

Contents

1	Asymptotic Estimates	1
1.1	Estimates of Functions	1
1.1.1	Basic Definitions	1
1.1.2	Operations with Symbols	3
1.1.3	Exercises	4
1.2	Asymptotic Series	5
1.2.1	Definitions	5
1.2.2	Properties of Asymptotic Series	7
1.2.3	Exercises	9
1.3	Newton Polygons	9
1.3.1	Introduction	9
1.3.2	Statement of the Problem	10
1.3.3	Newton Polyhedra	16
1.3.4	Exercises	18
1.4	Variation Index of a Function	18
1.4.1	Definitions	18
1.4.2	Auxiliary Definitions	20
1.4.3	Exercises	21
1.5	Asymptotic Solution of Transcendental Equations.	21
1.5.1	Exercises	26
1.6	Solution of Systems of Linear Algebraic Equations.	27
1.6.1	Regular Unperturbed Systems	27
1.6.2	Singular Unperturbed Systems in Special Cases.	28
1.6.3	Singular Unperturbed Systems in General Cases	29
1.6.4	Exercises	30
1.7	Eigenvalue Problems	31
1.7.1	Multiple Eigenvalues	32
1.7.2	Generalized Eigenvalue Problems.	33
1.7.3	Spectrum of a Bundle of Operators	34
1.7.4	Exercises	36
1.8	Answers and Solutions	37

2	Asymptotic Estimates for Integrals	51
2.1	Series Expansion of Integrands.	51
2.1.1	Exercises	54
2.2	Integration by Parts	55
2.2.1	Exercises	58
2.3	Laplace Method	58
2.3.1	Exercises	63
2.4	Stationary Phase Method	64
2.4.1	Integrals Without Stationary Points	64
2.4.2	Erdélyi's Lemma	67
2.4.3	Integrals with Stationary Points	68
2.4.4	Complete Asymptotic Expansions	71
2.4.5	Exercises	73
2.5	Saddle Point Method	73
2.5.1	Description of the Method	73
2.5.2	Asymptotics of Airy's Functions	74
2.5.3	Exercises	77
2.6	Answers and Solutions	77
3	Regular Perturbation of Ordinary Differential Equations	89
3.1	Introduction	89
3.2	Cauchy Problems	91
3.2.1	Motion of a Material Point in a Gravity Field	91
3.2.2	Duffing Equation	93
3.2.3	Exercises	95
3.3	Periodic Solutions	96
3.3.1	Solution of Non-autonomous Quasilinear Equations Without Resonance	97
3.3.2	Solution of Non-autonomous Quasilinear Equations with Resonance.	99
3.3.3	Poincaré's Method	103
3.3.4	Exercises	109
3.4	Transient Regimes	109
3.4.1	Van der Pol Method.	110
3.4.2	Stability of Stationary Solutions.	113
3.4.3	Multiscale Method	116
3.4.4	Exercises	120
3.5	Boundary Value Problems	121
3.5.1	Non-homogeneous Boundary Value Problems	121
3.5.2	Eigenvalue Problems	125
3.5.3	Boundary Value Problems for Equations with Highly Oscillating Coefficients.	129
3.5.4	Exercises	132
3.6	Answers and Solutions	135

4 Singularly Perturbed Linear Ordinary Differential Equations. 155

4.1 Solutions of Linear Ordinary Differential Equations 155

 of the n th Order 155

 4.1.1 Simple Roots of the Characteristic Equation 156

 4.1.2 Multiple Roots of the Characteristic Equation 159

 4.1.3 Asymptotic Solutions of Parameter-Free Equations. 161

 4.1.4 Asymptotic Solutions of Non-homogeneous Equations . . . 163

 4.1.5 Exercises 164

4.2 Solutions of Systems of Linear Ordinary Differential Equations 165

 4.2.1 Simple Roots of the Characteristic Equation 165

 4.2.2 Multiple Roots of the Characteristic Equation 168

 4.2.3 Asymptotic Solutions of Parameter-Free Systems. 170

 4.2.4 Asymptotic Solutions of Non-homogeneous Systems 171

 4.2.5 Equations of the Theory of Shells 171

 4.2.6 The Case $m \sim \mu^{-1}$, $\Lambda \sim 1$ 173

 4.2.7 The Case $m \sim \mu^{-1/2}$, $\Lambda \sim 1$ 174

 4.2.8 Low Frequency Vibrations of Shells of Revolution of Zero Gaussian Curvature 175

 4.2.9 Low Frequency Vibrations of Shells of Revolution of Negative Gaussian Curvature 176

 4.2.10 Exercises 177

4.3 Non-homogeneous Boundary Value Problems 178

 4.3.1 Statement of Boundary Value Problems 178

 4.3.2 Classification of Solution Types. 180

 4.3.3 The Simplest Case 180

 4.3.4 Deflection of a Beam on an Elastic Foundation 182

 4.3.5 Regular Degeneracy 186

 4.3.6 Non-absolutely Flexible String. 188

 4.3.7 Axisymmetric Deformation of a Shell of Revolution 190

 4.3.8 Shell Deformation Under External Pressure. 193

 4.3.9 Shell Deformations Under Axial Force 195

 4.3.10 Exercises 196

4.4 Eigenvalue Problems 197

 4.4.1 Asymptotics Solutions of Eigenvalue Problems 197

 4.4.2 Vibrations of Non-absolutely Flexible Strings 200

 4.4.3 Vibrations of Strings with Variable Density. 202

 4.4.4 Vibrations of Beams with Variable Cross-Section 203

 4.4.5 Axisymmetric Vibrations of Cylindrical Shells. 204

 4.4.6 Exercises 208

4.5 Eigenfunctions Localized in a Neighborhood of One End of the Interval 209

 4.5.1 Vibrations of Rectangular Plates 209

 4.5.2 Vibrations and Buckling of Shells 211

4.5.3	Vibrations of Cylindrical Panels	214
4.5.4	Buckling of Cylindrical Panels	216
4.5.5	Exercises	217
4.6	Answers and Solutions	218
5	Singularly Perturbed Linear Ordinary Differential Equations with Turning Points	239
5.1	Airy Functions	239
5.1.1	Exercises	240
5.2	Solutions of Second-Order Ordinary Differential Equations with Turning Points	241
5.2.1	Asymptotic Expansion of Solutions	242
5.2.2	Turning Points at the Ends of Integration Intervals	243
5.2.3	Interior Turning Points	245
5.2.4	Vibrations of Strings on Elastic Foundations	246
5.2.5	Asymptotic Expansions of Bessel Functions	249
5.2.6	Exercises	251
5.3	Solutions of Systems of Linear Ordinary Differential Equations with Turning Points	252
5.3.1	Splitting Theorem	253
5.3.2	Vibrations of Circular Plates	255
5.3.3	Vibrations of Shells of Revolution	258
5.3.4	Exercises	260
5.4	Localized Eigenfunctions	261
5.4.1	Existence Conditions for Localized Solutions	262
5.4.2	Construction of Localized Solutions	262
5.4.3	Vibrations of Prolate Ellipsoidal Shells of Revolution	264
5.4.4	Buckling of Cylindrical Shells Under Non-uniform Compression	267
5.4.5	Exercises	269
5.5	Answers and Solutions	271
6	Asymptotic Integration of Nonlinear Differential Equations	291
6.1	Cauchy Problems for Ordinary Differential Equations with a Small Parameter	291
6.1.1	Problem Statement	291
6.1.2	Construction of a Formal Asymptotic Solution	293
6.1.3	Exercises	298
6.2	Perturbation of Nonlinear Boundary Value Problems with a Small Parameter	299
6.2.1	Introduction	299
6.2.2	Exercises	304

- 6.3 Bifurcation of Solutions of Nonlinear Equations 304
 - 6.3.1 Statement of the Problem 305
 - 6.3.2 Solution of Nonlinear Problems 306
 - 6.3.3 Exercises 307
- 6.4 Answers and Solutions 308

- Bibliography** 319

- Index** 323



<http://www.springer.com/978-3-319-18310-7>

Asymptotic methods in mechanics of solids

Bauer, S.M.; Filippov, S.B.; Smirnov, A.L.; Tovstik, P.E.;
Vaillancourt, R.

2015, XXI, 325 p. 88 illus., Hardcover

ISBN: 978-3-319-18310-7

A product of Birkhäuser Basel