If a picture is worth a thousand words, how much is a video worth? To answer this question, please take a look at a video demonstrating the capabilities of a Field-Programmable Gate Array (FPGA) for implementing chaotic systems: https://www.youtube.com/watch?v=wwa7aylrLGo&index=1&list=PLr9kJRBrySkf3P4yiWAxz-CdzaWhTof-PW.

Now that we have you convinced about the robustness of twenty-first century FPGAs, let us talk about the book. In a single sentence, the purpose of this volume is to expose the reader to the exciting world of chaos (science) using FPGAs (engineering). This book differs from other texts on chaos because of a variety of reasons. First, our experimental approach toward chaos theory attempts to provide the reader with a “physical-feel” for the mathematics involved. Second, the medium that we use for physical realization and experiments is the FPGA. These devices are massively parallel digital architectures that can be configured to realize a variety of logic functions. Hence unlike microcontrollers that run sequential compiled code, FPGAs can be configured to execute systems of discrete difference equations in parallel. Moreover, ever since the early twenty-first century, one could realize design specifications in a mathematical simulation software such as Simulink directly onto an FPGA. Also, twenty-first century FPGA development boards have digital-to-analog converters, hence the signals viewed on the oscilloscope are analog waveforms from our digital chaotic realization!

Nevertheless, maximizing the capabilities of an FPGA requires the user to deeply understand the underlying hardware and also the software used to realize the differential equations. This is achieved by another feature in this book: a lab component (along with exercises) in each chapter. In the lab component, readers are asked to investigate chaotic phenomena via MATLAB (or Simulink), design digital logic systems on FPGAs, and also implement a variety of chaotic systems. The specific objective of the lab depends obviously on the particular chapter. Note that one could use FPGAs and development platforms from other manufacturers to understand the concepts in this book. But, for the sake of brevity, we use the Altera Cyclone IV FPGA on a Terasic DE2-115 board which includes an onboard ADA (Analog-to-Digital and Digital-to-Analog) from the audio codec (coder/decoder).
Details on procuring hardware are in Chap. 2. However, understand that FPGA technology is changing rapidly and the hardware (software) used in this book will become quickly outdated. Thus from this book, one has to learn the concepts used in implementing nonlinear ordinary differential equations on FPGAs.

This text is intended for final-year undergraduate or graduate electrical engineering students who are interested in a scientific application of an engineered product. Knowledge of digital logic system (combinational and sequential) realization on FPGAs and an integral calculus course is necessary. A first-year undergraduate course in FPGA-based digital logic and a first-year undergraduate calculus course is necessary and sufficient. However, the only prerequisite for understanding this book is a thirst for knowledge and the willingness to overcome failure. To quote Albert Einstein, “Anyone who has never made a mistake has never tried anything new.”

This book is organized as follows: Chapter 1 is an introduction to both chaos theory and FPGAs. Some mathematical foundations for working with chaos are discussed. Chapter 2 gives an overview of the FPGA hardware platform and includes tutorials on utilizing the DE2-115. Chapter 3 shows how to simulate and realize some chaotic ODEs on FPGAs. This chapter combines the ideas in Chaps. 1 and 2 to show the reader how to implement chaotic systems on FPGAs. Chapters 4 and 5 are more mathematical in nature and serve as a precursor to Volume II (Theoretical Methods). Chapter 4 shows how to study bifurcation mechanisms in chaotic systems using FPGAs. Chapter 5 covers time-delayed systems. The FPGA is particularly suited for implementing time-delayed systems because one can implement the necessary delay using \( n \) flip-flops (\( n \) could be 4096!) by using only five lines in a hardware description language!

There is also a companion website: http://www.harpgroup.org/muthuswamy/ARouteToChaosUsingFPGAs/ARouteToChaosUsingFPGAs.html for the book that has recorded video solutions (maximum 20 minutes) to all book exercises+labs, FPGA reference designs and corresponding videos (maximum 20 minutes), forums for discussing other hardware platforms, etc. It would be prudent to have access to the internet as you read through this book.

Note that this volume is an “engineering cookbook” that is full of examples for implementing nonlinear dynamical (chaotic) systems on FPGAs. The second volume (theoretical methods) is more rigorous and covers concepts for digital emulation of chaotic dynamics. However, either volumes can be used as stand-alone texts for understanding FPGA-based chaotic system implementation.

This book was typesetted using \LaTeX. Image processing softwares used were GIMP 2.8, Inkscape 0.48, and Xfig 3.2. An iPhone 5 camera was used for pictures.
NCH Debut Professional v1.64 was used for screen recordings. Oscilloscopes used were the Agilent 54645D and the Tektronix 2205.

Happy chaos via FPGAs!

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About the book title: it is intended to be a pun on the mathematical concept of a route to chaos.
About the cover art: we show a chaotic attractor from the Ikeda DDE.
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