A Neural Model of Number Interval Position Effect (NIPE) in Children

Michela Ponticorvo[1(✉)], Francesca Rotondaro[2], Fabrizio Doricchi[2], and Orazio Miglino[1]

[1] Department of Humanistic Studies, University of Naples "Federico II", Naples, Italy

[2] Department of Psychology, Sapienza, University of Rome, Rome, Italy

michela.ponticorvo@unina.it

Abstract. In the present paper we describe an artificial neural model of the Number Interval Position Effect (NIPE;[5]) that has been observed in the mental bisection of number intervals both in adults and in children. In this task a systematic error bias in the mental setting of the subjective midpoint of number intervals is found, so that for intervals of equal size there is a shift of the subjective midpoint towards numbers higher than the true midpoint for intervals at the beginning of decades while for intervals at the end of decades the error bias is directionally reversed towards numbers lower than the true midpoint. This trend of the bisection error is recursively present across consecutive decades.
Here we show that a neural-computational model based on information spread by energy gradients towards accumulation points based on the logarithimic compressed representation of number magnitudes that has been observed at the single cell level in rhesus monkeys [9] effectively simulates the performance of adults and children in the mental bisection of number intervals, in particular replicating the data observed in children.

Keywords: Artificial Neural Models · Numerical Cognition · Mental Number Line · Bisection of Number Intervals · NIPE effect

1 Introduction

Numbers are everywhere around us and dealing with them covers an important part of our cognitive activity throughout our life. A number of studies have suggested that when left/right response codes must be associated to number magnitudes, healthy participants belonging to western cultures with left-to-right reading habits map numbers upon a mental number line (MNL) with small integers positioned to the left of larger ones. This is reflected in the SNARC effect, (Spatial-Numerical Association of Response Codes) first demonstrated by Dehaene, Bossini, and Giraux [4] who argued that A representation of number magnitude is automatically accessed during parity judgments of Arabic digits. This representation may be likened to a mental number line, because it bears a natural and seemingly irrepressible correspondence with the left/right

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coordinates of external space (p. 394). More recently a inherent spatial and spatial-response-code independent nature of the MNL was suggested by the finding that during the mental bisection of number intervals right brain damaged patients with attentional neglect for the left side of space shift the subjective midpoint of number intervals toward numbers higher than the true midpoint, i.e. supposedly to the right of the true midpoint [14].

However, several ensuing studies have demonstrated that this numerical bias is unrelated to left spatial neglect and that it is rather linked to a deficit in the abstract representation of small numerical magnitudes [1,2]; for a review see Rossetti and collegues [11]. This conclusion was suggested by the finding that in right brain damaged patient the pathological bias toward numbers higher than the midpoint in the mental bisection of number interval is correlated to a similar bias in the bisection of time intervals on an imagined clock face where higher number are positioned to the left, rather than to the right, of the mental display [2,11]. In a recent study, Doricchi and colleagues [5] have discovered an new interesting psychophisical property of the number interval bisection task. It was found that in this task, human participants show a systematic error bias which is linked to the position occupied by the number interval in a decade (Number Interval Position Effect, NIPE). The subjective midpoint of number intervals of the same length is placed on numbers higher than the true midpoint the closer the interval is to the beginning of a decade and on numbers lower than the midpoint the closer the interval is to the end of the same decade. For example, in case of 7 units intervals the bias is positive for the intervals at the beginning of the decade (1-7) and negative for the intervals at the end of the decade (3-9). This effect has been observed in healthy adults [1,5], right brain damaged patients [1,5] and in pre-school children [12] thus suggesting that it is not related to learning of formal arithmetics and that it could be linked to some fundamental properties of the neural representation of number magnitudes.

Neurophysiological studies have demonstrated a neuronal representations of numerosity in the prefrontal and parietal cortex of rhesus monkeys [9]. In these areas different neuronal populations code for different numerosities. For small numerosities, the neural discharge is narrowly tuned, according to a gaussian function, to the preferred numerosity of the neuron so that the discharge is weak for adjacent numerosities. This gaussian tuning becomes progressively larger, i.e. less selective, for increasing numerosities, so that neurons tuned to larger numerosities show some discharge also for numerosities that are immediately adjacent to the preferred one. The organisation of the gaussian curves linked to the different and progressively increasing numerosities is best described by a nonlinearly logarithmic compressed scaling of numerical information.

In what follows we shall propose that the NIPE observed in the mental bisection of number intervals can be simulated by a neural-computational model based on infomation spread by energy gradients towards accumulation points based on the logarithimic compressed representation of number magnitudes that has been observed at the single cell level in the rhesus monkey [9].

2 Materials and Method

2.1 The Task

In order to investigate neuro-cognitive structures and mechanisms underlying basic arithmetics, namely neural coding of natural numbers and simple arithmetics operations, it is often proposed a task in which the participant has to identify the natural number that divides equally, bisects, a numerical series that is delimited by two natural numbers. For example, if we consider only the series of the first natural ten (1-10), the partecipant can be asked to identify the middle number between 1 (lower bound) and 7 (upper bound) or between 2 (lower bound) and 6 (upper bound) and so on. This task includes various forms: some of them permit one single solution, the ones whose limits sum is an even number, some others, the ones whose limits sum is an odd number, permit two solutions. This latter case is exemplified by the identification of the middle number between 1 and 8: the solutions are 4 and 5. To reply univocally the partecipant must choose the number that is closer to the lower bound, rounding down, or the upper, rounding up. For this reason, it is preferred to propose the task form with even sum.

2.2 The Model

We propose a neural model where no linear spatial representation is present. For this reason we start from two general principles about neural mechanisms which are strongly funded:

a. **Natural numbers neural coding**: basic numbers in a certain notation are coded in an amodal way by distinct neural groups. In other words, if we consider the decimal notation, there is a neural group whose activation is more probable when the number 1 is presented regardless of the presentation form, another one for number 2 and so on up to 10.
b. **Neural accumulation mechanisms**: neural elaboration takes place by energy transfer between neural groups and arrives to its conclusion when some neural group accumulates a certain energy level.

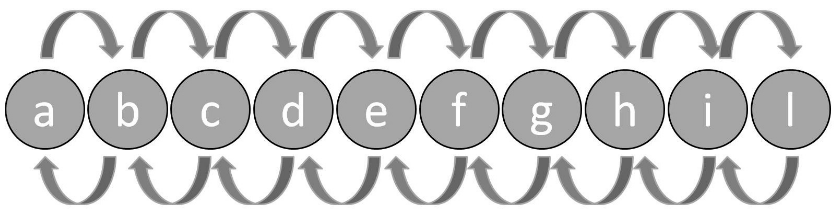

Fig. 1. The neural network architecture with nodes connections

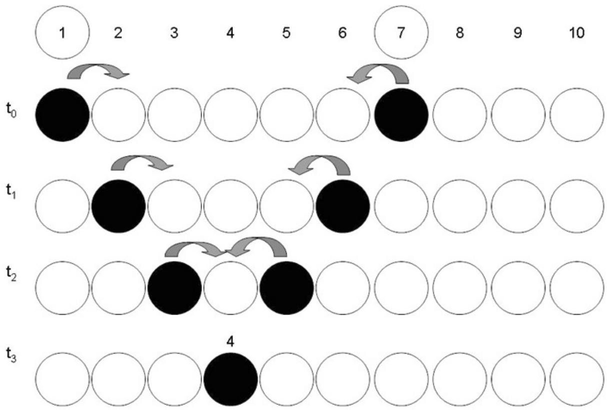

Fig. 2. The bisection problem: the even sum case

Let us consider that *m* neural groups code *n* natural numbers: m=n. In decimal notation, we will therefore have ten neural groups *a,b,c,d,e,f,g,h,i,l* which code natural numbers from 1 to 10. Let us imagine that neural groups are communicating vessels that transfer from one to another their energy level and the transfer dynamic ends when one neural group goes beyond a certain accumulation threshold. The various groups are connected in such a way that the neural group who presents its biggest activation probability when the number n is presented, is connected with the groups representing *n-1* and *n+2*. The groups representing the bounds 1 and 10 are an exception. Number 1 is connected only with *n+1* group and 10 is connected only with *n-1*. This architecture is represented in figure 1. Please note that each node does not represent a single neuron, but a group of neurons, a network.

We dictate the following dynamic to our network:

1. A neural group is univocally associated to a natural number. It therefore activates when this number is presented. The node *a,b,c,d,e,f,g,h,i,l* are associated with numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.
2. At time t_0, nodes coding upper and lower bound have 1.0 activation whereas other nodes have 0.0.
3. An highly active node transfers its energy to a node to whom it is connected with lower energy.
4. At time t_1 energy flows between nodes according to the following constraint: the node that codes the lower limit transfers its energy to the node representing the number immediately superior; the node coding the upper bound transfers its energy to the node representing the immediately inferior number.
5. If two contiguous nodes have the same energy level the energy flow interrupts.

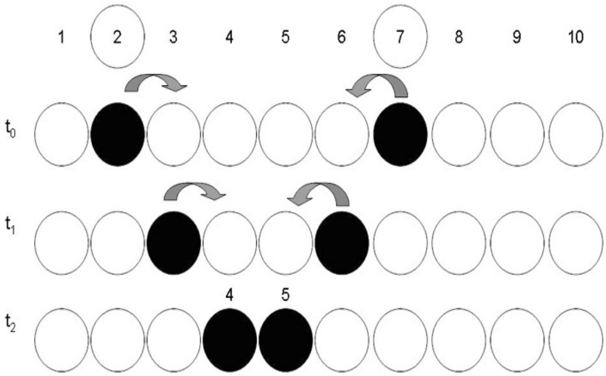

Fig. 3. The bisection problem: the odd sum case

6. One single node activation is the sum of all energies that collects.
7. Every node has an accumulation threshold that, when overcome, interrupts the network dynamic. Let us dictate that this threshold is equal to 1.5.

Such a neural network is able to calculate the intermediate number between two limits whose sum is an even number and the two intermediate numbers when the sum is an odd number. Figure 2 and 3 represent two examples: in figure 2 we have the intermediate number between 1-7 (even sum), figure 3 represents the odd sum (2-7).

To obtain in every case an univocal result, even if the limit numbers sum is odd, it is necessary hypothesize that various neural groups at moment t_0 present different energetic levels in order to have a convergence toward a single neural group. In other words, it is necessary to hypothesize that to each neural group is associated an activation coefficient or energy gradient that reinforce or soften stimulation coming from outside. For example, at time t_0, the energy level associated is inversely proportional to the numerical value to which it is associated. This relation is shown in figure 4. It is necessary to underline that this relation is arbitrary.

Let us modify the point 2. of the above described neural dynamic in the following way:

2. At time t_0, nodes coding upper and lower bound have an activation that varies in function of an activation coefficient that is specific for each neuronal group. All the other nodes have activation 0.0.

This condition is the equivalent of defining a neuro-cognitive bias that, depending on the activation coefficient associated to lower and upper bound produces a solution of rounding up or down. Obviously this bias is valid only for "odd sum" problems. The example in figure 3, with this change in neural dynamic and of parameters defined in figure 4, produces a new solution, as illustrated in figure 5.

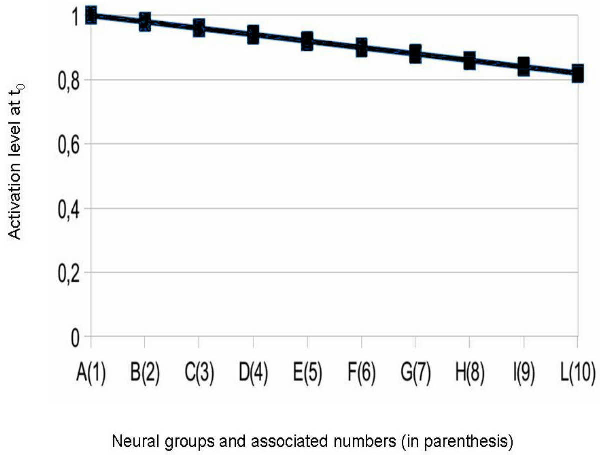

Fig. 4. The bisection problem: the odd sum case

The relation defined in figure 4 imposes to the network to select an univocal intermediate point also in cases where the limit numbers is an odd number (see figg. 3 and 5). It is worth noting that the relation between initial activation level of neural groups reported in fig.4 produces rounding up solutions, the intermediate point of "odd sum" problem is placed toward the upper bound. Obviuosly if we change the relation, the rounding changes too. For example, if the relation is directly proportional we have a rounding down. If the relation is non-linear we would observe sometimes a rounding up, some others down.

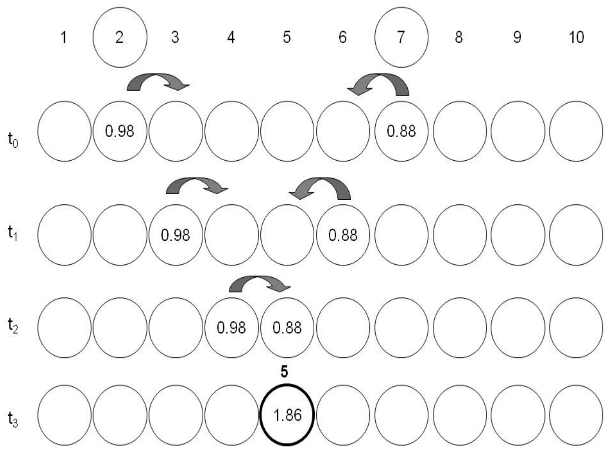

Fig. 5. The bisection problem: the odd sum case with the modified neural dynamic

This network is able to calculate the natural number between two limit numbers. In even sum case, it produces solutions without any mistake. On the contrary, human beings committ systematic errors, as shown in the introduction. They select, in some conditions, the intermediate number toward the lower limit (rounding down) and, in some others, toward the upper bound (rounding up).

This means that the initial activation gradient, shown in figure 4, is non linear, as shown by already cited studies. One possible explanation can be that when a certain number is presented a particular neural group activates selectively. Small numbers (1, 2, 3) are always associated to the same neural group, whereas bigger numbers are associated with neural groups with a certain probabilty. In other words a given neural group can activate more probably than others, but other groups, devoted to coding numbers that are close to the presented one, can activate too. For example, ift number 8 is presented, neural groups for 7 and 9 can activate too. If we introduce in our network dynamic this phenomen, the item 1 becomes:

1. A neural group is **probabilistically** associated to a natural numbers. Each neural group has a probability distribution where the association between the neural group and the natural number is defined.

Behavioral and neurophysiological evidence show that representations of increasing numerosities increasingly overlap, thereby becoming progressively less discriminable from adjacent ones [9]. Let us now imagine to adopt the probability distribution as reported in figure 6, deduced from cited studies.

Applying the relation illustrated in figure 6, a subject to whom is proposed to identify the intermediate number between 1 and 7 can accomplish the task sometimes as the demand is with 1 and 6 limits and some others as it is with 1 and 8 limits. Data indicate that the systematic error is non-linear: sometimes we observe a overestimation, in other cases an underestimation. For these reasons we modify figure 4 giving it a non-linear trend, as shown in figure 7.

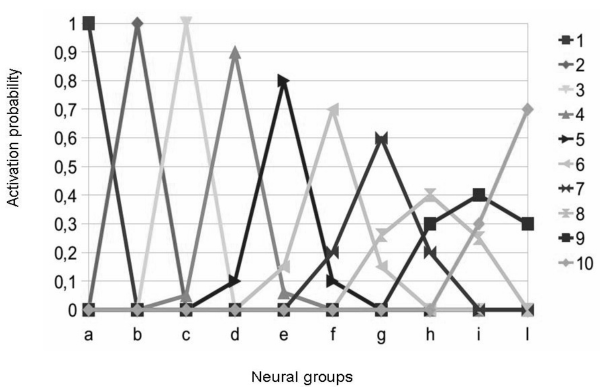

Fig. 6. Probability distribution of each neural group activation for number from 1 to 10

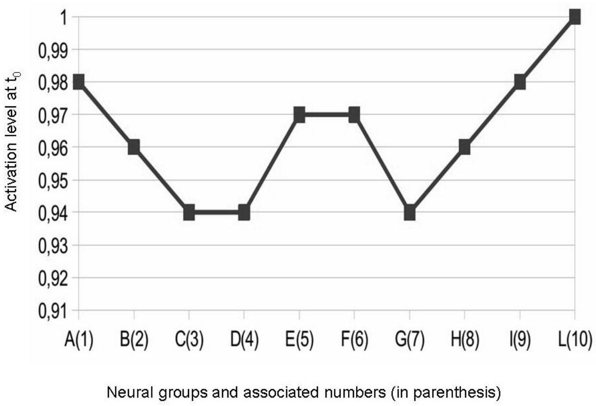

Fig. 7. Probability distribution of each neural group activation for number from 1 to 10

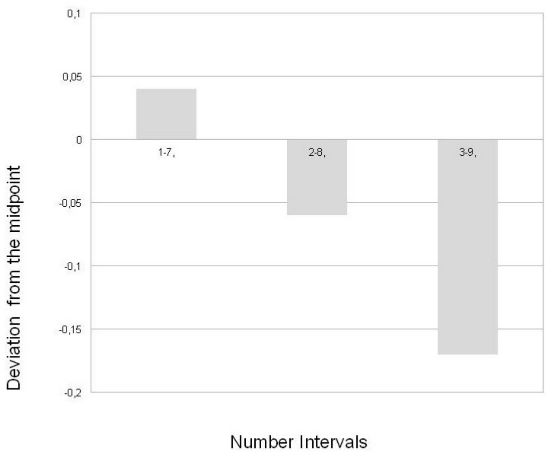

Fig. 8. Data about Number Intervals Bisection with 7 units derived from the neural model

Please not that the parameters related to the model have been obtained using a genetic algorithm [3]. For more detailsl, please look at additional materials section.

3 Results

The proposed model is able to replicate the data observed with children [12] indicating the presence of NIPE effect. In figure 8 the data about the 7 units task are reported. The results reported in figure 8 indicate that the model shows the NIPE effect too, displayng the same trend as the children. The artificial neural network in fact commits a systematic error that is consistent with the NIPE effect, in fact the closer one boundary of the interval was to the border of a ten, the more its midpoint was shifted.

4 Conclusions and Future Directions

In the present paper we have described a neural model that replicates data observed in mental bisection of numerical intervals in children. This model is exclusively based on energy transfer and accumulation and, despite of this, it can replicate data observed in children.

These results support the idea that the mental number line does not represents numbers in a spatial guise and the arithmetics module can, at least in principle, work on energy transfer rather than on number spatial representation.

Numbers are a fundamental part of our cognitive environment and it is worth interrogating on how they are represented in the brain. Feigenson and collegues [6] underline that there are two core systems that underlie the ability to think and reason about number: one system that is devoted to represent large, approximate numerical magnitudes, and another system that precisely represent small numbers of individual objects. These systems are shared across different developmental stages and different species and represent the basis on which the sophisticated human numerical ability is built.

The reported studies and the presented results indicate that the NIPE effect is widespread too. It can be observed in human adult and children and in our artificial model too. What does this tell us about number representation? The NIPE effect can mirror the logarithmic central representation of numerical magnitudo that is independent from school education and that is shared by non-human species too.

In this context a computational model can be an interesting way to approach cognitive issues [7,10]. Artificial models, in fact, can give us the chance to produce an artifact to be included in the list of species to be studied. If comparative sciences can give us insights about cognition, artificial models can give further insights in reproducing a certain phenomenon. In this case the scientific challenge is buiding a new artificial species with its own specific features. These artificial networks can reproduce phenomena at various levels: behavioural, physiological, neural with different granularity from the single neuron to whole structures. This approach has been already used in modelling neuropsychological phenomena,[13] linking these phenomena with neural representation as well as organisms interaction with the environment [8] giving useful insights to this research field.

The next step will be to build an extended model with more different layers to reproduce not only the behavioural side of NIPE effect but also the supposed corresponding neural circuitry.

Additional Materials

More details about the model and the related code can be provided to whom is interested by emailing the author Orazio Miglino (orazio.miglino@unina.it) or Michela Ponticorvo (michela.ponticorvo@unina.it).

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