{0j}\right), \quad \chi_{ij} = \gamma_{ij} - A\left(x_3 H_{03}^2\right)\delta_i^j - A\left(H_{0j}\frac{\partial d_3}{\partial x_i}\right),$$

$$F_{ij} = b_{33}\delta_i^j - \int_{-h}^{h} \frac{\partial a_3}{\partial x_i} H_{0j}\mathrm{d}x_3,$$

$$G_{ij} = d_{ij} - \delta_i^j \int_{-h}^{h} x_3 H_{03}^2 \mathrm{d}x_3 - \int_{-h}^{h} H_{0j}\frac{\partial d_3}{\partial x_i}\mathrm{d}x_3.$$

Let us now employ the as-yet-unused third equation of system (2.1.1). Substituting into it (2.1.9), (2.1.12), (2.1.16), and (2.1.17) and integrating the obtained as a result equation with respect to x_3 from $x_3 = h$ up to $x_3 = -h$, taking into account the third condition from (1.6.17), the following equation is obtained in addition to systems (2.1.13) and (2.1.18):

$$\begin{aligned}
&D\Delta^2 w + 2\rho h\frac{\partial^2 w}{\partial t^2} - \frac{2\rho h^3}{3}\frac{\partial^2 \Delta w}{\partial t^2} - N_{11}^0\frac{\partial^2 w}{\partial x_1^2} - 2N_{12}^0\frac{\partial^2 w}{\partial x_1 \partial x_2} - N_{22}^0\frac{\partial^2 w}{\partial x_2^2} \\
&\quad = -\frac{\sigma}{c}\left\{\left(b_1 + \frac{\partial l_3}{\partial x_1}\right)\psi - \left(b_2 + \frac{\partial l_3}{\partial x_2}\right)\varphi\right. \\
&\qquad + \left(g_2\frac{\partial}{\partial x_1} - g_1\frac{\partial}{\partial x_2}\right)\left(\frac{\partial \varphi}{\partial x_1} + \frac{\partial \psi}{\partial x_2}\right) \\
&\qquad + l_3\left(\frac{\partial \psi}{\partial x_1} - \frac{\partial \varphi}{\partial x_2}\right) + \frac{1}{c}\left[\left(\frac{\partial L_{12}}{\partial x_2} - \frac{\partial L_{22}}{\partial x_1} - b_{13}\right)\frac{\partial u}{\partial t}\right. \\
&\qquad + \left(\frac{\partial L_{21}}{\partial x_1} - \frac{\partial L_{11}}{\partial x_2} - b_{23}\right)\frac{\partial v}{\partial t} \\
&\qquad + \left(b_{11} + b_{22} + \frac{\partial l_{13}}{\partial x_1} + \frac{\partial l_{23}}{\partial x_2}\right)\frac{\partial w}{\partial t} \\
&\qquad - \left(\frac{\partial N_{22}}{\partial x_1} - \frac{\partial N_{21}}{\partial x_2}\right)\frac{\partial^2 w}{\partial x_1 \partial t} + \left(\frac{\partial N_{12}}{\partial x_1} - \frac{\partial N_{11}}{\partial x_2}\right)\frac{\partial^2 w}{\partial x_2 \partial t} \\
&\qquad - N_{22}\frac{\partial^3 w}{\partial x_1^2 \partial t} - N_{11}\frac{\partial^3 w}{\partial x_2^2 \partial t} + \left(N_{12} + N_{21}\right)\frac{\partial^3 w}{\partial x_1 \partial x_2 \partial t} \\
&\qquad \left. - \frac{\partial}{\partial t}\left(L_{22}\frac{\partial u}{\partial x_1} - L_{12}\frac{\partial u}{\partial x_2}\right) + \frac{\partial}{\partial t}\left(L_{21}\frac{\partial v}{\partial x_1} - L_{11}\frac{\partial v}{\partial x_2}\right)\right] \\
&\qquad + \frac{1}{c}\left[\left(\frac{\partial a_{32}}{\partial x_1} - \frac{\partial a_{31}}{\partial x_2}\right)\frac{\partial}{\partial t}\left(\frac{\partial v}{\partial x_1} - \frac{\partial u}{\partial x_2}\right) + a_{31}\frac{\partial^3 u}{\partial x_2^2 \partial t}\right. \\
&\qquad \left.\left. + a_{32}\frac{\partial^3 v}{\partial x_1^2 \partial t} - \frac{\partial}{\partial t}\left(a_{31}\frac{\partial^2 v}{\partial x_1 \partial x_2} + a_{32}\frac{\partial^2 u}{\partial x_1 \partial x_2}\right)\right]\right\}.
\end{aligned} \tag{2.1.19}$$

Here the following notations are performed:

$$D = \frac{2Eh^3}{3(1-\nu^2)}, \quad N_{ij}^0 = \int_{-h}^{h} s_{ij}^H \mathrm{d}x_3, \quad \Delta = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2},$$

$$l_i = \int_{-h}^{h} a_i \mathrm{d}x_3, \quad g_i = \int_{-h}^{h} d_i \mathrm{d}x_3, \quad l_{ij} = \int_{-h}^{h} \alpha_{ij} \mathrm{d}x_3,$$

$$a_{ij} = \int_{-h}^{h} \gamma_{ij} \mathrm{d}x_3, \quad L_{ij} = \int_{-h}^{h} \beta_{ij} \mathrm{d}x_3, \quad N_{ij} = \int_{-h}^{h} \chi_{ij} \mathrm{d}x_3.$$

Thus, using the hypothesis of the magnetoelasticity of thin bodies, the two-dimensional Eqs. (2.1.13), (2.1.18), and (2.1.19) are obtained. If assume here that $N_{ij}^0 = 0$, then the equations of free magnetoelastic vibrations of the examined plates will be obtained. In addition to this $\left(N_{ij}^0 = 0\right)$, when we add the term $P_3(x_1, x_3, t)$ characterizing the transverse surface load to the right-hand side of Eq. (2.1.19), the equation of forced vibrations of conducting plates in a magnetic field will be obtained. The theory of the magnetoelasticity of thin bodies possessing more complicate physical properties is constructed in works [11, 62, 81, 99].

To the system of two-dimensional differential Eqs. (2.1.13), (2.1.18), and (2.1.19) it is necessary to also add the boundary conditions and conditions for components of the electromagnetic field at the surfaces and bounds of the plate.

Conditions at the facial surfaces $(x_3 = \pm h)$, according to the Eq. (1.6.17) and $\mu = 1$, have the form

$$h_3^{(e)} = f(x_1, x_2, t), \quad e_1^{(e)} = \varphi(x_1, x_2, t), \quad e_2^{(e)} = \psi(x_1, x_2, t). \tag{2.1.20}$$

In an analogous way, the conditions at the boundary surfaces of the plate can be written. For example, if the boundary surface is plane with the external normal parallel to the axis $0x_1$, then $x_1 = \text{const}$ at this surface and boundary conditions for the components of electromagnetic field can be written as

$$\begin{aligned} h_1^{(e)} &= h_1, \quad h_2^{(e)} = h_2, \quad h_3^{(e)} = f, \\ e_1^{(e)} &= \varepsilon\varphi, \quad e_2^{(e)} = \psi, \quad e_3^{(e)} = e_3. \end{aligned} \tag{2.1.21}$$

For a complete definition of displacements and electromagnetic field in the plate, as shown by Eqs. (2.1.12), (2.1.13), (2.1.18), (2.1.19), and (2.1.21), it is necessary to also have the values of the tangential components of the induced magnetic field at the plate's surface. Therefore, in general, the obtained equations should be studied together with Maxwell equations (1.6.16) for the external area with the boundary conditions (2.1.20) and (2.1.21) and conditions at infinity [5]. This means that the problem of magnetoelasticity is still three-dimensional despite the fact that the obtained equations are two-dimensional with respect to the unknown functions.

However, there are a number of problems for which either (1) the components of the induced electromagnetic field are not included into the brought equations, or (2) the boundary conditions do not contain the values of components of the induced magnetic field in the surroundings electromagnetic field, or (3) they were included but are given before.

Such problems arise, for example, if the following conditions are considered [5].

1. If the plate has infinitely far bound, then it is sufficient to use only the boundless of the solution.
2. In the case when the plate is partially contacted with the perfectly conducting body, the motion of which is given, then the components of the external induced magnetic field are known and can be defined by way of the formulas

$$\begin{aligned} \mathbf{e}^{(\mathbf{e})} &= \frac{1}{c}\left(\mathbf{H}_{\mathbf{0}}^{(\mathbf{e})} \times \frac{\partial \mathbf{u_0}}{\partial t}\right), \\ \mathbf{h}^{(\mathbf{e})} &= \operatorname{rot}\left(\mathbf{u_0} \times \mathbf{H}_{\mathbf{0}}^{(\mathbf{e})}\right), \end{aligned} \tag{2.1.22}$$

where $\mathbf{u_0}$ is the given vector of displacement of the perfectly conductor, and $\mathbf{H}_{\mathbf{0}}^{(\mathbf{e})}$ is the vector of the given external magnetic field intensity in the perfectly conductor, for which $\mu = 1$.

On the basis of Eq. (2.1.22), from Eq. (2.1.21), in this case the following boundary conditions are obtained:

$$\begin{aligned} \psi &= \frac{1}{c}\left[H_{01}^{(e)}\frac{\partial u_{03}}{\partial t} - H_{03}^{(e)}\frac{\partial u_{01}}{\partial t}\right], \\ e_3 &= \frac{1}{c}\left[H_{02}^{(e)}\frac{\partial u_{01}}{\partial t} - H_{01}^{(e)}\frac{\partial u_{02}}{\partial t}\right], \\ h_1 &= \left[\operatorname{rot}\left(\mathbf{u_0} \times \mathbf{H}_{\mathbf{0}}^{(\mathbf{e})}\right)\right]_1. \end{aligned} \tag{2.1.23}$$

3. If the bound of the plate is fixed then at this bound, it can be assumed that the components of the induced electric current are equal to zero because as a rule the electric current at the point is conditioned by the motion speed of the point.
4. In the case when the given magnetic field is perpendicular to the middle plane of the plate (so it should be a constant), the induced electromagnetic field is not included in the equation of transverse vibrations of the plate as shown in Eq. (2.1.19). In this case the problem of transverse magnetoelastic vibrations is brought to the solution of Eq. (2.1.19) with the usual boundary conditions.

Let us note also that if the boundary surfaces of the plate are placed in the vacuum, then the condition (2.1.14) can be used, because it does not contain the solution of the external problem. Then, for example, for the edge $x_1 = \text{const}$ we have

$$e_1 = \varphi = \frac{1}{c}\left[H_{02}\frac{\partial w}{\partial t} - H_{03}\frac{\partial v}{\partial t}\right]. \tag{2.1.24}$$

The obtained equations can be simplified essentially in some particular cases of the external magnetic field. Let us indicate some of them having assumed that only the transverse load $P(x_1, x_2, t)$ acts on the plate.

- The plate is immersed in the external constant magnetic field $\mathbf{H_0}(H_{01}, H_{02}, 0)$. In this case from Eqs. (2.1.13), (2.1.18), and (2.1.19), it is easy to note, that the equation of transverse vibrations is separated from the equations of longitudinal vibrations. Moreover, for the longitudinal vibrations the classical equations of the theory of elasticity are true and for the transverse vibrations the following equations are obtained:

$$\begin{aligned}
&\frac{\partial \psi}{\partial x_1} - \frac{\partial \varphi}{\partial x_2} + \frac{1}{c}\frac{\partial f}{\partial t} = 0,\\
&\frac{\partial f}{\partial x_1} + \frac{4\pi\sigma}{c}\left(\psi + \frac{H_{01}}{c}\frac{\partial w}{\partial t}\right) = \frac{h_1^+ - h_1^-}{2h},\\
&\frac{\partial f}{\partial x_2} - \frac{4\pi\sigma}{c}\left(\varphi - \frac{H_{02}}{c}\frac{\partial w}{\partial t}\right) = \frac{h_2^+ - h_2^-}{2h},\\
&D\Delta^2 w + 2\rho h\frac{\partial^2 w}{\partial t^2}\\
&\quad = P - \frac{2\sigma h}{c}\left[H_{01}\left(\psi + \frac{H_{01}}{c}\frac{\partial w}{\partial t}\right) - H_{02}\left(\varphi - \frac{H_{02}}{c}\frac{\partial w}{\partial t}\right)\right].
\end{aligned} \tag{2.1.25}$$

In particular, when the form of vibrations of the plate is a cylindrical surface $x_3 = w(x_1, t)$ (plane problem), then Eq. (2.1.25) can be more simplified and depending on the orientation of the external magnetic field have the form:

in the case of magnetic field, parallel to the axis $0x_1$

$$\begin{aligned}
&\frac{\partial \psi}{\partial x_1} + \frac{1}{c}\frac{\partial f}{\partial t} = 0,\\
&\frac{\partial f}{\partial x_1} + \frac{4\pi\sigma}{c}\left(\psi + \frac{H_{01}}{c}\frac{\partial w}{\partial t}\right) = \frac{h_1^+ - h_1^-}{2h},
\end{aligned} \tag{2.1.26}$$

$$D\frac{\partial^4 w}{\partial x_1^4} + 2\rho h\frac{\partial^2 w}{\partial t^2} = P - \frac{2\sigma h}{c}H_{01}\left(\psi + \frac{H_{01}}{c}\frac{\partial w}{\partial t}\right);$$

and in the case of magnetic field, parallel to the axis $0x_2$

$$
\begin{aligned}
&\frac{4\pi\sigma}{c}\left(\varphi-\frac{H_{02}}{c}\frac{\partial w}{\partial t}\right)+\frac{h_2^+-h_2^-}{2h}=0,\\
&D\frac{\partial^4 w}{\partial x_2^4}+2\rho h\frac{\partial^2 w}{\partial t^2}=P+\frac{2\sigma h}{c}H_{02}\left(\varphi-\frac{H_{02}}{c}\frac{\partial w}{\partial t}\right).
\end{aligned}
\tag{2.1.27}
$$

- The plate is immersed in a transversal magnetic field $\mathbf{H_0}(0,0,H_{03})$. In this case, systems (2.1.13), (2.1.18), and (2.1.19) also split, and for the longitudinal vibrations the following equations are obtained

$$
\begin{aligned}
&\frac{\partial\psi}{\partial x_1}-\frac{\partial\varphi}{\partial x_2}+\frac{1}{c}\frac{\partial f}{\partial t}=0,\\
&\frac{\partial f}{\partial x_1}+\frac{4\pi\sigma}{c}\left(\psi-\frac{H_{03}}{c}\frac{\partial u}{\partial t}\right)=\frac{h_1^+-h_1^-}{2h},\\
&\frac{\partial f}{\partial x_2}-\frac{4\pi\sigma}{c}\left(\varphi+\frac{H_{03}}{c}\frac{\partial v}{\partial t}\right)=\frac{h_2^+-h_2^-}{2h},\\
&\frac{\partial^2 u}{\partial x_1^2}+\frac{1-\nu}{2}\frac{\partial^2 u}{\partial x_2^2}+\frac{1+\nu}{2}\frac{\partial^2 v}{\partial x_1\partial x_2}\\
&\quad+\frac{1-\nu^2}{E}\frac{\sigma H_{03}}{c}\left(\psi-\frac{H_{03}}{c}\frac{\partial u}{\partial t}\right)=\frac{\rho(1-\nu^2)}{E}\frac{\partial^2 u}{\partial t^2},\\
&\frac{\partial^2 v}{\partial x_2^2}+\frac{1-\nu}{2}\frac{\partial^2 v}{\partial x_1^2}+\frac{1+\nu}{2}\frac{\partial^2 u}{\partial x_1\partial x_2}\\
&\quad-\frac{1-\nu^2}{E}\frac{\sigma H_{03}}{c}\left(\varphi+\frac{H_{03}}{c}\frac{\partial v}{\partial t}\right)=\frac{\rho(1-\nu^2)}{E}\frac{\partial^2 v}{\partial t^2}.
\end{aligned}
\tag{2.1.28}
$$

The equation of transversal vibrations has the form:

$$
D\Delta^2 w+2\rho h\frac{\partial^2 w}{\partial t^2}=P+\frac{2\sigma h^3}{3c^2}H_{03}^2\frac{\partial\Delta w}{\partial t}. \tag{2.1.29}
$$

To investigate this equation it is necessary to have only the initial magnetic field and usual boundary conditions for w. From here it follows also that the hypotheses of magnetoelasticity of thin bodies do not have an effect on the character of the magnetoelastic vibrations.

2.1.2 Perfectly Conducting Plates

Using the Kirchhoff hypothesis, the two-dimensional equations of magnetoelasticity of thin perfectly conducting plates in a constant magnetic field are obtained here on the basis of the works [13, 69]. It is assumed that the load $P(x_1, x_2, t)$ of nonelectromagnetic origin $(F_1 = F_2 = 0, F_3 = P)$ acts normally to the surface $x_3 = h$ of the plate, and the magnetic susceptibility of the plate's material is equal to the unit.

On the basis of the accepted assumptions from Eqs. (1.3.1) and (1.2.6), using Eqs. (1.3.7) and (1.3.9), after linearization the following three-dimensional equations are obtained to characterize the behavior of magnetoelastic quantities within the plate:

$$\frac{\partial s_{ik}}{\partial x_k} + \frac{1}{4\pi}(\operatorname{rot}\mathbf{h} \times \mathbf{H_0})_i = \rho\frac{\partial^2 u_i}{\partial t^2}, \tag{2.1.27}$$

$$\mathbf{h} = \operatorname{rot}(\mathbf{u} \times \mathbf{H_0}), \quad \mathbf{e} = \frac{1}{c}\left(\mathbf{H_0} \times \frac{\partial \mathbf{u}}{\partial t}\right). \tag{2.1.28}$$

According to the Kirchhoff hypothesis, relations (2.1.5) and (2.1.7) take place. Substituting (2.1.5) into Eq. (2.1.28), the following expressions with respect to the components of the induced electromagnetic field are obtained:

$$\begin{aligned}
h_1 &= -H_{01}\left(\frac{\partial v}{\partial x_2} - x_3\frac{\partial^2 w}{\partial x_2^2}\right) + H_{02}\left(\frac{\partial u}{\partial x_2} - x_3\frac{\partial^2 w}{\partial x_1 \partial x_2}\right) - H_{03}\frac{\partial w}{\partial x_1},\\
h_2 &= H_{01}\left(\frac{\partial v}{\partial x_1} - x_3\frac{\partial^2 w}{\partial x_1 \partial x_2}\right) - H_{02}\left(\frac{\partial u}{\partial x_1} - x_3\frac{\partial^2 w}{\partial x_1^2}\right) - H_{03}\frac{\partial w}{\partial x_2},\\
h_3 &= H_{01}\frac{\partial w}{\partial x_1} + H_{02}\frac{\partial w}{\partial x_2} - H_{03}\left(\frac{\partial u}{\partial x_1} + \frac{\partial v}{\partial x_2} - x_3\Delta w\right);
\end{aligned} \tag{2.1.29}$$

$$\begin{aligned}
e_1 &= \frac{1}{c}\left[H_{02}\frac{\partial w}{\partial t} - H_{03}\left(\frac{\partial v}{\partial t} - x_3\frac{\partial^2 w}{\partial x_2 \partial t}\right)\right],\\
e_2 &= \frac{1}{c}\left[-H_{01}\frac{\partial w}{\partial t} + H_{03}\left(\frac{\partial u}{\partial t} - x_3\frac{\partial^2 w}{\partial x_1 \partial t}\right)\right],\\
e_3 &= \frac{1}{c}\left[H_{01}\left(\frac{\partial v}{\partial t} - x_3\frac{\partial^2 w}{\partial x_2 \partial t}\right) - H_{02}\left(\frac{\partial u}{\partial t} - x_3\frac{\partial^2 w}{\partial x_1 \partial t}\right)\right].
\end{aligned} \tag{2.1.30}$$

On the basis of Eq. (2.1.5) from Eq. (2.1.27), the following expressions are obtained for the components of space force of electromagnetic origin:

$$
\begin{aligned}
X_1 &= \frac{H_{03}A_2 - H_{02}A_3}{4\pi} - \frac{x_3}{4\pi}\left[\left(H_{02}^2 + H_{03}^2\right)\frac{\partial \Delta w}{\partial x_1} - H_{01}H_{02}\frac{\partial \Delta w}{\partial x_2}\right],\\
X_2 &= \frac{H_{01}A_3 - H_{03}A_1}{4\pi} - \frac{x_3}{4\pi}\left[\left(H_{01}^2 + H_{03}^2\right)\frac{\partial \Delta w}{\partial x_2} - H_{01}H_{02}\frac{\partial \Delta w}{\partial x_1}\right],\\
X_3 &= \frac{H_{02}A_1 - H_{01}A_2}{4\pi} + \frac{x_3}{4\pi}\left[H_{02}H_{03}\frac{\partial \Delta w}{\partial x_2} + H_{01}H_{03}\frac{\partial \Delta w}{\partial x_1}\right],
\end{aligned}
\tag{2.1.31}
$$

where

$$
\begin{aligned}
A_1 &= -H_{02}\Delta w + 2\frac{\partial}{\partial x_2}\left(H_{01}\frac{\partial w}{\partial x_1} + H_{02}\frac{\partial w}{\partial x_2}\right) - H_{03}\frac{\partial}{\partial x_2}\left(\frac{\partial u}{\partial x_1} + \frac{\partial v}{\partial x_2}\right),\\
A_2 &= H_{01}\Delta w - 2\frac{\partial}{\partial x_1}\left(H_{01}\frac{\partial w}{\partial x_1} + H_{02}\frac{\partial w}{\partial x_2}\right) + H_{03}\frac{\partial}{\partial x_1}\left(\frac{\partial u}{\partial x_1} + \frac{\partial v}{\partial x_2}\right),\\
A_3 &= H_{01}\Delta v - H_{02}\Delta u.
\end{aligned}
$$

From the first two equations of system (2.1.27), in account of (2.1.7), (2.1.31), and boundary conditions (1.6.12), we can find

$$
\begin{aligned}
s_{13} = x_3 &\left[\rho\frac{\partial^2 u}{\partial t^2} - \frac{E}{1-\nu^2}\left(\frac{\partial^2 u}{\partial x_1^2} + \frac{1-\nu}{2}\frac{\partial^2 u}{\partial x_2^2} + \frac{1+\nu}{2}\frac{\partial^2 v}{\partial x_1 \partial x_2}\right) - \frac{H_{03}A_2 - H_{02}A_3}{4\pi}\right]\\
&+ \frac{H_{03}}{4\pi}\left[H_{03}\frac{\partial w}{\partial x_1} + H_{01}\frac{\partial v}{\partial x_2} - H_{02}\frac{\partial u}{\partial x_2} + \frac{1}{2}\left(h_1^{(e)+} - h_1^{(e)-}\right)\right]\\
&+ \frac{h^2 - x_3^2}{2}\left[\rho\frac{\partial^3 w}{\partial x_1 \partial t^2} - \left(\frac{E}{1-\nu^2} + \frac{H_{02}^2 + H_{03}^2}{4\pi}\right)\frac{\partial \Delta w}{\partial x_1} + \frac{H_{01}H_{02}}{4\pi}\frac{\partial \Delta w}{\partial x_2}\right],
\end{aligned}
\tag{2.1.32}
$$

$$
\begin{aligned}
s_{23} = x_3 &\left[\rho\frac{\partial^2 v}{\partial t^2} - \frac{E}{1-\nu^2}\left(\frac{\partial^2 v}{\partial x_2^2} + \frac{1-\nu}{2}\frac{\partial^2 v}{\partial x_1^2} + \frac{1+\nu}{2}\frac{\partial^2 u}{\partial x_1 \partial x_2}\right) - \frac{H_{01}A_3 - H_{03}A_1}{4\pi}\right]\\
&+ \frac{H_{03}}{4\pi}\left[H_{03}\frac{\partial w}{\partial x_2} + H_{02}\frac{\partial u}{\partial x_1} - H_{01}\frac{\partial v}{\partial x_1} + \frac{1}{2}\left(h_2^{(e)+} - h_2^{(e)-}\right)\right]\\
&+ \frac{h^2 - x_3^2}{2}\left[\rho\frac{\partial^3 w}{\partial x_2 \partial t^2} - \left(\frac{E}{1-\nu^2} + \frac{H_{01}^2 + H_{03}^2}{4\pi}\right)\frac{\partial \Delta w}{\partial x_2} + \frac{H_{01}H_{02}}{4\pi}\frac{\partial \Delta w}{\partial x_1}\right]
\end{aligned}
$$

Substituting (2.1.7), (2.1.31), and (2.1.32) into Eq. (2.1.27) and integrating the obtained equation with respect to x_3 from $x_3 = -h$ up to $x_3 = h$, taking into account the third condition from (1.6.12), the following system of differential equations are obtained with respect to functions u, v, w:

$$\begin{gathered}
\frac{\partial^2 u}{\partial x_1^2} + \frac{1-\nu}{2}\frac{\partial^2 u}{\partial x_2^2} + \frac{1+\nu}{2}\frac{\partial^2 v}{\partial x_1 \partial x_2} \\
+ \frac{1-\nu^2}{4\pi E}\left[H_{02}^2 \Delta u - H_{01}H_{02}\Delta v + H_{03}^2 \frac{\partial}{\partial x_1}\left(\frac{\partial u}{\partial x_1} + \frac{\partial v}{\partial x_2}\right)\right. \\
\left. - H_{03}\left(H_{01}\frac{\partial^2 w}{\partial x_1^2} + H_{02}\frac{\partial^2 w}{\partial x_1 \partial x_2}\right)\right] \\
= \frac{\rho(1-\nu^2)}{E}\frac{\partial^2 u}{\partial t^2} - \frac{H_{03}}{4\pi}\frac{1-\nu^2}{2Eh}\left[h_1^{(e)+} - h_1^{(e)-}\right], \\
\frac{\partial^2 v}{\partial x_2^2} + \frac{1-\nu}{2}\frac{\partial^2 v}{\partial x_1^2} + \frac{1+\nu}{2}\frac{\partial^2 u}{\partial x_1 \partial x_2} \\
+ \frac{1-\nu^2}{4\pi E}\left[H_{01}^2 \Delta v - H_{01}H_{02}\Delta u + H_{03}^2 \frac{\partial}{\partial x_2}\left(\frac{\partial u}{\partial x_1} + \frac{\partial v}{\partial x_2}\right)\right. \\
\left. - H_{03}\left(H_{01}\frac{\partial^2 w}{\partial x_1 \partial x_2} + H_{02}\frac{\partial^2 w}{\partial x_2^2}\right)\right] \\
= \frac{\rho(1-\nu^2)}{E}\frac{\partial^2 v}{\partial t^2} - \frac{H_{03}}{8\pi}\frac{1-\nu^2}{Eh}\left[h_2^{(e)+} - h_2^{(e)-}\right], \\
D_*\Delta^2 w + 2\rho h\frac{\partial^2 w}{\partial t^2} - \frac{2\rho h^3}{3}\frac{\partial^2 \Delta w}{\partial t^2} \\
- \frac{2h}{4\pi}\left[H_{01}^2\frac{\partial^2 w}{\partial x_1^2} + 2H_{01}H_{02}\frac{\partial^2 w}{\partial x_1 \partial x_2} + H_{02}^2\frac{\partial^2 w}{\partial x_2^2} + H_{03}^2\Delta w\right. \\
\left. - H_{03}\left(H_{01}\frac{\partial}{\partial x_1} + H_{02}\frac{\partial}{\partial x_2}\right)\left(\frac{\partial u}{\partial x_1} + \frac{\partial v}{\partial x_2}\right)\right] \\
= P - \frac{H_{01}}{4\pi}\left[h_1^{(e)+} - h_1^{(e)-}\right] - \frac{H_{02}}{4\pi}\left[h_2^{(e)+} - h_2^{(e)-}\right] \\
+ \frac{hH_{03}}{4\pi}\left[\frac{\partial\left(h_1^{(e)+} + h_1^{(e)-}\right)}{\partial x_1} + \frac{\partial\left(h_2^{(e)+} + h_2^{(e)-}\right)}{\partial x_2}\right],
\end{gathered} \tag{2.1.33}$$

where

$$D_* = \frac{2h^3}{3}\left(\frac{E}{1-\nu^2} + \frac{H_{01}^2 + H_{02}^2 + H_{03}^2}{4\pi}\right).$$

To define all displacements and electromagnetic field in the plate, as Eq. (2.1.33) shows, it is also necessary to have the tangential components of the induced electromagnetic field at the plate's surface magnetic field. Therefore, in general the problem of magnetoelasticity is still three-dimensional and Eq. (2.1.33) should be studied together with Maxwell equations (1.6.16) at the external area and with the general boundary conditions (1.6.17). The issues of determination of $h_i^{(e)\pm}$ and, for the final reduction of the three-dimensional problem of magnetoelasticity to the two-dimensional problem in the case of perfectly conducting plates mentioned in Sect. 2.3 (Reduction of the three-dimensional problem of magnetoelasticity of thin plates to the two-dimensional), will be studied using the asymptotic method.

Let us note that in the case of longitudinal magnetic field $\mathbf{H_0}(H_{01}, H_{02}, 0)$, the system of differential Eq. (2.1.33) is split. In particular, the equation of transverse vibration of the plate has the form

$$\begin{gathered} D_*\Delta^2 w + 2\rho h \frac{\partial^2 w}{\partial t^2} \\ -\frac{2h}{4\pi}\left[\left(H_{01}^2 + H_{02}^2\right)\Delta w + 2H_{01}H_{02}\frac{\partial^2 w}{\partial x_1 \partial x_2} - H_{01}^2\frac{\partial^2 w}{\partial x_2^2} - H_{02}^2\frac{\partial^2 w}{\partial x_1^2}\right] \\ = P - \frac{H_{01}}{4\pi}\left[h_1^{(e)+} - h_1^{(e)-}\right] - \frac{H_{02}}{4\pi}\left[h_2^{(e)+} - h_2^{(e)-}\right]. \end{gathered} \tag{2.1.34}$$

The system is also split in the case of plane problem, when the plate does experience vibrations in the form of a cylindrical surface $x_3 = w(x_1, t)$, and the magnetic field has the origin $\mathbf{H_0}(H_{01}, 0, H_{03})$. The equation of transverse vibrations in this case has the form

$$\begin{gathered} D(1+\alpha)\frac{\partial^4 w}{\partial x_1^4} + 2\rho h\frac{\partial^2 w}{\partial t^2} - \frac{2h}{4\pi}\left[H_{03}^2 + \frac{H_{01}^2}{1+\alpha}\right]\frac{\partial^2 w}{\partial x_1^2} \\ = P - \frac{H_{01}}{4\pi(1+\alpha)}\left[h_1^{(e)+} - h_1^{(e)-}\right] + \frac{hH_{03}}{4\pi}\frac{\partial}{\partial x_1}\left(h_1^{(e)+} + h_1^{(e)-}\right), \\ D = \frac{2Eh^3}{3(1-\nu^2)}, \quad \alpha = \frac{1-\nu^2}{E}\frac{H_{03}^2}{4\pi}. \end{gathered} \tag{2.1.35}$$

In each particular case, in addition to the above-mentioned conditions, the fixing conditions for the plate's bounds should be attached to the system of obtained equations.

2.2 Two-Dimensional Equations of Magnetoelasticity of Thin Conducting Shells

Let the middle plane of the shell be referred to as the curvilinear orthogonal coordinate system α_1, α_2 and the main coordinate lines α_1 and α_2 coincide with the lines of principal curvature of the middle surface of the shell (Fig. 2.2). In the chosen coordinate system, the middle surface of the shell is characterized by way of the principal curvatures $k_1 = k_1(\alpha_1, \alpha_2), k_2 = k_2(\alpha_1, \alpha_2)$ corresponding to the curvature radiuses $R_1 = R_1(\alpha_1, \alpha_2), R_2 = R_2(\alpha_1, \alpha_2)$ of the curvature lines $\alpha_1 = \text{const}$, $\alpha_2 = \text{const}$ and by way of the coefficients of the first quadratic form $A_1 = A_1(\alpha_1, \alpha_2), A_2 = A_2(\alpha_1, \alpha_2)$.

Thus the position of any point of the middle plane will be characterized by way of the two curvilinear coordinates α_1 and α_2. To define the position of any point out of the middle plane, let us introduce the third coordinate line α_3, normal to the lines $\alpha_1 = \text{const}$, $\alpha_2 = \text{const}$. The coordinate α_3 is the distance along the normal between the points (α_1, α_2) and $(\alpha_1, \alpha_2, \alpha_3)$.

In the future, in general, very flat shells will be considered, i.e., the shells for which it is approximately assumed that the internal geometry of the middle surface is not different from Euclidean geometry on the plane. For such shells,with the accuracy of the accepted geometrical assumptions it is assumed that the coefficients of the first quadratic form A_1, A_2 and the main curvatures k_1 and k_2 behave as constants when performing differentiation [3, 60].

Let the isotropic, thin, very flat shell of constant thickness $2h$ be made of a material with the finite electroconductivity σ, equal to the unit of magnetic susceptibility and be placed in an external stationary magnetic field $\mathbf{H_0}$.

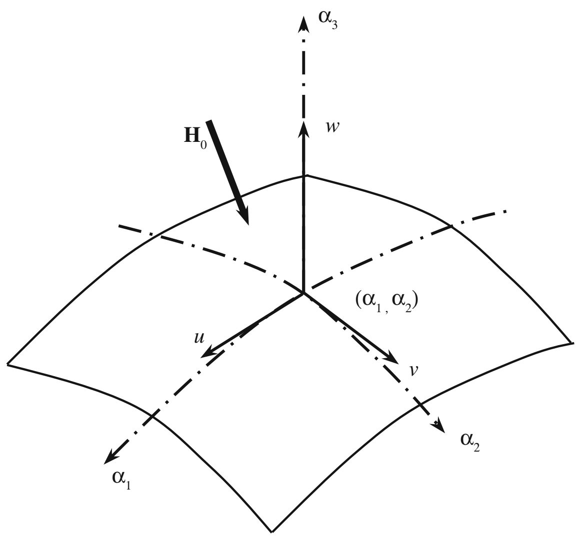

Fig. 2.2 Geometrical interpretation of the problem. Shell in a magnetic field

In the work [5], the asymptotic integration of the three-dimensional equations of magnetoelasticity for thin shells is drafted. As a result, the following hypotheses of magnetoelasticity of thin shells is addressed based on the change in character of the electromagnetic field and elastic displacements along the thickness of the shell:

- normal to the shell's middle surface rectilinear element after deformation remains rectilinear and normal to the shell's deformed middle surface and keeps its length; and
- tangential components of intensity vector of the induced electric field and normal component of intensity vector of the induced magnetic field remain unchanged along the thickness of the shell.

Within the accuracy of the first assumption it is also assumed that in the Hooke generalized law, the term s_{33} can be neglected.

Taking into account the expressions for ε_{ij} the addressed hypotheses have the following analytical form:

$$u_1 = u - \frac{\alpha_3}{A_1}\frac{\partial w}{\partial \alpha_1}, \quad u_2 = v - \frac{\alpha_3}{A_2}\frac{\partial w}{\partial \alpha_2}, \quad u_3 = w(\alpha_1, \alpha_2, t); \tag{2.2.1}$$

$$e_1 = \varphi(\alpha_1, \alpha_2, t), \quad e_2 = \psi(\alpha_1, \alpha_2, t), \quad h_3 = f(\alpha_1, \alpha_2, t). \tag{2.2.2}$$

Herein $u(\alpha_1, \alpha_2, t), v(\alpha_1, \alpha_2, t), w(\alpha_1, \alpha_2, t)$ are unknown tangential and normal displacements of points of the middle surface of the shell; φ, ψ, f are unknown components of the induced magnetic field in the shell's electromagnetic field.

On the basis of these hypotheses from the three-dimensional equations of magnetoelasticity, discussed in Sect. 1.1, the main two-dimensional equations of magnetoelasticity of thin shells are obtained in the work [5]. Therefore, let us bring here the final results only, which are devoted to very flat shells.

1. The system of differential equations, with respect to the unknown functions, is obtained from the equations of electrodynamics:

$$\begin{gathered}
\frac{1}{A_1}\frac{\partial \psi}{\partial \alpha_1} - \frac{1}{A_2}\frac{\partial \varphi}{\partial \alpha_2} + \frac{1}{c}\frac{\partial f}{\partial t} = 0, \\
\frac{1}{A_1}\frac{\partial f}{\partial \alpha_1} + \frac{4\pi\sigma}{c}\left[\psi + \frac{1}{2hc}\left(b_1\frac{\partial w}{\partial t} - b_3\frac{\partial u}{\partial t} + \frac{c_3}{A_1}\frac{\partial^2 w}{\partial \alpha_1 \partial t}\right)\right] = \frac{h_1^+ - h_1^-}{2h}, \\
\frac{1}{A_2}\frac{\partial f}{\partial \alpha_2} + \frac{4\pi\sigma}{c}\left[\varphi - \frac{1}{2hc}\left(b_2\frac{\partial w}{\partial t} - b_3\frac{\partial v}{\partial t} + \frac{c_3}{A_2}\frac{\partial^2 w}{\partial \alpha_2 \partial t}\right)\right] = \frac{h_2^+ - h_2^-}{2h}.
\end{gathered} \tag{2.2.3}$$

2. The system of differential equations, obtained from the first two equations of motion of the medium, is:

$$
\begin{aligned}
&\frac{1}{A_1^2}\frac{\partial^2 u}{\partial \alpha_1^2}+\frac{1-\nu}{2A_2^2}\frac{\partial^2 u}{\partial \alpha_2^2}+\frac{1+\nu}{2A_1A_2}\frac{\partial^2 v}{\partial \alpha_1 \partial \alpha_2}+\frac{k_1+\nu k_2}{A_1}\frac{\partial w}{\partial \alpha_1}\\
&\quad+\frac{1-\nu^2}{2Eh}\frac{\sigma}{c}\left[b_3\psi+c_2\left(\frac{1}{A_1}\frac{\partial \varphi}{\partial \alpha_1}+\frac{1}{A_2}\frac{\partial \psi}{\partial \alpha_2}\right)\right.\\
&\quad-\frac{1}{c}\left(F_{22}\frac{\partial u}{\partial t}-F_{12}\frac{\partial v}{\partial t}\right)+\frac{d_{32}}{c}\frac{\partial}{\partial t}\left(\frac{1}{A_1}\frac{\partial v}{\partial \alpha_1}-\frac{1}{A_2}\frac{\partial u}{\partial \alpha_2}\right)\\
&\quad\left.+\frac{b_{13}}{c}\frac{\partial w}{\partial t}-\frac{1}{c}\frac{\partial}{\partial t}\left(\frac{G_{22}}{A_1}\frac{\partial w}{\partial \alpha_1}-\frac{G_{12}}{A_2}\frac{\partial w}{\partial \alpha_2}\right)\right]=\frac{\rho(1-\nu^2)}{E}\frac{\partial^2 u}{\partial t^2},
\end{aligned}
\tag{2.2.4}
$$

$$
\begin{aligned}
&\frac{1}{A_2^2}\frac{\partial^2 v}{\partial \alpha_2^2}+\frac{1-\nu}{2A_1^2}\frac{\partial^2 v}{\partial \alpha_1^2}+\frac{1+\nu}{2A_1A_2}\frac{\partial^2 u}{\partial \alpha_1 \partial \alpha_2}+\frac{k_2+\nu k_1}{A_2}\frac{\partial w}{\partial \alpha_2}\\
&\quad+\frac{1-\nu^2}{2Eh}\frac{\sigma}{c}\left[-b_3\varphi-c_1\left(\frac{1}{A_1}\frac{\partial \varphi}{\partial \alpha_1}+\frac{1}{A_2}\frac{\partial \psi}{\partial \alpha_2}\right)\right.\\
&\quad-\frac{1}{c}\left(F_{11}\frac{\partial v}{\partial t}-F_{21}\frac{\partial u}{\partial t}\right)+\frac{d_{31}}{c}\frac{\partial}{\partial t}\left(\frac{1}{A_2}\frac{\partial u}{\partial \alpha_2}-\frac{1}{A_1}\frac{\partial v}{\partial \alpha_1}\right)\\
&\quad\left.+\frac{b_{23}}{c}\frac{\partial w}{\partial t}-\frac{1}{c}\frac{\partial}{\partial t}\left(\frac{G_{11}}{A_2}\frac{\partial w}{\partial \alpha_2}-\frac{G_{21}}{A_1}\frac{\partial w}{\partial \alpha_1}\right)\right]=\frac{\rho(1-\nu^2)}{E}\frac{\partial^2 v}{\partial t^2}.
\end{aligned}
$$

3. The differential equation, obtained from the third equation of motion of the medium, is:

$$
\begin{aligned}
&D\left\{\Delta^2 w+\frac{3}{h^2}\left[\frac{k_1+\nu k_2}{A_1}\frac{\partial u}{\partial \alpha_1}\right.\right.\\
&\quad\left.\left.+\frac{k_2+\nu k_1}{A_2}\frac{\partial v}{\partial \alpha_2}+\left(k_1^2+2\nu k_1k_2+k_2^2\right)w\right]\right\}\\
&\quad+2\rho h\frac{\partial^2 w}{\partial t^2}-\frac{N_{11}^0}{A_1^2}\frac{\partial^2 w}{\partial \alpha_1^2}-\frac{2N_{12}^0}{A_1A_2}\frac{\partial^2 w}{\partial \alpha_1 \partial \alpha_2}-\frac{N_{22}^0}{A_2^2}\frac{\partial^2 w}{\partial \alpha_2^2}\\
&=\frac{\sigma}{c}\left\{\left(\frac{1}{A_1}\frac{\partial c_3}{\partial \alpha_1}-b_1\right)\psi+\left(b_2-\frac{1}{A_2}\frac{\partial c_3}{\partial \alpha_2}\right)\varphi\right.\\
&\quad+c_3\left(\frac{1}{A_1}\frac{\partial \psi}{\partial \alpha_1}-\frac{1}{A_2}\frac{\partial \varphi}{\partial \alpha_2}\right)\\
&\quad+\left(\frac{g_2}{A_1}\frac{\partial}{\partial \alpha_1}-\frac{g_1}{A_2}\frac{\partial}{\partial \alpha_2}\right)\left(\frac{1}{A_1}\frac{\partial \varphi}{\partial \alpha_1}+\frac{1}{A_2}\frac{\partial \psi}{\partial \alpha_2}\right)\\
&\quad+\frac{1}{c}\left[\left(b_{13}-\frac{1}{A_1}\frac{\partial L_{22}}{\partial \alpha_1}+\frac{1}{A_2}\frac{\partial L_{12}}{\partial \alpha_2}\right)\frac{\partial u}{\partial t}\right.
\end{aligned}
$$

$$
\begin{aligned}
&+\left(b_{23}+\frac{1}{A_1}\frac{\partial L_{21}}{\partial \alpha_1}-\frac{1}{A_2}\frac{\partial L_{11}}{\partial \alpha_2}\right)\frac{\partial v}{\partial t}\\
&-\left(b_{11}+b_{22}-\frac{1}{A_1}\frac{\partial c_{13}}{\partial \alpha_1}-\frac{1}{A_2}\frac{\partial c_{23}}{\partial \alpha_2}\right)\frac{\partial w}{\partial t}\\
&-\left(\frac{1}{A_1}\frac{\partial N_{22}}{\partial \alpha_1}-\frac{1}{A_2}\frac{\partial N_{21}}{\partial \alpha_2}\right)\frac{1}{A_1}\frac{\partial^2 w}{\partial \alpha_1 \partial t}\\
&+\left(\frac{1}{A_1}\frac{\partial N_{12}}{\partial \alpha_1}-\frac{1}{A_2}\frac{\partial N_{11}}{\partial \alpha_2}\right)\frac{1}{A_2}\frac{\partial^2 w}{\partial \alpha_2 \partial t}\\
&-\frac{L_{22}}{A_1}\frac{\partial^2 u}{\partial \alpha_1 \partial t}+\frac{L_{12}}{A_2}\frac{\partial u}{\partial \alpha_2 \partial t}+\frac{L_{21}}{A_1}\frac{\partial^2 v}{\partial \alpha_1 \partial t}-\frac{L_{11}}{A_2}\frac{\partial^2 v}{\partial \alpha_2 \partial t}\\
&\left.-\frac{N_{11}}{A_2^2}\frac{\partial^3 w}{\partial \alpha_2^2 \partial t}-\frac{N_{22}}{A_1^2}\frac{\partial^3 w}{\partial \alpha_1^2 \partial t}+\frac{N_{12}+N_{21}}{A_1 A_2}\frac{\partial^3 w}{\partial \alpha_1 \partial \alpha_2 \partial t}\right]\\
&+\frac{1}{c}\left[\frac{a_{31}}{A_2^2}\frac{\partial^3 u}{\partial \alpha_2^2 \partial t}+\frac{a_{32}}{A_1^2}\frac{\partial^3 v}{\partial \alpha_1^2 \partial t}\right.\\
&-\frac{1}{A_1 A_2}\frac{\partial}{\partial t}\left(a_{31}\frac{\partial^2 v}{\partial \alpha_1 \partial \alpha_2}+a_{32}\frac{\partial^2 u}{\partial \alpha_1 \partial \alpha_2}\right)\\
&\left.\left.+\left(\frac{1}{A_1}\frac{\partial a_{32}}{\partial \alpha_1}-\frac{1}{A_2}\frac{\partial a_{31}}{\partial \alpha_2}\right)\left(\frac{1}{A_1}\frac{\partial^2 v}{\partial \alpha_1 \partial t}-\frac{1}{A_2}\frac{\partial u}{\partial \alpha_2 \partial t}\right)\right]\right\}.
\end{aligned}
\tag{2.2.5}
$$

Coefficients in Eqs. (2.2.3)–(2.2.5) are calculated by way of the same formulas as in the case of plates replacing x_3 into α_3. In these equations, N_{ik}^0 are the forces of the unperturbed state, which are calculated from the solutions of the problem (1.6.10)–(1.6.13), and Δ is the two-dimensional Laplace operator: $\Delta = A_1^{-2}\partial^2/\partial\alpha_1^2 + A_2^{-2}\partial^2/\partial\alpha_2^2$. If we take $N_{ik}^0 = 0$ in Eq. (2.2.5), then Eqs. (2.2.3)–(2.2.5) will present the equations of magnetoelastic vibrations of conducting very flat shells in a magnetic field.

When solving certain problems, both equations of the electrodynamics for the surroundings and surface conditions should be attached to Eqs. (2.2.3)–(2.2.5). In addition, the fixing conditions at the edges of the sell and conditions at infinity must also be added.

From the above-mentioned equations, the basic equations of magnetoelasticity for several types of shells can be obtained. For example, from Eqs. (2.2.3)–(2.2.5) for $A_1 = 1, A_2 = R$, the two-dimensional equations of the technical theory of thin cylindrical shells of the radius R, made of a material with finite electroconductivity, will be obtained.

2.3 Reduction of the Three-Dimensional Problem of Magnetoelasticity of Thin Plates to the Two-Dimensional One

In Sect. 2.1 (Two-dimensional equations of the magnetoelasticity of thin conducting plates), on the basis of the hypotheses of magnetoelasticity of thin bodies, the two-dimensional equations (2.1.13), (2.1.18), and (2.1.19) of the magnetoelasticity of thin plates were obtained. In these equations, the unknown boundary values of tangential components of the induced magnetic field h_1 and h_2 are included. Therefore, the obtained equations should be investigated together with the Maxwell equation (1.6.16) for the surroundings of the plate with the general boundary conditions (2.1.20) and (2.1.21) at the contact surface between the two media. Hence, the problem of magnetoelasticity is still three-dimensional. To reduce the three-dimensional problem to the two-dimensional one, it is necessary to add the additional relations, which will close system (2.1.13), (2.1.18), and (2.1.19). These relations are introduced having determined the introduced electromagnetic field in the external area. Here, when obtaining the additional relations, for the sake of simplicity the case of constant external magnetic field ($H_{0i} = \text{const}$) is examined. For this case, according to Eqs. (2.1.13), (2.1.18), and (2.1.19), the equations of magnetoelastic vibrations have the form:

$$\begin{aligned}
&\frac{\partial\psi}{\partial x_1} - \frac{\partial\varphi}{\partial x_2} + \frac{1}{c}\frac{\partial f}{\partial t} = 0,\\
&\frac{\partial f}{\partial x_1} + \frac{4\pi\bar{\sigma}}{c}\left[\psi + \frac{1}{c}\left(H_{01}\frac{\partial w}{\partial t} - H_{03}\frac{\partial u}{\partial t}\right)\right] = \frac{h_1^+ - h_1^-}{2h},\\
&\frac{\partial f}{\partial x_2} - \frac{4\pi\bar{\sigma}}{c}\left[\varphi - \frac{1}{c}\left(H_{02}\frac{\partial w}{\partial t} - H_{03}\frac{\partial v}{\partial t}\right)\right] = \frac{h_2^+ - h_2^-}{2h},
\end{aligned} \tag{2.3.1}$$

$$\begin{aligned}
&\frac{\partial^2 u}{\partial x_1^2} + \frac{1-\nu}{2}\frac{\partial^2 u}{\partial x_2^2} + \frac{1+\nu}{2}\frac{\partial^2 v}{\partial x_1 \partial x_2}\\
&\quad + \frac{1-\nu^2}{E}\frac{\sigma H_{03}}{c}\left(\psi - \frac{H_{03}}{c}\frac{\partial u}{\partial t} + \frac{H_{01}}{c}\frac{\partial w}{\partial t}\right) = \frac{\rho(1-\nu^2)}{E}\frac{\partial^2 u}{\partial t^2},\\
&\frac{\partial^2 v}{\partial x_2^2} + \frac{1-\nu}{2}\frac{\partial^2 v}{\partial x_1^2} + \frac{1+\nu}{2}\frac{\partial^2 u}{\partial x_1 \partial x_2}\\
&\quad - \frac{1-\nu^2}{E}\frac{\sigma H_{03}}{c}\left(\varphi + \frac{H_{03}}{c}\frac{\partial v}{\partial t} - \frac{H_{02}}{c}\frac{\partial w}{\partial t}\right) = \frac{\rho(1-\nu^2)}{E}\frac{\partial^2 v}{\partial t^2},
\end{aligned} \tag{2.3.2}$$

$$\begin{aligned}
&D\Delta^2 w + 2\rho h \frac{\partial^2 w}{\partial t^2} + 2\rho h \varepsilon \frac{\partial w}{\partial t} \\
&\quad + \frac{2\sigma h}{c}\left[H_{01}\left(\psi + \frac{H_{01}}{c}\frac{\partial w}{\partial t} - \frac{H_{03}}{c}\frac{\partial u}{\partial t}\right)\right. \\
&\quad \left. - H_{02}\left(\varphi - \frac{H_{02}}{c}\frac{\partial w}{\partial t} + \frac{H_{03}}{c}\frac{\partial v}{\partial t}\right)\right] \\
&\quad - \frac{2\sigma h^3 H_{03}^2}{3c^2}\frac{\partial \Delta w}{\partial t} = P(x_1, x_2, t).
\end{aligned} \tag{2.3.3}$$

In the first three equations $-\infty < x_1, x_2 < \infty$ and in the rest of equations $(x_1, x_2) \in Q$. In the last equation of system (2.3.5) ε is the damping coefficient in absence of magnetic field.

When system (2.3.5) was obtained, the surface conditions (1.6.17) were used, and it was assumed that the force, having nonelectromagnetic origin, has only the normal component $P(x_1, x_2, t)$.

Thus, the problem of magnetoelastic vibrations of the electroconducting isotropic plate in the external constant magnetic field is reduced to the joint solution of two-dimensional differential Eq. (2.3.5) and equations of electrodynamics in the areas $x_3 < -h$ and $x_3 > h$. In addition to the fixing conditions (1.6.17) of the plate's edges and conditions,

$$h_3^{(e)} = f(x_1, x_2, t), \quad e_1^{(e)} = \varphi(x_1, x_2, t), \quad e_2^{(e)} = \psi(x_1, x_2, t) \tag{2.3.4}$$

on the surfaces $x_3 = \pm h$ are the conditions of attenuation of perturbations at infinity

$$\lim_{x_1, x_2 \to \pm\infty} \varphi = \lim_{x_1, x_2 \to \pm\infty} \psi = \lim_{x_1, x_2 \to \pm\infty} f = 0. \tag{2.3.5}$$

To define all of the displacements and electromagnetic field in the plate, as Eqs. (2.3.1)–(2.3.3) show, it is necessary to calculate the components h_1 and h_2 of the induced magnetic field at the surfaces $x_3 < -h$ and $x_3 > h$. Let us calculate them solving the following quasi-static equations of electrodynamics for a vacuum:

$$\operatorname{rot} \mathbf{h}^{(\mathbf{e})} = 0, \quad \operatorname{div} \mathbf{h}^{(\mathbf{e})} = 0 \tag{2.3.6}$$

at the areas $x_3 < -h$ and $x_3 > h$ with the boundary conditions

$$h_3^{(e)}\Big|_{x_3 = \pm h} = f(x_1, x_2, t), \tag{2.3.7}$$

which are defined from Eq. (1.6.17) using (2.1.9).

Introducing the potential function $\Phi(x_1, x_2, t)$ in the form

$$\mathbf{h}^{(\mathbf{e})} = \nabla\Phi, \tag{2.3.8}$$

the problem of definition of magnetic field $\mathbf{h}^{(\mathbf{e})}$ out of the strip ($|x_3| > h$), according to Eqs. (2.3.6) and (2.3.7), is brought to the following Neumann problem in the half-spaces $x_3 < -h$ and $x_3 > h$:

$$\Delta\Phi = 0, \quad \left.\frac{\partial\Phi}{\partial x_3}\right|_{x_3=\pm h} = f(x_1, x_2, t). \tag{2.3.9}$$

Solution of problem (2.3.9) can be represented in the form of single-layer potential [74, 107]

$$\Phi = \mp\frac{1}{2\pi}\iint_{-\infty}^{\infty}\frac{f(\xi_1, \xi_2, t)\mathrm{d}\xi_1\mathrm{d}\xi_2}{\sqrt{(x_1 - \xi_1)^2 + (x_2 - \xi_2)^2 + (x_3 \mp h)^2}}. \tag{2.3.10}$$

In particular, when the vibrational form of the plate is a cylindrical surface $x_3 = w(x_1, t)$, then the solution to the Neumann problem can be presented by way of the logarithmic potential of a simple layer [74].

From Eq. (2.3.10), on the basis (2.3.8), one can find

$$h_i^{\pm} = h_i^{\pm(e)} = \mp\frac{1}{2\pi}\frac{\partial}{\partial x_i}\iint_{-\infty}^{\infty}\frac{f(\xi_1, \xi_2, t)\mathrm{d}\xi_1\mathrm{d}\xi_2}{\sqrt{(x_1 - \xi_1)^2 + (x_2 - \xi_2)^2}}(i = 1, 2). \tag{2.3.11}$$

Substituting Eq. (2.3.11) into Eqs. (2.3.1)–(2.3.3), we obtain the following closed system of two-dimensional singular integral-differential equations with the Cauchy-type kernel [10]:

$$\begin{aligned}
&\frac{\partial\psi}{\partial x_1} - \frac{\partial\varphi}{\partial x_2} + \frac{1}{c}\frac{\partial f}{\partial t} = 0,\\
&\frac{\partial F}{\partial x_1} + \frac{4\pi\bar{\sigma}}{c}\left[\psi + \frac{1}{c}\left(H_{01}\frac{\partial w}{\partial t} - H_{03}\frac{\partial u}{\partial t}\right)\right] = 0,\\
&\frac{\partial F}{\partial x_2} - \frac{4\pi\bar{\sigma}}{c}\left[\varphi - \frac{1}{c}\left(H_{02}\frac{\partial w}{\partial t} - H_{03}\frac{\partial v}{\partial t}\right)\right] = 0,\\
&\frac{\partial^2 u}{\partial x_1^2} + \frac{1-\nu}{2}\frac{\partial^2 u}{\partial x_2^2} + \frac{1+\nu}{2}\frac{\partial^2 v}{\partial x_1\partial x_2}\\
&\qquad + \frac{1-\nu^2}{E}\frac{\sigma H_{03}}{c}\left(\psi - \frac{H_{03}}{c}\frac{\partial u}{\partial t} + \frac{H_{01}}{c}\frac{\partial w}{\partial t}\right) = \frac{\rho(1-\nu^2)}{E}\frac{\partial^2 u}{\partial t^2},
\end{aligned}$$

$$
\begin{aligned}
&\frac{\partial^2 v}{\partial x_2^2}+\frac{1-\nu}{2}\frac{\partial^2 v}{\partial x_1^2}+\frac{1+\nu}{2}\frac{\partial^2 u}{\partial x_1 \partial x_2}\\
&\quad-\frac{1-\nu^2}{E}\frac{\sigma H_{03}}{c}\left(\varphi+\frac{H_{03}}{c}\frac{\partial v}{\partial t}-\frac{H_{02}}{c}\frac{\partial w}{\partial t}\right)=\frac{\rho(1-\nu^2)}{E}\frac{\partial^2 v}{\partial t^2},\\
&D\Delta^2 w+2\rho h\frac{\partial^2 w}{\partial t^2}+2\rho h\varepsilon\frac{\partial w}{\partial t}\\
&\quad+\frac{2\sigma h}{c}\left[H_{01}\left(\psi+\frac{H_{01}}{c}\frac{\partial w}{\partial t}-\frac{H_{03}}{c}\frac{\partial u}{\partial t}\right)\right.\\
&\quad\left.-H_{02}\left(\varphi-\frac{H_{02}}{c}\frac{\partial w}{\partial t}+\frac{H_{03}}{c}\frac{\partial v}{\partial t}\right)\right]\\
&\quad-\frac{2\sigma h^3}{3c^2}H_{03}^2\frac{\partial^2 \Delta w}{\partial t^2}=P(x_1,x_2,t).
\end{aligned}
\tag{2.3.12}
$$

In the first three equations $-\infty < x_1, x_2 < \infty$, and in the rest of equations $(x_1, x_2) \in Q$. In Eq. (2.3.12), the following notation is performed:

$$F=f+\frac{1}{2\pi h}\iint_{-\infty}^{\infty}\frac{f(\xi_1,\xi_2,t)\mathrm{d}\xi_1\mathrm{d}\xi_2}{\sqrt{(x_1-\xi_1)^2+(x_2-\xi_2)^2}}.$$

Thus, the problem of magnetoelastic vibrations of the plate is brought to the solution of the two-dimensional Eqs. (2.3.12) with the usual fixing conditions at the edges of the plate and conditions at infinity:

$$\lim_{x_1,x_2\to\pm\infty}\varphi=\lim_{x_1,x_2\to\pm\infty}\psi=\lim_{x_1,x_2\to\pm\infty}f=0.$$

In the case when the plate-strip of the length $2a$ is placed in an external constant magnetic field $\mathbf{H_0}(H_{01},0,0)$ and perturbations do not depend on the coordinate x_2, the following system of singular integral-differential equations is obtained with respect to the unknown functions ψ, f, w [10]:

$$
\begin{aligned}
&\frac{\partial\psi}{\partial x_1}+\frac{1}{c}\frac{\partial f}{\partial t}=0,\\
&\frac{\partial f}{\partial x_1}+\frac{4\pi\bar{\sigma}}{c}\left(\psi+\frac{H_{01}}{c}\frac{\partial w}{\partial t}\right)=\frac{1}{\pi h}\int_{-\infty}^{\infty}\frac{f(\xi_1,t)}{x_1-\xi_1}\mathrm{d}\xi_1;\\
&D\frac{\partial^4 w}{\partial x_1^4}+2\rho h\frac{\partial^2 w}{\partial t^2}+2\rho h\varepsilon\frac{\partial w}{\partial t}\\
&\quad+\frac{2\sigma h H_{01}}{c}\left(\psi+\frac{H_{01}}{c}\frac{\partial w}{\partial t}\right)=P(x_1,t).
\end{aligned}
\tag{2.3.13}
$$

where in the first two equations $-\infty < x_1 < \infty$, in the third equation $|x_1| < a$, and

$$\bar{\sigma}(x_1) = \begin{cases} \sigma & \text{for} \quad |x_1| \le a, \\ 0 & \text{for} \quad |x_1| > a \end{cases} \tag{2.3.14}$$

In Eq. (2.3.12), in addition to the components u, v, w of displacement vector of the middle plane of the plate, the components φ, ψ, f of the induced electromagnetic field are included. Below, with the help of Fourier integral transformations, the values of φ, ψ, f are calculated expressed by way of the main unknowns u, v, w. On thid basis, the problem of magnetoelastic vibrations of thin plates is addressed as a dynamic boundary value problem with respect to functions u, v, w in the area Q.

Having eliminated functions φ and ψ from the second, third, and the last equation of system (2.3.12), it is easy to obtain the following equation with respect to function f:

$$\left(H_{01}\frac{\partial}{\partial x_1} + H_{02}\frac{\partial}{\partial x_2}\right)\left(f + \frac{1}{2\pi h}\iint_{-\infty}^{\infty}\frac{f}{r}\mathrm{d}\xi_1\mathrm{d}\xi_2\right) = \frac{2\pi\gamma}{h}W \tag{2.3.15}$$

where

$$\begin{aligned} &r^2 = (x_1 - \xi_1)^2 + (x_2 - \xi_2)^2, \\ &\gamma = \begin{cases} 1 & \text{for} \quad (x_1, x_2) \notin Q \\ 0 & \text{for} \quad (x_1, x_2) \notin Q, \end{cases} \\ &W = D\Delta^2 w + 2\rho h\frac{\partial^2 w}{\partial t^2} - \frac{2\sigma h^3}{3c^2}\frac{\partial \Delta w}{\partial t}H_{03}^2 - P. \end{aligned} \tag{2.3.16}$$

Applying the two-dimensional Fourier exponential transformation with respect to variables x_1 and x_2, taking into account (2.3.16) and conditions of attenuation of perturbations at infinity ($f \to 0$ for $x_1 \to \pm\infty, x_2 \to \pm\infty$), we obtain

$$\begin{aligned} &\iint_{-\infty}^{\infty} f\,\mathrm{e}^{i(\alpha x_1 + \beta x_2)}\mathrm{d}x_1\mathrm{d}x_2 \\ &\quad = \frac{2\pi i\sqrt{\alpha^2 + \beta^2}}{\left(1 + h\sqrt{\alpha^2 + \beta^2}\right)(H_{01}\alpha + H_{02}\beta)}\iint_Q W\,\mathrm{e}^{i(\alpha x_1 + \beta x_2)}\mathrm{d}x_1\mathrm{d}x_2 \end{aligned}$$

from which, according to the Fourier inverse transformations, it follows

$$\begin{aligned} f(x_1, x_2, t) &= \frac{1}{2\pi}\iint_Q W(\xi_1, \xi_2, t)\,K_1(x_1, x_2; \xi_1, \xi_2)\mathrm{d}\xi_1\mathrm{d}\xi_2 \\ K_1 &= \frac{1}{2\pi i}\iint_{-\infty}^{\infty}\frac{\mathrm{e}^{i[\alpha(x_1-\xi_1)+\beta(x_2-\xi_2)]}}{1 + h\sqrt{\alpha^2 + \beta^2}}\frac{\sqrt{\alpha^2 + \beta^2}}{H\alpha_{01}\alpha + H_{02}\beta}\mathrm{d}\alpha\,\mathrm{d}\beta. \end{aligned} \tag{2.3.17}$$

Substituting (2.3.17) into the second and third equations of system (2.3.12), functions φ and ψ are found in the area Q:

$$\begin{aligned} \frac{4\pi\sigma}{c}\varphi &= \frac{4\pi\sigma}{c^2}\left(H_{02}\frac{\partial w}{\partial t} - H_{03}\frac{\partial v}{\partial t}\right) \\ &\quad + \frac{\partial}{\partial x_2}\left(f + \frac{1}{2\pi h}\iint_{-\infty}^{\infty}\frac{f}{r}\mathrm{d}\xi_1\mathrm{d}\xi_2\right), \\ \frac{4\pi\sigma}{c}\psi &= \frac{4\pi\sigma}{c^2}\left(H_{03}\frac{\partial u}{\partial t} - H_{01}\frac{\partial w}{\partial t}\right) \\ &\quad - \frac{\partial}{\partial x_1}\left(f + \frac{1}{2\pi h}\iint_{-\infty}^{\infty}\frac{f}{r}\mathrm{d}\xi_1\mathrm{d}\xi_2\right). \end{aligned} \tag{2.3.18}$$

Substituting now (2.3.17) and (2.3.18) into the as-yet-unused equations (first, fourth, and fifth) of system (2.3.12), the following final system of integral-differential equations with respect to functions u, v, w will be obtained in the area Q:

$$\begin{aligned} &\Delta\omega - \frac{1}{c^2}\frac{\partial^2\omega}{\partial t^2} = 0, \\ &\Delta e - \frac{1}{c^2}\frac{\partial^2 e}{\partial t^2} - \frac{\sigma H_{03}^2}{c_l^2 c^2}\frac{\partial e}{\partial t} \\ &\quad = \frac{\sigma H_{03}}{c_l^2 c^2}\frac{\partial}{\partial t}\left[H_{01}\frac{\partial w}{\partial x_1} + H_{02}\frac{\partial w}{\partial x_2} + \iint_Q W K_1 \mathrm{d}\xi_1\mathrm{d}\xi_2\right], \\ &\Delta W - \frac{4\pi\sigma}{c^2}\iint_Q \frac{\partial W}{\partial t} K_1(x_1, x_2; \xi_1, \xi_2)\mathrm{d}\xi_1\mathrm{d}\xi_2 \\ &\quad + \frac{2\sigma h}{c^2}\frac{\partial}{\partial t}\left[H_{01}^2\frac{\partial^2 w}{\partial x_1^2} + 2H_{01}H_{02}\frac{\partial^2 w}{\partial x_1\partial x_2}\right. \\ &\quad \left. + H_{02}^2\frac{\partial^2 w}{\partial x_2^2} - H_{03}\left(H_{01}\frac{\partial e}{\partial x_1} + H_{02}\frac{\partial e}{\partial x_2}\right)\right], \end{aligned} \tag{2.3.19}$$

where

$$\begin{aligned} &\omega = \frac{\partial v}{\partial x_1} - \frac{\partial u}{\partial x_2}, \quad e = \frac{\partial u}{\partial x_1} + \frac{\partial v}{\partial x_2}, \\ &c_t^2 = \frac{E}{2\rho(1+\nu)}, \quad c_l^2 = \frac{E}{\rho(1-\nu^2)}, \\ &K = \delta(x_1 - \xi_1)\delta(x_2 - \xi_2) \\ &\quad - \frac{1}{\pi^2}\iint_0^{\infty}\frac{\cos\alpha(x_1-\xi_1)\cos\beta(x_2-\xi_2)}{1 + h\sqrt{\alpha^2+\beta^2}}\mathrm{d}\alpha\mathrm{d}\beta, \end{aligned} \tag{2.3.20}$$

$\delta(x_2 - \xi_2)$ is the Dirac function.

System (2.3.19) shows the following: (1) shear waves are propagated independently and the presence of magnetic field does not affect the characteristics of their propagation; (2) dilatation waves are connected with the wave of transversal vibrations and are propagated with attenuation, which is proportional to the transversal component of the external magnetic field; (3) the plane problem is merged with the problem of bending vibrations; and (4) if the given magnetic field is a longitudinal ($H_{03} = 0$), then, the noted waves are propagated independently, and the problem of longitudinal vibrations is split from the problem of transversal vibrations. Moreover, for longitudinal vibrations, the usual equations of the theory of elasticity take place, and for transversal vibrations, the following integral-differential equation is obtained with respect to the plate's deflection w:

$$\Delta W_1 - \frac{4\pi\sigma}{c^2}\iint_Q \frac{\partial W_1}{\partial t} K \mathrm{d}\xi_1 \mathrm{d}\xi_2 + \frac{2\sigma h}{c^2}\frac{\partial}{\partial t}\left[H_{01}^2 \frac{\partial^2 w}{\partial x_1^2} + 2H_{01}H_{02}\frac{\partial^2 w}{\partial x_1 \partial x_2} + H_{02}^2 \frac{\partial^2 w}{\partial x_2^2}\right], \tag{2.3.21}$$

where

$$W_1 = D\Delta^2 w + 2\rho h \frac{\partial^2 w}{\partial t^2} - P.$$

Thus, the problem of transversal vibrations of conducting plates in a longitudinal magnetic field is brought to the solution of Eq. (2.3.21) with the usual boundary conditions for the function $w(x_1, x_2, t)$ and with the condition

$$\left(\mathbf{e} + \frac{1}{c}\frac{\partial \mathbf{u}}{\partial t} \times \mathbf{H_0}\right) \cdot \mathbf{N_0} = 0 \tag{2.3.22}$$

on the lateral surface of the plate. In (2.3.22), the vector $\mathbf{e}$ is the intensity of the induced electric field.

At the end, let us present one more expression for the kernel K of (2.3.21), which was obtained from (2.3.20) using the integral presentations of cylindrical functions:

$$K = \frac{1}{4h^2}\left[H_0\left(\frac{r}{h}\right) - Y_0\left(\frac{r}{h}\right)\right] - \frac{1}{2\pi hr}, \tag{2.3.23}$$

where H_0 is the Stroofie function, and Y_0 is the Bessel function of the second order.

The work [45] is also devoted to the reduction of the three-dimensional problem of magnetoelasticity of thin plates to the two-dimensional problem. In this chapter, equations with respect to the boundary values of the induced magnetic field are obtained, and these equations close the two-dimensional system of magnetoelasticity of thin plates.

2.4 Reduction of the Three-Dimensional Problem of Magnetoelasticity of Cylindrical Shells to the Two-Dimensional One

One of the basic difficulties within the reduction of three-dimensional problem of magnetoelasticity of cylindrical shells to the two-dimensional problem is—calculation of the boundary values of tangential components of the induced magnetic field on the shell's surface including in Eqs. (2.2.3)–(2.2.5). In this paragraph, such as to the case of thin plates, the noted boundary values are calculated in a quasi-static approximation and on the basis of them the closed two-dimensional system is obtained characterizing the behavior of conducting thin cylindrical shells in a stationary magnetic field.

Let us consider a thin isotropic electroconducting elastic cylindrical medium of an open structure referring to the cylindrical system of coordinates (x, r, θ) (Fig. 2.3). In the chosen system of coordinates the considered medium occupies the area G_0 ($G_0: -l \le x \le l, R-h \le r \le R+h, -\theta_0 \le \theta \le \theta_0$, where $2l$ is the length, $2h$ is the thickness, R is the radius of the middle surface of the shell, θ_0 is the opening of the shell).

In addition to this area, let us also consider the area G $(G: -\infty < x < \infty, R-h \le r \le R+h, -\pi \le \theta \le \pi)$ characterizing the infinitely long closed cylindrical layer with the thickness $2h$.

Analogous to the case of thin plates here, we will assume that electrodynamic part of hypotheses of magnetoelasticity of thin bodies takes place in the whole cylindrical strip G, i.e., instead of Eqs. (2.2.1) and (2.2.2), the following relations are accepted [38, 39]:

$$\left.u_x = u - \gamma\frac{\partial w}{\partial x}, u_\theta = v - \frac{\gamma}{R}\frac{\partial w}{\partial \theta}, u_r = w(x,\theta,t)\right\} \quad \text{for} \quad (x,r,\theta)\in G_0,$$
$$\left.e_x = \varphi(x,\theta,t), e_\theta = \psi(x,\theta,t), h_r = f(x,\theta,t)\right\} \quad \text{for} \quad (x,r,\theta)\in G. \tag{2.4.1}$$

where $\gamma = r - R$.

For the rest of components h_x and h_θ of the induced electromagnetic field in the cylindrical strip G magnetic field from Eqs. (1.6.15) and (1.6.16) written with respect to the cylindrical coordinates [using Eqs. (1.1.13)–(1.1.20)], having

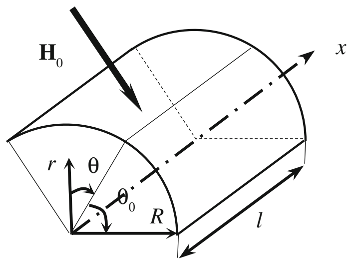

Fig. 2.3 Geometrical interpretation of the problem. Cylindrical medium of an open structure in a magnetic field

integrated with respect to γ in the range from 0 to γ, in account of (2.4.1) and continuity conditions for h_x and h_θ at the surface $\gamma = \pm h$ we obtain

$$\begin{aligned} h_x &= \frac{h_x^+ + h_x^-}{2} + \gamma\left(\frac{\partial f}{\partial x} + \frac{4\pi\bar{\sigma}}{c}\psi\right) \\ &\quad + \frac{4\pi\bar{\sigma}}{c}\left(a_{x0}\frac{\partial w}{\partial t} - a_{r0}\frac{\partial u}{\partial t} + a_{r1}\frac{\partial^2 w}{\partial x \partial t}\right), \\ h_\theta &= \frac{h_\theta^+ + h_\theta^-}{2} + \gamma\left(\frac{1}{R}\frac{\partial f}{\partial \theta} - \frac{4\pi\bar{\sigma}}{c}\varphi\right) \\ &\quad + \frac{4\pi\bar{\sigma}}{c}\left(a_{\theta 0}\frac{\partial w}{\partial t} - a_{r0}\frac{\partial v}{\partial t} + \frac{a_{r1}}{R}\frac{\partial^2 w}{\partial \theta \partial t}\right), \end{aligned} \tag{2.4.2}$$

where the following notations are performed

$$a_{ik} = \int_0^\gamma \gamma^k H_{0i} \mathrm{d}\gamma - \frac{1}{2}\left(\int_0^h \gamma^k H_{0i}\mathrm{d}\gamma + \int_0^{-h} \gamma^k H_{0i}\mathrm{d}\gamma\right),$$

$$\bar{\sigma}(x,\theta) = \begin{cases} \sigma & \text{for} \quad -\theta_0 \le \theta \le \theta_0,\ |x| \le l, \\ & \qquad\quad \theta_0 < \theta \le \pi,\ |x| < l, \\ 0 & \text{for} \quad -\pi \le \theta < -\theta_0,\ |x| < l, \\ & \qquad\quad -\pi \le \theta \le \pi,\ |x| > l, \end{cases}$$

$$(i = x, r, \theta;\ k = 0, 1, 2).$$

Indices "+" and "−" correspond to the appropriate quantities at $\gamma = h$ and $\gamma = -h$.

When obtaining the Eq. (2.4.2) the condition was taken into account that the normal component of the density of conductivity current is equal to zero at the surface $\gamma = \pm h$ (as the shell places in the vacuum).

Doing the above-mentioned operations with respect to Eqs. (1.6.15) and (1.6.16) in addition to the relation (2.4.2) the following differential equations are also obtained with respect to the unknown functions $u, v, w, \varphi, \psi, f$

$$\begin{aligned} &\frac{\partial \psi}{\partial x} - \frac{1}{R}\frac{\partial \varphi}{\partial \theta} + \frac{1}{c}\frac{\partial f}{\partial t} = 0, \\ &\frac{\partial f}{\partial x} + \frac{4\pi\bar{\sigma}}{c}\left[\psi + \frac{1}{2hc}\left(b_{x0}\frac{\partial w}{\partial t} - b_{r0}\frac{\partial u}{\partial t} + b_{r1}\frac{\partial^2 w}{\partial x \partial t}\right)\right] = \frac{h_x^+ - h_x^-}{2h}, \\ &\frac{1}{R}\frac{\partial f}{\partial \theta} + \frac{4\pi\bar{\sigma}}{c}\left[\varphi + \frac{1}{2hc}\left(-b_{\theta 0}\frac{\partial w}{\partial t} + b_{r0}\frac{\partial v}{\partial t} - b_{r1}\frac{1}{R}\frac{\partial^2 w}{\partial \theta \partial t}\right)\right] = \frac{h_\theta^+ - h_\theta^-}{2h}, \end{aligned} \tag{2.4.3}$$

where

$$b_{ik} = \int_{-h}^{h} \gamma^k H_{0i} \mathrm{d}\gamma \quad (i = x, r, \theta;\ k = 0, 1, 2).$$

Substituting (2.4.1) and (2.4.2) into the rest of equations of system (1.6.15) and averaging the obtained equations along the thickness of the shell the following system of differential equations is obtained with respect to the unknown functions [38, 39]:

$$\begin{aligned}
&\frac{\partial^2 u}{\partial x^2} + \frac{1-\nu}{2R^2}\frac{\partial^2 u}{\partial \theta^2} + \frac{1+\nu}{2R}\frac{\partial^2 v}{\partial x \partial \theta} + \frac{\nu}{R}\frac{\partial w}{\partial x} \\
&\quad = \frac{\rho(1-\nu^2)}{E}\frac{\partial^2 u}{\partial t^2} + L_1, \\
&\frac{1}{R^2}\frac{\partial^2 v}{\partial \theta^2} + \frac{1-\nu}{2}\frac{\partial^2 v}{\partial x^2} + \frac{1+\nu}{2R}\frac{\partial^2 u}{\partial x \partial \theta} + \frac{1}{R^2}\frac{\partial w}{\partial x} \\
&\quad = \frac{\rho(1-\nu^2)}{E}\frac{\partial^2 v}{\partial t^2} + L_2, \\
&D\Delta^2 w + 2\rho h \frac{\partial^2 w}{\partial t^2} + \frac{2Eh}{(1-\nu^2)R}\left(\frac{1}{R}\frac{\partial v}{\partial \theta} + \nu \frac{\partial u}{\partial x} + \frac{w}{R}\right) = L_3.
\end{aligned} \tag{2.4.4}$$

In (2.4.4) the operators L_i of magnetic origin characterize the influence of the induced electromagnetic field on the vibrations of the shell and have the following form:

$$\begin{aligned}
L_1 = -\frac{1-\nu^2}{2Eh}\frac{\sigma}{c}\Bigg[& b_{r0}\psi + b_{\theta 1}\left(\frac{\partial \varphi}{\partial x} + \frac{1}{R}\frac{\partial \psi}{\partial \theta}\right) + \frac{d_{r\theta}^{(0)}}{c}\frac{\partial}{\partial t}\left(\frac{\partial v}{\partial x} - \frac{1}{R}\frac{\partial u}{\partial \theta}\right) \\
& + \frac{C_{xr}^{(0)}}{c}\frac{\partial w}{\partial t} - \frac{1}{c}\left(F_{\theta\theta}^{0}\frac{\partial u}{\partial t} - F_{x\theta}^{0}\frac{\partial v}{\partial t}\right) - \frac{1}{c}\frac{\partial}{\partial t}\left(G_{\theta\theta}^{(0)}\frac{\partial w}{\partial x} - \frac{G_{x\theta}^{0}}{R}\frac{\partial w}{\partial \theta}\right)\Bigg];
\end{aligned}$$

The operator L_2 turns out from cyclic rearrangement $(x \to \theta, \theta \to x, r = r, u \to v, v \to u, w = w, \varphi \to -\psi, \psi \to -\varphi$; the top indexes are saved):

$$\begin{aligned}
L_3 = -\frac{\sigma}{c}\Bigg\{ & b_{x0}\psi + b_{\theta 1}\varphi - \frac{1}{c}\Bigg[C_{xr}^{(0)}\frac{\partial u}{\partial t} + C_{r\theta}^{(0)}\frac{\partial v}{\partial t} \\
& - \left(C_{xx}^{(0)} + C_{\theta\theta}^{(0)} - \frac{\partial c_{xr}^{(1)}}{\partial x} - \frac{1}{R}\frac{\partial C_{r\theta}^{(1)}}{\partial \theta}\right)\frac{\partial w}{\partial t}\Bigg] + \frac{\partial A_1}{\partial x} + \frac{\partial A_2}{\partial \theta}\Bigg\},
\end{aligned}$$

where

$$A_1 = -b_{r1}\psi - b_{\theta 2}\left(\frac{\partial \phi}{\partial x} + \frac{1}{R}\frac{\partial \psi}{\partial \theta}\right) + \frac{1}{c}\frac{\partial}{\partial t}\left[F_{\theta\theta}^{(1)}u - F_{\theta x}^{(1)}v\right.$$
$$\left. + G_{\theta\theta}^{(1)}\frac{\partial w}{\partial t} - \frac{G_{x\theta}^{(1)}}{R}\frac{\partial w}{\partial \theta} + d_{x\theta}^{(1)}\frac{\partial}{\partial t}\left(\frac{1}{R}\frac{\partial u}{\partial \theta} - \frac{\partial v}{\partial x}\right)\right.;$$

and A_2 turns out from the A_1 by way of above-mentioned cyclic rearrangement.

The coefficients of operators L_i are determined in the following form:

$$C_{ij}^{(k)} = \int_{-h}^{h} \gamma^k H_{0i} H_{oj} \mathrm{d}\gamma, \quad d_{ij}^{(k)} = \int_{-h}^{h} \gamma^k H_{oj} a_{oi} \mathrm{d}\gamma,$$
$$G_{ij}^{(k)} = 2d_{ij}^{(k)} - C_{rr}^{(k+1)}\delta_{ij} - C_{ij}^{(k+1)} + hC_i^- b_{jk},$$
$$F_{ij}^{(k)} = C_{ij}^{(k)} + C_{rr}^{(k)}\delta_{ij} - b_{ik}C_j^+,$$
$$C_i^+ = \frac{H_{0i}^+ + H_{0i}^-}{2}, \quad C_i^- = \frac{H_{0i}^+ - H_{0i}^-}{2}.$$

Thus, the problem of magnetoelastic vibrations of electroconducting isotropic cylindrical shell of an open structure in an external stationary magnetic field is reduced to the joint solution of two-dimensional differential equations (2.4.3) in the area $(-\pi \le \theta \le \pi, -\infty < x < \infty)$ and to the (2.4.4) in the area $(-\theta_0 \le \theta \le \theta_0, -l < x < l)$ and also to the equations of electrodynamics in the areas $r > R + h$ and $0 < r < R - h$. In addition to the usual fixing conditions of the shell and continuity conditions of quantities e_x, e_θ and h_r on the surfaces $\gamma = \pm h$ the boundary conditions for the problem are also the conditions of attenuation of perturbations at infinity in the area $\gamma > h$ and conditions of limitation of perturbations in the area $\gamma < -h$.

For the complete definition of displacements of points of the shell in the area G_0 and induced electromagnetic field in the whole space, as it is show Eqs. (2.4.3) and (2.4.4), it is necessary to have the values of components h_x and h_θ of the induced magnetic field on the surfaces $\gamma = \pm h$.

Let us define them having solved the equations

$$\mathrm{rot}\,\mathbf{h}^{(\mathbf{e})} = 0, \quad \mathrm{div}\,\mathbf{h}^{(\mathbf{e})} = 0 \tag{2.4.5}$$

in the areas $|\gamma| > h$ for the following boundary conditions:

$$h_r^{(e)}\big|_{\gamma=\pm h} = f(x, \theta, t), \tag{2.4.6}$$

where the index "e" corresponds to the area $|\gamma| > h$, moreover e = 1 corresponds to the area $\gamma > h$, and e = 2 to the area $\gamma < -h$.

Introducing the potential function $\Phi^{(e)}$ by way of the form

$$\mathbf{h}^{(\mathbf{e})} = \operatorname{grad} \Phi^{(e)} \tag{2.4.7}$$

the definition of $\mathbf{h}^{(\mathbf{e})}$ in account of Eqs. (2.4.5) and (2.4.6) is brought to the solution of the following Neumann problems in the areas $|\gamma| > h$:

$$\Delta\Phi^{(e)} = 0, \quad \left.\frac{\partial \Phi^{(e)}}{\partial r}\right|_{\gamma=\pm h} = f(x, \theta, t). \tag{2.4.8}$$

Representing the unknown functions in the form

$$\begin{aligned} f(x, \theta, t) &= \sum_{n=0}^{\infty} [f_{n1}(x, t)\sin n\theta + f_{n2}(x, t)\cos n\theta], \\ \Phi^{(e)}(x, r, \theta, t) &= \sum_{n=0}^{\infty} \left[\Phi_{n1}^{(e)}(x, r, t)\sin n\theta + \Phi_{n1}^{(e)}(x, r, t)\cos n\theta\right], \end{aligned} \tag{2.4.9}$$

and applying the Fourier transformation [74, 117] with respect to x, using the conditions at infinity, the problem (2.4.8) is brought to the following problems:

$$\frac{\mathrm{d}^2\bar{\Phi}_{ni}^{(e)}}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}\bar{\Phi}_{ni}^{(e)}}{\mathrm{d}r} - \left(\alpha^2 + \frac{n^2}{r^2}\right)\bar{\Phi}_{ni}^{(e)} = 0, \tag{2.4.10}$$

$$\left.\frac{\mathrm{d}\bar{\Phi}_{ni}^{(e)}}{\mathrm{d}r}\right|_{r=R\pm h} = \bar{f}_{ni}(\alpha, t), \tag{2.4.11}$$

where

$$\begin{aligned} \bar{\Phi}_{ni}^{(e)} &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \Phi_{ni}^{(e)}(x, r, t)\mathrm{e}^{i\alpha x}\mathrm{d}x, \\ \bar{f}_{ni} &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f_{ni}(x, t)\mathrm{e}^{i\alpha x}\mathrm{d}x. \end{aligned} \tag{2.4.12}$$

The common solution of the Eq. (2.4.10) has the form [74]

$$\bar{\Phi}_{ni}^{(e)} = A_{ni}^{(e)}(t)K_n(|\alpha|r) + B_{ni}^{(e)}(t)I_n(|\alpha|r), \tag{2.4.13}$$

where I_n, K_n are Bessel functions of pure imaginary argument of the order n.

Because the function I_n increases rapidly for $r \to \infty$ and the function K_n has singularity at the origin of coordinates, so $A_{ni}^{(2)} = B_{ni}^{(1)} = 0$. Satisfying the boundary conditions (2.4.11) let us determine the rest of integration constants and, hence, the functions $\bar{\Phi}_{ni}^{(e)}$.

Applying the transverse Fourier transformation for the originals $\Phi_{ni}^{(e)}$ we'll find the following expressions:

$$\Phi_{ni}^{(1)} = \frac{1}{\pi}\int\limits_{-\infty}^{\infty} f_{ni}(s,t)\mathrm{d}s \int\limits_{0}^{\infty} \frac{K_n(\alpha r)\cos\alpha(s-x)}{\alpha K_n'(\alpha r)}\mathrm{d}\alpha,$$
$$\Phi_{ni}^{(2)} = \frac{1}{\pi}\int\limits_{-\infty}^{\infty} f_{ni}(s,t)\mathrm{d}s \int\limits_{0}^{\infty} \frac{I_n(\alpha r)\cos\alpha(s-x)}{\alpha I_n'(\alpha r)}\mathrm{d}\alpha, \qquad (2.4.14)$$
$$g'(p) = \frac{\mathrm{d}g}{\mathrm{d}p}.$$

Substituting (2.4.14) into the (2.4.9) one can find $\Phi^{(e)}$ and by way of it from the Eq. (2.4.7) the induced magnetic field $\mathbf{h}^{(\mathbf{e})}$ can be calculated. From the found expression for $\mathbf{h}^{(\mathbf{e})}$ the following combinations of boundary values are obtained:

$$h_x^+ - h_x^- = \frac{1}{\pi R}\sum_{n=0}^{\infty}\int\limits_{-\infty}^{\infty} F_n(s,\theta,t)\mathrm{d}s \int\limits_{0}^{\infty}\frac{\partial R_n}{\partial x}\mathrm{d}x;$$
$$h_\theta^+ - h_\theta^- = \frac{1}{\pi R^2}\sum_{n=0}^{\infty} n \int\limits_{-\infty}^{\infty}\frac{\partial F_n}{\partial \theta}\mathrm{d}s \int\limits_{0}^{\infty} R_n(s-x)\mathrm{d}x; \qquad (2.4.15)$$

where

$$R_n(s-x) = \int\limits_{0}^{\infty}\frac{\cos\alpha(s-x)}{\alpha^2 K_n'(\alpha r) I_n'(\alpha r)}\mathrm{d}\alpha,$$
$$F_n = f_{n1}(s,t)\sin n\theta + f_{n2}(s,t)\cos n\theta.$$

Substituting Eq. (2.4.15) into system (2.4.3) and joining to it Eq. (2.4.4), the resolution system with respect to unknown functions $u, v, w, \varphi, \psi, f$ is obtained. Thus, the problem of magnetoelastic vibrations of cylindrical panel is brought to the solution of singular integral-differential equations with the usual fixing conditions of shell's edges and for $\lim\limits_{x\to\pm\infty}\varphi = \lim\limits_{x\to\pm\infty}\psi = \lim\limits_{x\to\pm\infty} f = 0$.

Taking $\theta_0 = \pi$ from the results of this paragraph one can obtain the basic equations and relations of magnetoelastic vibrations of a closed cylindrical shell [38].

The brought system can be simplified essentially in some particular cases. Let us bring this system for two cases only: (a) the case of axisymmetric problem when the shell is placed in the constant external magnetic field $\mathbf{H_0}(H_0, 0, 0)$ directed along the axis x; (b) the case of magnetic field of constant linear current $\mathbf{J}(J, 0, 0)$ directed along the axis of the cylinder of opened profile, when perturbations are depend on the coordinate x. In the case a) we have [38]

$$
\begin{aligned}
&\frac{\partial^2\psi}{\partial x^2} - \frac{4\pi\bar{\sigma}}{c^2}\frac{\partial}{\partial t}\left(\psi + \frac{H_0}{c}\frac{\partial w}{\partial t}\right) = \frac{1}{2\pi Rh}\int\limits_{-\infty}^{\infty}\frac{\partial\psi}{\partial s}K(s,x)\mathrm{d}x \\
&\quad(-\infty < x < \infty), \\
&D\frac{\partial^4 w}{\partial x^4} + \frac{2Eh}{R^2}w + 2\rho h\frac{\partial^2 w}{\partial t^2} + \frac{2\sigma h H_0}{c}\left(\psi + \frac{H_0}{c}\frac{\partial w}{\partial t}\right) = 0 \\
&\quad(|x| < l),
\end{aligned}
\tag{2.4.16}
$$

where the kernel $K(s,x)$ is calculated by way of the formula

$$
K(s,x) = \int\limits_0^{\infty}\frac{\sin\alpha(x-s)}{\alpha I_1(\alpha R)K_1(\alpha R)}\mathrm{d}\alpha;
$$

In the case (b) we have [39]

$$
\begin{aligned}
&-\frac{1}{R}\frac{\partial\varphi}{\partial\theta} + \frac{1}{c}\frac{\partial f}{\partial t} = 0, \\
&\frac{1}{R}\frac{\partial f}{\partial\theta} - \frac{4\pi\bar{\sigma}}{c}\left(\varphi - \frac{H_\theta}{c}\frac{\partial w}{\partial t}\right) = \frac{1}{2\pi h}\int\limits_{-\infty}^{\infty} f(\xi, t)\mathrm{ctg}\frac{\theta - \xi}{2}\mathrm{d}\xi \\
&\quad(-\pi < x < \pi), \\
&\frac{D}{R^4}\left(\frac{\partial^4 w}{\partial\theta^4} + 2\frac{\partial^2 w}{\partial\theta^2} + w\right) + 2\rho h\frac{\partial^2 w}{\partial t^2} \\
&-\frac{2\sigma h H_\theta}{c}\left(\varphi - \frac{H_\theta}{c}\frac{\partial w}{\partial t}\right) = 0 \\
&\quad(-\theta_0 < x < \theta_0),
\end{aligned}
\tag{2.4.17}
$$

where

$$
H_\theta = \frac{2J}{cR}, \quad \bar{\sigma}(\theta) = \begin{cases} \sigma & \text{for} \quad -\theta_0 \le \theta \le \theta_0, \\ 0 & \text{for} \quad -\pi \le \theta < -\theta_0, \\ & \qquad \theta_0 < \theta \le \pi. \end{cases}
$$

2.5 Two-Dimensional Equations of Magnetoelasticity of Thin Spherical Shells

Let the isotropic closed spherical shell of constant thickness $2h$ and radius R of the middle surface made of a material with the finite electroconductivity σ is placed in the stationary magnetic field.

The shell is immersed in the orthogonal coordinate system $(\alpha_1, \alpha_2, \alpha_3)$, plate's middle plane coincides with the plane α_1, α_2 (α_1 is the angle of width, α_2 is the angle of length), α_3 is directed along the normal to the middle plane (Fig. 2.4). Then for the coefficients of the first quadratic form and for the curvature of the middle plane we'll have $A_1 = R, A_2 = R\sin\alpha_1, k_1 = k_2 = R^{-1}$.

The problem is solved in an assumption that for the shell's surroundings the Maxwell equations for the vacuum are true. It is assumed also that the influence of displacement currents on the characteristics of elastic vibrations can be neglected.

From Eqs. (1.6.15) and (1.6.16) in account of (1.1.13)–(1.1.20) and accepted assumptions the following equations are obtained for the examined case:

(a) equations of magnetoelasticity in the area occupied by the shell $(-h < \alpha_3 < h)$,

$$\begin{aligned} &\operatorname{rot}\mathbf{h} = \frac{4\pi\sigma}{c}\left(\mathbf{e} + \frac{1}{c}\frac{\partial\mathbf{u}}{\partial t}\times\mathbf{H_H}\right), \quad \operatorname{div}\mathbf{h} = 0, \\ &\operatorname{rot}\mathbf{e} = -\frac{1}{c}\frac{\partial\mathbf{h}}{\partial t}, \quad \operatorname{div}\mathbf{e} = 4\pi\rho_e; \end{aligned} \tag{2.5.1}$$

$$\begin{aligned} &\frac{\partial}{\partial\alpha_1}(H_2 s_{11}) + \frac{\partial}{\partial\alpha_2}(H_1 s_{12}) + \frac{\partial}{\partial\alpha_3}(H_1 H_2 s_{13}) \\ &\quad - s_{22}\frac{\partial H_2}{\partial\alpha_1} + s_{13}H_2\frac{\partial H_1}{\partial\alpha_3} = \rho\frac{\partial^2 u_1}{\partial t^2} - H_1 H_2 X_1, \\ &\frac{\partial}{\partial\alpha_2}(H_1 s_{22}) + \frac{\partial}{\partial\alpha_3}(H_1 H_2 s_{23}) + \frac{\partial}{\partial\alpha_1}(H_2 s_{12}) \\ &\quad + s_{12}\frac{\partial H_2}{\partial\alpha_1} + s_{23}H_1\frac{\partial H_2}{\partial\alpha_3} = \rho\frac{\partial^2 u_2}{\partial t^2} - H_1 H_2 X_2, \\ &\frac{\partial}{\partial\alpha_3}(H_1 H_2 s_{33}) + \frac{\partial}{\partial\alpha_1}(H_2 s_{13}) + \frac{\partial}{\partial\alpha_2}(H_1 s_{23}) \\ &\quad - s_{11}H_2\frac{\partial H_1}{\partial\alpha_3} - s_{22}H_1\frac{\partial H_2}{\partial\alpha_3} = \rho\frac{\partial^2 u_3}{\partial t^2} - H_1 H_2 X_3, \end{aligned} \tag{2.5.2}$$

where $\mathbf{h}$ and $\mathbf{e}$ are the vectors of intensity of induced magnetic and electric fields in the area occupied by the shell $(0 \le \alpha_1 \le \pi, 0 \le \alpha_2 \le 2\pi, -h \le \alpha_3 \le h)$, respectively, $\mathbf{u}(u_1, u_2, u_3)$ is displacement vector of shell's particles, ρ_e is the density of electric charges, $H_1 = A_1(1 + k_1\alpha_3), H_2 = A_2(1 + k_2\alpha_3)$ are Lame coefficients, $\mathbf{X}(X_1, X_2, X_3)$ are forces of electromagnetic origin

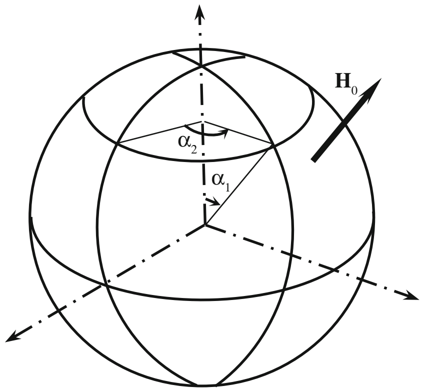

Fig. 2.4 Geometrical interpretation of the problem. Spherical shell in a magnetic field

$$\mathbf{X} = \frac{\sigma}{c}\left(\mathbf{e} + \frac{1}{c}\frac{\partial \mathbf{u}}{\partial t} \times \mathbf{H_0}\right) \times \mathbf{H_0}; \tag{2.5.3}$$

(b) equations of electrodynamics for vacuum in the areas $(-R < \alpha_3 < -h;\ \alpha_3 > h)$,

$$\begin{aligned} &\operatorname{rot}\mathbf{h}^{(\mathbf{e})} = 0, \quad \operatorname{div}\mathbf{h}^{(\mathbf{e})} = 0, \\ &\operatorname{rot}\mathbf{e}^{(\mathbf{e})} = -\frac{1}{c}\frac{\partial \mathbf{h}^{(\mathbf{e})}}{\partial t}, \quad \operatorname{div}\mathbf{e}^{(\mathbf{e})} = 0, \end{aligned} \tag{2.5.4}$$

where $\mathbf{h}^{(\mathbf{e})}$ and $\mathbf{e}^{(\mathbf{e})}$ are the vectors of intensity of induced magnetic and electric fields in the external area, respectively, moreover, the index $e = 1$ corresponds to the area $\alpha_3 > h$, and $e = 2 - k$ to the area $\alpha_3 < -h$.

Thus, three-dimensional problem of magnetoelastic vibrations of a spherical shell is brought to the joint integration of the system of Eqs. (2.5.1)–(2.5.4), the solutions of which must satisfy the conditions of continuity of electromagnetic field on the shell's vibrating surfaces and the conditions of attenuation of perturbations at infinity and limitedness in the area $\alpha_3 < -h$.

To bring the three-dimensional equations of magnetoelasticity of a spherical shell (2.5.1)–(2.5.3) to the two-dimensional the hypotheses of magnetoelasticity of thin bodies is accepted. According to these hypotheses

$$\begin{aligned} &u_1 = u - \frac{\alpha_3}{A_1}\frac{\partial w}{\partial \alpha_1}, \quad u_2 = v - \frac{\alpha_3}{A_2}\frac{\partial w}{\partial \alpha_2}, \quad u_3 = w; \\ &e_1 = \varphi(\alpha_1, \alpha_2, t), \quad e_2 = \psi(\alpha_1, \alpha_2, t), \quad h_3 = f(\alpha_1, \alpha_2, t), \end{aligned} \tag{2.5.5}$$

where $u(\alpha_1, \alpha_2, t), v(\alpha_1, \alpha_2, t), w(\alpha_1, \alpha_2, t)$ are unknown displacements of the middle surface of the shell; φ, ψ are unknown tangential components of the induced electromagnetic field in the shell's electric field $\mathbf{e}(e_1, e_2, e_3)$; f is unknown normal component of induced in the the shell's magnetic field $\mathbf{h}(h_1, h_2, h_3)$.

On the basis of the accepted hypotheses from the Eq. (2.5.1) the rest of components of induced in the shell's electromagnetic field are defined depending on the included in the Eq. (2.5.5) main six unknowns. Then substituting these expressions for the induced electromagnetic field into the equations of vibrations (2.5.2) and having averaged along the thickness of the shell (as it was performed in the previous paragraphs in the case of plates and cylindrical shells) the system of differential equations is obtained with respect to the main six unknowns $(u, v, w, \varphi, \psi, f)$. The noted system is obtained in the work [32] and here is not brought because it is huge enough.

Let us introduce the above-mentioned system in the case of nonhomogeneous magnetic field $\mathbf{H_0}$, which intensity vector is perpendicular to the nondeformed surface of the shell. When the shell is in the unperturbed state the vector is defined as

$$\mathbf{H_0} = \frac{H_0}{(1+\alpha_3/R)^2}\mathbf{N_0}. \tag{2.5.6}$$

Here H_0 is the intensity of magnetic field on the middle surface $(\alpha_3 = 0)$, $\mathbf{N_0}$ is unit external vector.

In the examined case the following system of singular integral-differential equations is obtained with respect to the unknown functions $(u, v, w, \varphi, \psi, f)$ [32]:

$$\begin{aligned}
&\frac{1}{A_1}\left[\frac{\partial\Theta}{\partial\alpha_1} - \frac{h^2}{3R^2}\frac{\partial}{\partial\alpha_1}(\Delta+2)w\right] \\
&\quad - \frac{1-\nu}{A_2}\frac{\partial X}{\partial\alpha_2} - \frac{1-\nu}{R}\left(\frac{u}{R} - \frac{1}{A_1}\frac{\partial w}{\partial\alpha_1}\right) \\
&\quad + \frac{(1-\nu^2)\sigma H_0}{Ec}\left[\psi - \frac{H_0}{c}\frac{\partial}{\partial t}\left(u - \frac{h^2}{3R}\frac{1}{A_1}\frac{\partial w}{\partial\alpha_1}\right)\right] = 0, \\
&\frac{1}{A_2}\left[\frac{\partial\Theta}{\partial\alpha_2} - \frac{h^2}{3R^2}\frac{\partial}{\partial\alpha_2}(\Delta+2)w\right] \\
&\quad + \frac{1-\nu}{A_1}\frac{\partial X}{\partial\alpha_1} + \frac{1-\nu}{R}\left(\frac{v}{R} - \frac{1}{A_2}\frac{\partial w}{\partial\alpha_2}\right) \\
&\quad - \frac{(1-\nu^2)\sigma H_0}{Ec}\left[\varphi + \frac{H_0}{c}\frac{\partial}{\partial t}\left(v - \frac{h^2}{3R}\frac{1}{A_2}\frac{\partial w}{\partial\alpha_2}\right)\right] = 0, \\
&\frac{1}{A_1A_2}\left[\frac{\partial(A_2\psi)}{\partial\alpha_1} - \frac{\partial(A_1\varphi)}{\partial\alpha_2}\right] + \frac{1}{c}\frac{\partial f}{\partial t} = 0, \\
&\frac{1}{A_1}\frac{\partial f}{\partial\alpha_1} + \frac{4\pi\sigma}{c}\left(\psi - \frac{H_0}{c}\frac{\partial u}{\partial t}\right) + \frac{R}{h}\frac{1}{A_1}\frac{\partial F}{\partial\alpha_1} = 0, \\
&\frac{1}{A_2}\frac{\partial f}{\partial\alpha_2} - \frac{4\pi\sigma}{c}\left(\varphi + \frac{H_0}{c}\frac{\partial v}{\partial t}\right) + \frac{R}{h}\frac{1}{A_2}\frac{\partial F}{\partial\alpha_2} = 0.
\end{aligned} \tag{2.5.7}$$

Let us note that in the analogous way the three-dimensional problem of magnetoelastisity of thin incomplete shell can be brought to the two-dimensional, also.

2.6 Influence of the Induced Electromagnetic Field in Problems of Vibrations of Conducting Plates in Transversal Magnetic Field

On the basis of hypotheses of magnetoelasticity of thin bodies the system of two-dimensional equations of perturbed motion of thin conducting plates in a stationary magnetic field was obtained in Sect. 2.1. In the case when the given magnetic field is perpendicular to the middle plane of the plate the noted system is split and according to Eq. (2.1.26) the problem of investigation of transversal vibrations is brought to the solution of the following equation:

$$D\Delta^2 w + 2\rho h \frac{\partial^2 w}{\partial t^2} - \frac{2\sigma h^3 H_{03}^2}{3c^2}\frac{\partial^2 \Delta w}{\partial t^2} = P \qquad (2.6.1)$$

with the usual boundary conditions.

From Eq. (2.6.1), where the last term presents the effect of magnetic field, it follows that in the range of accuracy of hypotheses of magnetoelasticity of thin bodies the induced electromagnetic field do not effect on the character of magnetoelastic vibrations of the plate.

In the present paragraph in the case of transversal magnetic field the equations of magnetoelastic vibrations of conducting plate will be obtained taking into account the effect of the induced electromagnetic field [4]. These equations will be derived on the basis of the linear law of change (along the thickness of the plate) of tangential components of intensity vector of the induced electric field and normal components of intensity vector of the induced magnetic field. The noted assumption is the result of the hypotheses of magnetoelasticity of thin bodies.

Let the elastic conducting plate of the thickness $2h$ is referred to the Cartesian coordinate system x_1, x_2, x_3 and plate's middle plane coincides with the coordinate system x_1, x_2. The plate is made of a material with the finite electroconductivity σ, and placed in the given transversal constant magnetic field $\mathbf{H_0}(0, 0, H_0)$.

The following assumptions are accepted [4]:

(a) The hypothesis on nondeformable normal, according to which

$$u_1 = u - x_3\frac{\partial w}{\partial x_1}, \quad u_2 = v - x_3\frac{\partial w}{\partial x_2}, \quad u_3 = w(x_1, x_2, t)$$
$$\text{for} \quad |x_3| < h \quad (x_1, x_2) \in G, \qquad (2.6.2)$$

where G is the area of the plane $x_3 = 0$, limited by the plate's contour Γ;

(b) The linear law of change of e_1, e_2 and h_3 along the thickness of the strip R $(R : |x_3| < h, -\infty < x_1, x_2 < h)$ is true

$$e_1 = \varphi + x_3\Phi, \quad e_2 = \psi + x_3\Psi, \quad h_h = f + x_3F, \quad \text{for } (x_1, x_2, x_3) \in R, \tag{2.6.3}$$

where e_1, e_2 are tangential components of the induced electromagnetic field in the strip R electric field $\mathbf{e}(e_1, e_2, e_3)$; h_3 is the normal component of the induced electromagnetic field in the strip R magnetic field $\mathbf{h}(h_1, h_2, h_3)$; $\varphi, \Phi, \psi, \Psi, f, F$ are unknown functions depending on x_1, x_2 and time t.

It is assumed also that the effect of shift currents on the characteristics of elastic vibrations can be neglected.

In the Eq. (2.6.3) if assume $\Phi = \Psi = F \equiv 0((x_1, x_2) \in G)$ then Eqs. (2.6.2) and (2.6.3) will coincide with the relations (2.3.2) and (2.3.3). Let us note again that in the case of the perfectly conducting plate $(\sigma \to \infty)$ Eq. (2.6.3) for $(x_1, x_2) \in G$ become the result from (2.6.2) [69].

The three-dimensional problem of magnetoelasticity in the chosen coordinate system, according to Eqs. (2.1.1)–(2.1.3) and (1.6.16), is brought to the joint solution of the following system of differential equations.

In the internal area: equations of electrodynamics

$$\operatorname{rot} \mathbf{h} = \frac{4\pi\sigma}{c}\left(\mathbf{e} + \frac{1}{c}\frac{\partial \mathbf{u}}{\partial t} \times \mathbf{H_0}\right), \tag{2.6.4}$$

$$\operatorname{div} \mathbf{h} = 0, \tag{2.6.5}$$

$$\operatorname{rot} \mathbf{e} = -\frac{1}{c}\frac{\partial \mathbf{h}}{\partial t}; \tag{2.6.6}$$

motion equations

$$\begin{gathered} \frac{\partial s_{ik}}{\partial x_k} + X_i = 0, \\ Q(X_1, X_2, X_3) = \frac{\sigma}{c}\left(\mathbf{e} + \frac{1}{c}\frac{\partial \mathbf{u}}{\partial t} \times \mathbf{H_0}\right) \times \mathbf{H_0} - \rho\frac{\partial^2 \mathbf{u}}{\partial t^2}. \end{gathered} \tag{2.6.7}$$

In the external area: equations of electrodynamics for vacuum

$$\operatorname{rot} \mathbf{h}^{(\mathbf{e})} = 0, \quad \operatorname{div} \mathbf{h}^{(\mathbf{e})} = 0, \quad \operatorname{rot} \mathbf{e}^{(\mathbf{e})} = -\frac{1}{c}\frac{\partial \mathbf{h}^{(\mathbf{e})}}{\partial t}. \tag{2.6.8}$$

To system (2.6.4)–(2.6.8) it is necessary to add the fixing conditions and conditions of attenuation of perturbations at infinity and, also, the conditions for the components of the tensor of elastic stresses and electromagnetic field on the plate's surface. The last conditions can be written as follows:

$$\hat{s} \cdot \mathbf{N_0} = \mathbf{P}, \tag{2.6.9}$$

$$\mathbf{h} \cdot \mathbf{N_0} = \mathbf{h}^{(\mathbf{e})} \cdot \mathbf{N_0}, \tag{2.6.10}$$

$$\mathbf{N_0} \times \left[\mathbf{h} - \mathbf{h}^{(\mathbf{e})}\right] = 0, \tag{2.6.11}$$

where $\mathbf{N_0}$ is the unit vector of the outward normal to the underformed surface of the plate, $\mathbf{P} = P\mathbf{N_0}$ is the surface force of nonelectromagnetic origin.

According to the Eq. (2.6.2) we have the relations (2.1.7) in the area $|x_3| < h, (x_1, x_2) \in G$.

From the first two equations of system (2.6.7) in account of Eqs. (2.6.3), (2.6.17), and boundary conditions (2.6.9) for the stresses s_{13} and s_{23}, we can find

$$\begin{aligned} s_{13} &= \frac{x_3^2 - h^2}{2}\left[\frac{E}{1-\nu^2}\frac{\partial \Delta w}{\partial x_1} - \frac{\sigma H_0^2}{c^2}\frac{\partial^2 w}{\partial x_1 \partial t} - \frac{\sigma H_0}{c}\Psi\right], \\ s_{23} &= \frac{x_3^2 - h^2}{2}\left[\frac{E}{1-\nu^2}\frac{\partial \Delta w}{\partial x_2} - \frac{\sigma H_0^2}{c^2}\frac{\partial^2 w}{\partial x_2 \partial t} - \frac{\sigma H_0}{c}\Phi\right]. \end{aligned} \tag{2.6.12}$$

Substituting Eqs. (2.6.2) and (2.6.3) into Eqs. (2.6.4) and (2.6.8), and taking into account the boundary conditions (2.6.11), one can find the following expressions for the components h_1 and h_2 of magnetic field induced in the strip R

$$\begin{aligned} h_1 &= \frac{h_1^+ + h_1^-}{2} + x_3\left[\frac{\partial f}{\partial x_1} + \frac{4\pi\bar{\sigma}}{c}\left(\psi - \frac{H_0}{c}\frac{\partial u}{\partial t}\right)\right] \\ &\quad + \frac{x_3^2 - h^2}{2}\left[\frac{\partial F}{\partial x_1} + \frac{4\pi\bar{\sigma}}{c}\left(\Psi + \frac{H_0}{c}\frac{\partial^2 w}{\partial x_1 \partial t}\right)\right], \\ h_2 &= \frac{h_2^+ + h_2^-}{2} + x_3\left[\frac{\partial f}{\partial x_2} - \frac{4\pi\bar{\sigma}}{c}\left(\varphi + \frac{H_0}{c}\frac{\partial v}{\partial t}\right)\right] \\ &\quad + \frac{x_3^2 - h^2}{2}\left[\frac{\partial F}{\partial x_2} - \frac{4\pi\bar{\sigma}}{c}\left(\Phi - \frac{H_0}{c}\frac{\partial^2 w}{\partial x_2 \partial t}\right)\right]. \end{aligned} \tag{2.6.13}$$

where

$$\bar{\sigma} = \begin{cases} \sigma & \text{for} \quad (x_1, x_2) \in G, \\ 0 & \text{for} \quad (x_1, x_2) \notin G. \end{cases}$$

Here the indices "+" and "−" are correspond to the quantities at $x_3 = h$ and $x_3 = -h$.

Substituting Eqs. (2.6.2), (2.6.3), and (2.6.12) into Eq. (2.6.7) and doing integration with respect to x_3 from $x_3 = -h$ to $x_3 = h$, having into account the third condition of the Eq. (2.6.9), the following system of differential equations is obtained:

$$
\begin{aligned}
&\frac{\partial \psi}{\partial x_1} - \frac{\partial \varphi}{\partial x_2} + \frac{1}{c}\frac{\partial f}{\partial t} = 0,\\
&\frac{\partial f}{\partial x_1} + \frac{4\pi\bar{\sigma}}{c}\left[\psi - \frac{H_0}{c}\frac{\partial u}{\partial t}\right] = \frac{h_1^+ - h_1^-}{2h},\\
&\frac{\partial f}{\partial x_2} - \frac{4\pi\bar{\sigma}}{c}\left[\varphi + \frac{H_0}{c}\frac{\partial v}{\partial t}\right] = \frac{h_2^+ - h_2^-}{2h},
\end{aligned}
\tag{2.6.14}
$$

$$
\begin{aligned}
&\frac{\partial^2 u}{\partial x_1^2} + \frac{1+\nu}{2}\frac{\partial^2 v}{\partial x_1 \partial x_2} + \frac{1-\nu}{2}\frac{\partial^2 u}{\partial x_2^2}\\
&\quad + \frac{1-\nu^2}{E}\frac{\sigma H_0}{c}\left(\psi - \frac{H_0}{c}\frac{\partial u}{\partial t}\right) = \frac{\rho(1-\nu^2)}{E}\frac{\partial^2 u}{\partial t^2},\\
&\frac{\partial^2 v}{\partial x_2^2} + \frac{1+\nu}{2}\frac{\partial^2 u}{\partial x_1 \partial x_2} + \frac{1-\nu}{2}\frac{\partial^2 v}{\partial x_1^2}\\
&\quad - \frac{1-\nu^2}{E}\frac{\sigma H_0}{c}\left(\varphi + \frac{H_0}{c}\frac{\partial v}{\partial t}\right) = \frac{\rho(1-\nu^2)}{E}\frac{\partial^2 v}{\partial t^2};
\end{aligned}
$$

equations of transversal vibrations

$$
\begin{aligned}
&D\Delta^2 w + 2\rho h\frac{\partial^2 w}{\partial t^2} = P + \frac{2\sigma h^3}{3c^2}H_0\left(H_0\frac{\partial \Delta w}{\partial t} - \frac{\partial F}{\partial t}\right),\\
&\Delta F + \frac{4\pi\bar{\sigma}}{c^2}\frac{\partial}{\partial t}(H_0\Delta w - F)\\
&\quad = \frac{3}{h^2}\left[F + \frac{\partial}{\partial x_1}\left(\frac{h_1^+ + h_1^-}{2}\right) + \frac{\partial}{\partial x_2}\left(\frac{h_2^+ + h_2^-}{2}\right)\right],
\end{aligned}
\tag{2.6.15}
$$

moreover, in the first two equations of system (2.6.14) and in the second equation of system (2.6.15) $-\infty < x_1, x_2 < \infty$ and in the rest of equations $(x_1, x_2) \in G$.

From the Eq. (2.6.15) in the case of perfectly conducting material ($\sigma \to \infty$) it is easy to obtain the following equation:

$$
\begin{aligned}
&\left(D + \frac{h^3 H_0^2}{6\pi}\right)\Delta^2 w + 2\rho h\frac{\partial^2 w}{\partial t^2} - \frac{hH_0^2}{2\pi}\Delta w\\
&\quad = P + \frac{hH_0^2}{2\pi}\left[\frac{\partial}{\partial x_1}\left(\frac{h_1^+ + h_1^-}{2}\right) + \frac{\partial}{\partial x_2}\left(\frac{h_2^+ + h_2^-}{2}\right)\right],
\end{aligned}
$$

which is the same as the equation of transversal vibrations of perfectly conducting plates obtained in the works [13, 69] on the basis of Kirchhoff hypothesis.

When investigating Eqs. (2.6.14) and (2.6.15) one can note that they contain the unknown boundary values of tangential components of the induced magnetic field $h_1^{\pm}$ and $h_2^{\pm}$. Adding to these equations system (2.6.8) and boundary conditions (2.6.10) and (2.6.11) one can close the system.

Introducing the potential function $\varphi_0(x_1, x_2, x_3, t)$ by way of the formula

$$\mathbf{h}^{(\mathbf{e})} = \operatorname{grad}\varphi_0, \tag{2.6.16}$$

The problem of calculation of magnetic field $\mathbf{h}^{(\mathbf{e})}$ out of the strip R, according to Eqs. (2.6.8), (2.6.19), and (2.6.3), is brought to the solution of the following Neumann problem in the half-spaces $|x_3| > h$:

$$\Delta\varphi_0 = 0,$$
$$\left.\frac{\partial\varphi_0}{\partial x_3}\right|_{x_3=\pm h} = f(x_1, x_2, t) \pm hF(x_1, x_2, t).$$

Damping at infinity the solution of these problems has the form [74]

$$\varphi_0 = \mp\frac{1}{2\pi}\iint_{-\infty}^{\infty}\frac{[f(\xi,\eta,t) \pm hF(\xi,\eta,t)]\mathrm{d}\xi\mathrm{d}\eta}{\sqrt{(x_1-\xi)^2+(x_2-\eta)^2+(x_3\mp h)^2}}. \tag{2.6.17}$$

where the upper sign corresponds to the half-space $x_3 > h$ and the lower sign—to the $x_3 < -h$.

On the basis of (2.6.17) from the Eq. (2.6.16) we can find

$$\begin{aligned} \frac{h_i^+ - h_i^-}{2} &= -\frac{1}{2\pi}\frac{\partial}{\partial x_i}\iint_{-\infty}^{\infty}\frac{f(\xi,\eta,t)\mathrm{d}\xi\mathrm{d}\eta}{\sqrt{(x_1-\xi)^2+(x_2-\eta)^2}}, \\ \frac{h_i^+ + h_i^-}{2} &= -\frac{h}{2\pi}\frac{\partial}{\partial x_i}\iint_{-\infty}^{\infty}\frac{F(\xi,\eta,t)\mathrm{d}\xi\mathrm{d}\eta}{\sqrt{(x_1-\xi)^2+(x_2-\eta)^2}}. \end{aligned} \tag{2.6.18}$$

Substituting (2.6.8) into Eqs. (2.6.14) and (2.6.15), one can note that the problems of longitudinal and transversal magnetoelastic vibrations are wholly split. In particular, according to Eqs. (2.6.15) and (2.6.18) the problem of transversal vibrations is brought to the solution of the following system of integral-differential equations:

$$\begin{aligned} & D\Delta^2 w + 2\rho h\frac{\partial^2 w}{\partial t^2} = P + \frac{2\sigma h^3 H_0}{3c^2}\frac{\partial}{\partial t}(H_0\Delta w - F), \\ & \Delta F + \frac{4\pi\bar{\sigma}}{c^2}\frac{\partial}{\partial t}(H_0\Delta w - F) \\ & \quad = \frac{3}{h^2}\left[F - \frac{h}{2\pi}\Delta\left(\iint_{-\infty}^{\infty}\frac{F(\xi,\eta,t)\mathrm{d}\xi\mathrm{d}\eta}{\sqrt{(x_1-\xi)^2+(x_2-\eta)^2}}\right)\right], \\ & \quad -\infty < x_1, x_2 < \infty, \end{aligned} \tag{2.6.19}$$

with the conditions on $w(x_1, x_2, t)$ along the plate's contour and with the condition at infinity

$$F \to 0 \quad \text{for} \quad x_1^2 + x_2^2 \to \infty. \tag{2.6.20}$$

From system (2.6.19) it is easy to obtain the following nonhomogeneous equation with respect to the function F:

$$\Delta F - \frac{3}{h^2}\left[F - \frac{h}{2\pi}\Delta\left(\iint_{-\infty}^{\infty} \frac{F(\xi,\eta,t)\mathrm{d}\xi\mathrm{d}\eta}{\sqrt{(x_1-\xi)^2 + (x_2-\eta)^2}}\right)\right] = \gamma, \tag{2.6.21}$$

where

$$\gamma = \begin{cases} 0 & \text{for} \quad (x_1, x_2) \notin G, \\ -\frac{6\pi}{h^3 H_0}\left[D\Delta^2 w + 2\rho h \frac{\partial^2 w}{\partial t^2} - P\right] & \text{for} \quad (x_1, x_2) \in G. \end{cases}$$

Applying the two-dimensional exponential Fourier transformation [74, 75] with respect to the Eq. (2.6.2) in account of (2.6.20) we can find

$$\begin{aligned} &\iint_{-\infty}^{\infty} F(x_1, x_2, t)\mathrm{e}^{i(\alpha_1 x_1 + \alpha_2 x_2)}\mathrm{d}x_1\mathrm{d}x_2 \\ &\quad = \frac{6\pi}{hH_0\left[3 + 3h\sqrt{\alpha_1^2 + \alpha_2^2} + h^2\left(\alpha_1^2 + \alpha_2^2\right)\right]} \\ &\quad\times \iint_G \left(D\Delta^2 w + 2\rho h \frac{\partial^2 w}{\partial t^2} - P\right)\mathrm{e}^{i(\alpha_1 x_1 + \alpha_2 x_2)}\mathrm{d}x_1\mathrm{d}x_2. \end{aligned} \tag{2.6.22}$$

On the basis of integral representation of Bessel functions [75, 117]:

$$J_0(z) = \frac{1}{2\pi}\int_{\alpha}^{2\pi+\alpha} \mathrm{e}^{iz\sin\beta}\mathrm{d}\beta,$$

from the Eq. (2.6.22) according to the formula of transverse Fourier transformation [74] we can calculate the function:

$$F = \frac{3}{hH_0}\iint_G \left(D\Delta^2 w + 2\rho h \frac{\partial^2 w}{\partial t^2} - P\right)K(x_1, x_2; \xi, \eta)\mathrm{d}\xi\mathrm{d}\eta, \tag{2.6.23}$$

where

$$K = \int_0^\infty \frac{\rho J_0\left(\rho\left[(x_1-\xi)^2+(x_2-\eta)^2\right]^{\frac{1}{2}}\right)}{3+3h\rho+h^2\rho^2}\mathrm{d}\rho.$$

Substituting (2.6.23) into the first equation of system (2.6.19) the problem of magnetoelastic vibrations of conducting plate in a transversal magnetic field is brought to the solution of the following integral-differential equation [4]:

$$\begin{aligned} D\Delta^2 w + 2\rho h\frac{\partial^2 w}{\partial t^2} &= P + \frac{2\sigma h^3}{3c^2}H_0^2\frac{\partial \Delta w}{\partial t} \\ &- \frac{2\sigma}{c^2}\frac{\partial}{\partial t}\iint_G\left(D\Delta^2 w + 2\rho h\frac{\partial^2 w}{\partial t^2} - P\right)K\mathrm{d}\xi\mathrm{d}\eta \end{aligned} \tag{2.6.24}$$

with the appropriate fixing conditions of plate's edges.

At the end let us obtain the most approximate but simplified equation, thus characterizing the vibrations of the conducting plate in a transversal magnetic field. For this reason, when obtaining the boundary values for $h_i^{\pm}$, let us assume that the plate is long enough. In this case, representing the function $f(x_1, x_2, t)$ in the form

$$F = F_0(t)\exp[i(k_1x_1 + k_2x_2)]$$

(where k_1 and k_2 are wave numbers) and using the second equation of system (2.6.19), it is easy to note that the first and last terms have the orders $1+(kh)^2$ and $1+kh$, respectively $\left(k=\sqrt{k_1^2+k_2^2}\right)$. Thus, neglecting the noted terms brings to accuracy the order kh compared with the unit, which is a normal approximation within the theory of thin plates. Taking into account the above-mentioned from the second equation of system (2.6.19), we will obtain the approximate formula for the function F:

$$F = \frac{2\pi}{hH_0}\left[D\Delta^2 w + 2\rho h\frac{\partial^2 w}{\partial t^2} - P\right]. \tag{2.6.25}$$

Substituting (2.6.25) into the first equation of system (2.6.19), the problem of the magnetoelastic vibrations of the conducting plate in a transversal magnetic field is brought to the solution of [4]:

$$\begin{aligned} D\Delta^2 w + 2\rho h\frac{\partial^2 w}{\partial t^2} &= P + \frac{2\sigma h^3}{3c^2}H_0^2\frac{\partial \Delta w}{\partial t} \\ &- \frac{4\pi\sigma h^2}{3c^2}\frac{\partial}{\partial t}\left(D\Delta^2 w + 2\rho h\frac{\partial^2 w}{\partial t^2} - P\right) \end{aligned} \tag{2.6.26}$$

with the appropriate boundary conditions.

Having compared the Eq. (2.6.26) with the Eq. (2.6.1), one can note that the effect of the induced electromagnetic field is taken into account by way of the last term in the right-hand side of Eq. (2.6.26). From this equation for $(\sigma \to \infty)$, it is easy to obtain the following:

$$D\Delta^2 w + 2\rho h \frac{\partial^2 w}{\partial t^2} - \frac{hH_0^2}{2\pi}\Delta w = P. \tag{2.6.27}$$

In the range of the accepted approximation, Eq. (2.6.27) coincides with obtained [in 13, 69] equation of magnetoelastic vibrations of a perfectly conducting plate in a transversal magnetic field.

2.7 Two-Dimensional Equations of Magnetoelasticity of Perfectly Conducting Plates

Using Kirchhoff hypothesis, the two-dimensional equations of the magnetoelasticity of thin perfectly conducting plates in a constant magnetic field are obtained here on the basis of the works [13, 69]. It is assumed that the load $P(x_1, x_2, t)$ of nonelectromagnetic origin $(F_1 = F_2 = 0, F_3 = P)$ acts normally to the surface $x_3 = h$ of the plate, and the magnetic susceptibility of plate's material is equal to one unit.

On the basis of the accepted assumptions from Eqs. (1.3.1) and (1.2.6), using Eqs. (1.3.7) and (1.3.9), after linearization the following three-dimensional equations are obtained, thus characterizing the behavior of magnetoelastic quantities within the plate:

$$\frac{\partial s_{ik}}{\partial x_k} + \frac{1}{4\pi}(\mathrm{rot}\,\mathbf{h} \times \mathbf{H_0})_i = \rho \frac{\partial^2 u_i}{\partial t^2}, \tag{2.7.1}$$

$$\mathbf{h} = \mathrm{rot}(\mathbf{u} \times \mathbf{H_0}), \quad \mathbf{e} = \frac{1}{c}\left(\mathbf{H_0} \times \frac{\partial \mathbf{u}}{\partial t}\right). \tag{2.7.2}$$

According to Kirchhoff's hypothesis, relations (2.1.5) and (2.1.10) take place. Substituting Eq. (2.1.5) into Eq. (2.7.2), the following expressions with respect to the components of the induced electromagnetic field are obtained:

$$\begin{aligned}
h_1 &= -H_{01}\left(\frac{\partial v}{\partial x_2} - x_3\frac{\partial^2 w}{\partial x_2^2}\right) + H_{02}\left(\frac{\partial u}{\partial x_2} - x_3\frac{\partial^2 w}{\partial x_1 \partial x_2}\right) - H_{03}\frac{\partial w}{\partial x_1},\\
h_2 &= H_{01}\left(\frac{\partial v}{\partial x_1} - x_3\frac{\partial^2 w}{\partial x_1 \partial x_2}\right) - H_{02}\left(\frac{\partial u}{\partial x_1} - x_3\frac{\partial^2 w}{\partial x_1^2}\right) - H_{03}\frac{\partial w}{\partial x_2},\\
h_3 &= H_{01}\frac{\partial w}{\partial x_1} + H_{02}\frac{\partial w}{\partial x_2} - H_{03}\left(\frac{\partial u}{\partial x_1} + \frac{\partial v}{\partial x_2} - x_3\Delta w\right);
\end{aligned} \tag{2.7.3}$$

$$
\begin{aligned}
e_1 &= \frac{1}{c}\left[H_{02}\frac{\partial w}{\partial t} - H_{03}\left(\frac{\partial v}{\partial t} - x_3\frac{\partial^2 w}{\partial x_2 \partial t}\right)\right],\\
e_2 &= \frac{1}{c}\left[-H_{01}\frac{\partial w}{\partial t} + H_{03}\left(\frac{\partial u}{\partial t} - x_3\frac{\partial^2 w}{\partial x_1 \partial t}\right)\right],\\
e_3 &= \frac{1}{c}\left[H_{01}\left(\frac{\partial v}{\partial t} - x_3\frac{\partial^2 w}{\partial x_2 \partial t}\right) - H_{02}\left(\frac{\partial u}{\partial t} - x_3\frac{\partial^2 w}{\partial x_1 \partial t}\right)\right].
\end{aligned} \tag{2.7.4}
$$

On the basis of Eq. (2.1.5)—from Eq. (2.7.1)—the following expressions are obtained for the components of a space force of electromagnetic origin:

$$
\begin{aligned}
X_1 &= \frac{H_{03}A_2 - H_{02}A_3}{4\pi} - \frac{x_3}{4\pi}\left[\left(H_{02}^2 + H_{03}^2\right)\frac{\partial \Delta w}{\partial x_1} - H_{01}H_{02}\frac{\partial \Delta w}{\partial x_2}\right],\\
X_2 &= \frac{H_{01}A_3 - H_{03}A_1}{4\pi} - \frac{x_3}{4\pi}\left[\left(H_{01}^2 + H_{03}^2\right)\frac{\partial \Delta w}{\partial x_2} - H_{01}H_{02}\frac{\partial \Delta w}{\partial x_1}\right],\\
X_3 &= \frac{H_{02}A_1 - H_{01}A_2}{4\pi} + \frac{x_3}{4\pi}\left[H_{02}H_{03}\frac{\partial \Delta w}{\partial x_2} - H_{01}H_{03}\frac{\partial \Delta w}{\partial x_1}\right],
\end{aligned} \tag{2.7.5}
$$

where

$$
\begin{aligned}
A_1 &= -H_{02}\Delta w + 2\frac{\partial}{\partial x_2}\left(H_{01}\frac{\partial w}{\partial x_1} + H_{02}\frac{\partial w}{\partial x_2}\right) - H_{03}\frac{\partial}{\partial x_2}\left(\frac{\partial u}{\partial x_1} + \frac{\partial v}{\partial x_2}\right),\\
A_2 &= -H_{01}\Delta w - 2\frac{\partial}{\partial x_1}\left(H_{01}\frac{\partial w}{\partial x_1} + H_{02}\frac{\partial w}{\partial x_2}\right) + H_{03}\frac{\partial}{\partial x_1}\left(\frac{\partial u}{\partial x_1} + \frac{\partial v}{\partial x_2}\right),\\
A_3 &= H_{01}\Delta v - H_{02}\Delta u.
\end{aligned}
$$

From the first two equations of system (2.7.1), in account of Eqs. (2.1.10) and (2.7.5) and the boundary conditions (1.6.12), we can find

$$
\begin{aligned}
s_{13} = x_3 &\left[\rho\frac{\partial^2 u}{\partial t^2} - \frac{E}{1-\nu^2}\left(\frac{\partial^2 u}{\partial x_1^2} + \frac{1-\nu}{2}\frac{\partial^2 u}{\partial x_2^2} + \frac{1+\nu}{2}\frac{\partial^2 v}{\partial x_1 \partial x_2}\right) - \frac{H_{03}A_2 - H_{02}A_3}{4\pi}\right]\\
&+ \frac{H_{03}}{4\pi}\left[H_{03}\frac{\partial w}{\partial x_1} + H_{01}\frac{\partial v}{\partial x_2} - H_{02}\frac{\partial u}{\partial x_2} + \frac{1}{2}\left(h_1^{(e)+} - h_1^{(e)-}\right)\right]\\
&+ \frac{h^2 - x_3^2}{2}\left[\rho\frac{\partial^3 w}{\partial x_1 \partial t^2} - \left(\frac{E}{1-\nu^2} + \frac{H_{02}^2 + H_{03}^2}{4\pi}\right)\frac{\partial \Delta w}{\partial x_1} + \frac{H_{01}H_{02}}{4\pi}\frac{\partial \Delta w}{\partial x_2}\right],
\end{aligned} \tag{2.7.6}
$$

$$s_{23} = x_3\left[\rho\frac{\partial^2 v}{\partial t^2} - \frac{E}{1-\nu^2}\left(\frac{\partial^2 v}{\partial x_2^2} + \frac{1-\nu}{2}\frac{\partial^2 v}{\partial x_1^2} + \frac{1+\nu}{2}\frac{\partial^2 u}{\partial x_1 \partial x_2}\right) - \frac{H_{01}A_3 - H_{03}A_1}{4\pi}\right]$$
$$+\frac{H_{03}}{4\pi}\left[H_{03}\frac{\partial w}{\partial x_2} + H_{02}\frac{\partial u}{\partial x_1} - H_{01}\frac{\partial v}{\partial x_1} + \frac{1}{2}\left(h_2^{(e)+} - h_2^{(e)-}\right)\right]$$
$$+\frac{h^2 - x_3^2}{2}\left[\rho\frac{\partial^3 w}{\partial x_2 \partial t^2} - \left(\frac{E}{1-\nu^2} + \frac{H_{01}^2 + H_{03}^2}{4\pi}\right)\frac{\partial \Delta w}{\partial x_2} + \frac{H_{01}H_{02}}{4\pi}\frac{\partial \Delta w}{\partial x_1}\right].$$

Substituting Eqs. (2.1.10), (2.7.5), and (2.7.6) into Eq. (2.7.1) as well as integrating the obtained equation with respect to x_3 from $x_3 = -h$ up to $x_3 = h$, and having taken into account the third condition from (1.6.12), the following system of differential equations are obtained with respect to functions u, v, w:

$$\frac{\partial^2 u}{\partial x_1^2} + \frac{1-\nu}{2}\frac{\partial^2 u}{\partial x_2^2} + \frac{1+\nu}{2}\frac{\partial^2 v}{\partial x_1 \partial x_2}$$
$$+\frac{1-\nu^2}{4\pi E}\left[H_{02}^2\Delta u - H_{01}H_{02}\Delta v + H_{03}^2\frac{\partial}{\partial x_1}\left(\frac{\partial u}{\partial x_1} + \frac{\partial v}{\partial x_2}\right)\right.$$
$$\left. - H_{03}\left(H_{01}\frac{\partial^2 w}{\partial x_1^2} + H_{02}\frac{\partial^2 w}{\partial x_1 \partial x_2}\right)\right]$$
$$= \frac{\rho(1-\nu^2)}{E}\frac{\partial^2 u}{\partial t^2} - \frac{H_{03}}{4\pi}\frac{1-\nu^2}{2Eh}\left[h_1^{(e)+} - h_1^{(e)-}\right],$$
$$\frac{\partial^2 v}{\partial x_2^2} + \frac{1-\nu}{2}\frac{\partial^2 v}{\partial x_1^2} + \frac{1+\nu}{2}\frac{\partial^2 u}{\partial x_1 \partial x_2} \tag{2.7.7}$$
$$+\frac{1-\nu^2}{4\pi E}\left[H_{01}^2\Delta v - H_{01}H_{02}\Delta u + H_{03}^2\frac{\partial}{\partial x_2}\left(\frac{\partial u}{\partial x_1} + \frac{\partial v}{\partial x_2}\right)\right.$$
$$\left. - H_{03}\left(H_{01}\frac{\partial^2 w}{\partial x_1 \partial x_2} + H_{02}\frac{\partial^2 w}{\partial x_2^2}\right)\right]$$
$$= \frac{\rho(1-\nu^2)}{E}\frac{\partial^2 v}{\partial t^2} - \frac{H_{03}}{8\pi}\frac{1-\nu^2}{Eh}\left[h_2^{(e)+} - h_2^{(e)-}\right],$$

$$D_*\Delta^2 w + 2\rho h\frac{\partial^2 w}{\partial t^2} - \frac{2\rho h^3}{3}\frac{\partial^2 \Delta w}{\partial t^2}$$
$$-\frac{2h}{4\pi}\left[H_{01}^2\frac{\partial^2 w}{\partial x_1^2} + 2H_{01}H_{02}\frac{\partial^2 w}{\partial x_1 \partial x_2} + H_{02}^2\frac{\partial^2 w}{\partial x_2^2} + H_{03}^2\Delta w\right.$$
$$\left. - H_{03}\left(H_{01}\frac{\partial}{\partial x_1} + H_{02}\frac{\partial}{\partial x_2}\right)\left(\frac{\partial u}{\partial x_1} + \frac{\partial v}{\partial x_2}\right)\right]$$
$$= P - \frac{H_{01}}{4\pi}\left[h_1^{(e)+} - h_1^{(e)-}\right] - \frac{H_{02}}{4\pi}\left[h_2^{(e)+} - h_2^{(e)-}\right]$$
$$+\frac{hH_{03}}{4\pi}\left[\frac{\partial\left(h_1^{(e)+} + h_1^{(e)-}\right)}{\partial x_1} + \frac{\partial\left(h_2^{(e)+} + h_2^{(e)-}\right)}{\partial x_2}\right],$$

where

$$D_* = \frac{2h^3}{3}\left(\frac{E}{1-\nu^2} + \frac{H_{01}^2 + H_{02}^2 + H_{03}^2}{4\pi}\right).$$

For the complete definition of displacements and electromagnetic field in the plate, as Eq. (2.7.7) shows, it is necessary to also have the tangential components of the induced electromagnetic field at the plate's surface magnetic field. Therefore, in general, the problem of magnetoelasticity is still three-dimensional, and Eq. (2.7.7) should be studied together with the Maxwell equation (1.6.16) in the external area and with the general boundary conditions (1.6.17). In the case of perfectly conducting plates, the issues of calculation of $h_i^{(e)\pm}$, a final reduction of the three-dimensional problem of magnetoelasticity to the two-dimensional one will be studied in Sect. 2.3 (Reduction of the three-dimensional problem of magnetoelasticity of thin plates to the two-dimensional) using the asymptotic method.

Let us note that in the case of longitudinal magnetic field $\mathbf{H_0}(H_{01}, H_{02}, 0)$, the system of differential equations (2.7.7) splits. In particular, the equation of transverse vibration of the plate has the form

$$\begin{aligned} & D_*\Delta^2 w + 2\rho h\frac{\partial^2 w}{\partial t^2} \\ & \quad -\frac{2h}{4\pi}\left[\left(H_{01}^2 + H_{02}^2\right)\Delta w + 2H_{01}H_{02}\frac{\partial^2 w}{\partial x_1 \partial x_2}\right. \\ & \quad \left. -H_{01}^2\frac{\partial^2 w}{\partial x_2^2} - H_{02}^2\frac{\partial^2 w}{\partial x_1^2}\right] \\ & \quad = P - \frac{H_{01}}{4\pi}\left[h_1^{(e)+} - h_1^{(e)-}\right] - \frac{H_{02}}{4\pi}\left[h_2^{(e)+} - h_2^{(e)-}\right]. \end{aligned} \tag{2.7.8}$$

The system also splits in the case of the plane problem when the plate vibrates in the form of a cylindrical surface $x_3 = w(x_1, t)$ and when the magnetic field has the origin $\mathbf{H_0}(H_{01}, 0, H_{03})$. The equation of transverse vibrations in this case has the form

$$\begin{aligned} & D(1+\alpha)\frac{\partial^4 w}{\partial x_1^4} + 2\rho h\frac{\partial^2 w}{\partial t^2} - \frac{2h}{4\pi}\left[H_{03}^2 + \frac{H_{01}^2}{1+\alpha}\right]\frac{\partial^2 w}{\partial x_1^2} \\ & = P - \frac{H_{01}}{4\pi(1+\alpha)}\left[h_1^{(e)+} - h_1^{(e)-}\right] \\ & \quad + \frac{hH_{03}}{4\pi}\frac{\partial}{\partial x_1}\left(h_1^{(e)+} + h_1^{(e)-}\right), \\ & D = \frac{2Eh^3}{3(1-\nu^2)}, \quad \alpha = \frac{1-\nu^2}{E}\frac{H_{03}^2}{4\pi}. \end{aligned} \tag{2.7.9}$$

In each particular case, the system of the obtained equations, in addition to the above-mentioned conditions, the fixing conditions for the plate's edges should be attached.

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