Chapter 2
Interactions with the Atmosphere and Road

2.1 Introduction

The interactions of a car with its environment—gravity, the atmosphere, and the road surface—create forces which act on the car, usually opposing its motion. This chapter shows how these forces are related to the characteristics of the car under the designer's control, such as its shape and weight, and to the effort required to move it: the tractive force. A magnitude will be calculated to give the reader an idea of the importance of each interaction. The calculations will use the characteristics of actual solar racing cars to make the numbers realistic.

No attempt is made to present an exhaustive treatment of each interaction. However, the most important features have been presented. Additional details will be found in Chaps. 17, 18, 19 and 20. Readers who wish to pursue topics in even greater depth may consult the references at the end of the chapter.

Cruise Condition This chapter concentrates on the interactions going on most of the time, which will be called the *cruise condition*. The cruise condition plays the strongest role in setting the energy consumption of the car, which determines the range. In this condition, the car moves straight ahead or turns through a large-enough radius or at a slow-enough angular rate, such that inertia-related forces transverse to the car's direction of motion are relatively small. Also, the wind relative to the car, which interacts with the shape of the car to create aerodynamic forces, blows from nearly directly ahead. The aerodynamic side forces are therefore relatively small. Thus, in the cruise condition all inertial and aerodynamic forces acting transverse to the direction of motion, and their associated moments, are neglected.

Intermittent Conditions However, important interactions are associated with events that happen intermittently, such as side gusts and emergency maneuvers. The transverse forces generated in these events strongly influence the controllability, stability, and structural design of a solar-electric vehicle.

When passed by a large vehicle, such as a truck, or when a wind gust blows suddenly from the side, a car experiences wind forces which tend to blow it sideways and rotate it about the vertical axis through its center of gravity. Solar racing cars

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tend to be light and are thus more sensitive to side gusts than conventional vehicles. Chapter 21, *Stability Calculations,* presents a method for predicting the effect of a side gust.

A solar car maneuvering in an emergency may be required to accelerate or brake while turning. In this situation, the car should remain controllable[1] and hence must not skid nor roll over, and its structure must withstand the moments and forces developed by the maneuver. Chapter 21 presents an analysis of the stability of the vehicle.

2.2 Equivalent Interactions

The forces on the car are distributed over the car or some portion of it. Gravity acts on the entire mass of the car. The friction force of the road on the tires acts over the area of the tire in contact with the road. In general, each force tends to both translate the car in and rotate the car about, at least one coordinate direction. To conveniently model the dynamics and energetics of the vehicle, we replace each distributed force by an equivalent isolated force and its associated moment. That is, the isolated force and moment have the same translational and rotational effects as the distributed force which they replace. Each of the equivalent forces acts at a convenient point, such as the center of gravity. Each of the equivalent moments acts about a convenient axis (usually a coordinate axis). Henceforward when the terms "force" and "moment" are used, it will usually be the equivalent, isolated forces and moments that are meant.

2.3 Coordinate Systems

Figure 2.1 shows a set of coordinate axes attached to the center of gravity of a solar-electric car so that the axes always point in the same directions relative to the car. The positive direction of each axis is shown. A force acting in the positive direction of each axis is defined as positive. A semicircular arrow about each coordinate axis shows the positive direction of the moments about those axes.

Aerodynamic forces arise from the motion of the air relative to the car. Thus, it is natural when discussing this relative motion to think of the car as stationary with axes attached to it. This is exactly the situation when the aerodynamic forces are measured in a wind tunnel, for instance. When it is more convenient, we will revert to thinking of the car as moving with respect to a coordinate system fixed to the earth, such as at the starting line at the Indianapolis Motor Speedway.

[1] Within specified design limits; absolute stability cannot be achieved.

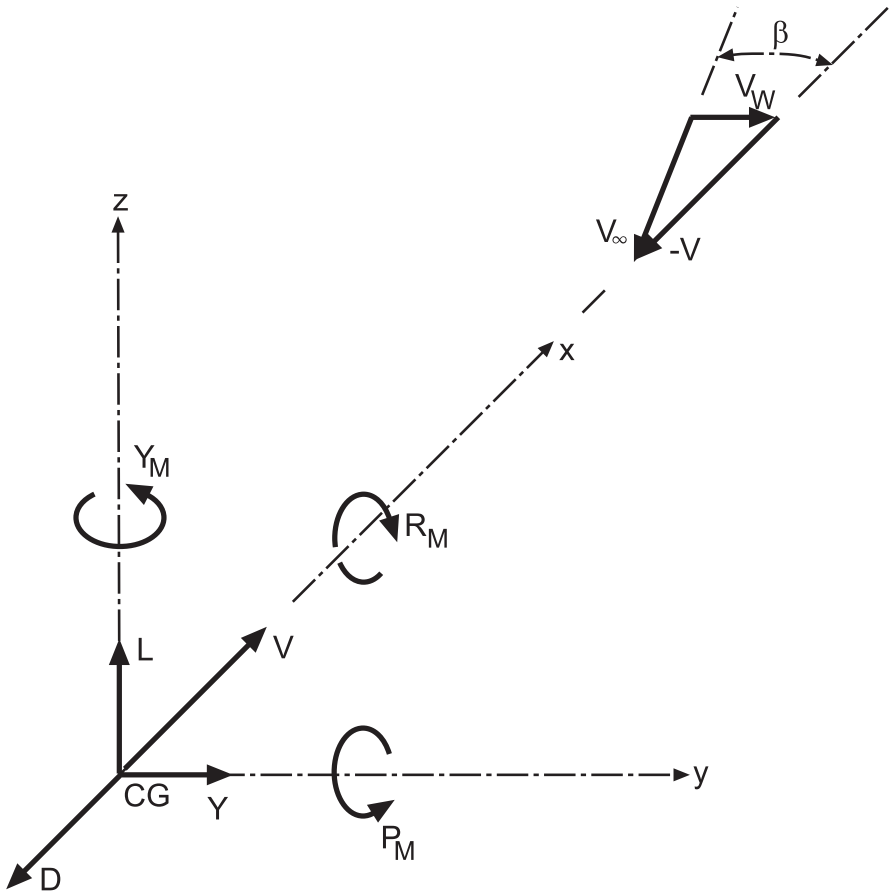

Fig. 2.1 Coordinate axes

2.4 Aerodynamic Interactions

We will select the point of action of aerodynamic forces as the center of gravity of the vehicle. Wind tunnel measurements of the moments of these forces are by convention often referenced to a point on the centerline of the car and halfway down the wheelbase. However, it is convenient when discussing their effects on motion to reference these moments to the center of gravity, as for the forces.

Figure 2.1 shows three aerodynamic forces and three aerodynamic moments, a force and moment for each coordinate axis, each named to suggest how it tends to affect the car's motion. The force acting along the x-axis is called *drag (D),* that acting along the y-axis is called *side force (Y),* and that acting along the z-axis is called *lift (L)*. The moment about the x-axis is called *roll* (R_M), that about the y-axis is called *pitch* (P_M), and that about the z-axis is called *yaw* (Y_M). In general, each of the forces and moments can be positive or negative.

Figure 2.1 also shows a vector representing the result of subtracting the car's velocity *(V)* from the true wind vector. This result is the *relative wind*, V_R, the motion of the air relative to the car, but sufficiently far upstream of the car so that it is undisturbed by the shape of the car. The relative wind blows from a *yaw angle*, *β,* measured from the *x*-axis and positive in the direction of positive Y_M.

In the cruise condition, the side force, yawing moment, rolling moment, and yaw angle are zero. Pitch, drag, and lift remain. How these arise from interactions between the flow field relative to the car and the car's shape, attitude, and internal flow passages will now be discussed.

Drag Experiment This section elaborates a bit on an example in Sherman (1990). Suppose you stir a mixture of small pepper grains and water in a white cup (so you can see the grains and thus visualize the flow) and then remove the spoon. The whirling motion of the mixture persists but eventually slows to a stop. The persistence depends upon the fluid's *momentum,* which in turn depends on both the mass of the fluid and its rotational speed. One would expect the liquid metal mercury in an identical cup to whirl for a longer time than water.

The mixture does not whirl forever but comes to rest because the friction force caused by the *viscosity* of the fluid opposes the rotation. Viscosity measures a fluid's resistance to flowing relative to itself, just like your hands resist being rubbed against each other. As in that case, the friction force is tangent to the flow. All fluids have viscosity; in some, such as air, it is small and in others, such as honey, it is large. (Try the experiment with a cup of honey. The pepper will stay on the honey's surface, but it will still help to visualize the flow. The friction force could rotate a light cup in this case. Would this show that the force is tangential to the cup's inner surface?)

If you observe the pepper, you will see (especially if you have been able to impart mostly circular motion to the mixture) that the grains near the inner surface of the cup slow down first. It turns out that the mixture actually contacting the cup's surface is at a speed of zero, which is called the *no slip* condition. So the rotational speed of the mixture is zero at the cup surface but increases toward the center. (You may observe other motions as well.) Because the friction force is created when the fluid resists flowing relative to itself, the speed difference (or *gradient*) must be present to give the friction force.

The foregoing discussion will be of use in understanding the friction drag on a car moving through air.

Parked Car A car parked along a road in still air, like the pepper grains in the cup or a fish motionless in a pond, is immersed in a fluid: the atmosphere. This mixture of gases (about 75 % nitrogen and 25 % oxygen) presses on every part of the outside (and inside) of the car. This pressure force distribution is called *static* because the atmosphere is not moving relative to the car at any point on it.

The static pressure is not uniformly distributed over the body of the fish, being greater underneath it because of the greater depth. This is also true for the parked car. However, the density of water is about 850 times greater than that of air at standard conditions (temperature 298.15 K, pressure 101.325 kPa). Consequently, the maximum pressure difference across the car is on the order of 0.0002 atm. So, differences in height between parts of the car may be neglected and the static pressure distribution taken as uniform over the car. Thus, the net static pressure force on the car is zero.

Moving Car As the car moves down the road, air flows over the surface of the car. This relative motion[2] changes the pressure distribution such that a net pressure force is created that opposes the car's motion. The external flow also applies a retarding tangential friction force to the car's surface, as in the stirred-cup experiment. Also, air flows through the car for ventilation. The net pressure loss in internal passages, caused by friction and the losses in ducting bends, dampers, and other components, also exerts a retarding force. It can be as much as 8–10 % of the total.

The total of the external and internal retarding forces we call *drag*. The magnitude of the drag is expressed by

$$D = c_{\mathrm{D}} A_D q. \tag{2.1}$$

The drag coefficient, c_{D}, a dimensionless quantity, characterizes the drag of the car and changes with the flow, in general. The *dynamic pressure* of the relative air speed far from the car, q, is given by

$$q = \frac{1}{2}\rho V_{\mathrm{R}}^2. \tag{2.2}$$

The dynamic pressure is the pressure increase above the ambient static pressure that would occur if the flow were brought to a halt with no losses (stagnate) against a surface. The air density (ρ) may be computed from the ideal gas equation

$$\rho = \frac{p}{R_{\mathrm{A}} T}. \tag{2.3}$$

The gas constant for air (R_{A}) is 0.287 kJ/kg·K. At standard temperature and pressure, Eq. (2.3) gives an air density of 1.184 kg/m^3. Note that for the same pressure, the drag is lower if the air is hotter and higher if the air is cooler.

In order to give units of force, the dynamic pressure must be multiplied by an area. By convention, the area used is the profile area (A_D), the area blocked out by the car when viewed from straight ahead. The product $c_{\mathrm{D}}A_D$ is called the *drag area*. Chapter 17 presents a means of estimating the drag area of a candidate body shape. Measurement of the drag area of a scale model or full-scale vehicle in a wind tunnel or by coast-down testing will be discussed in Chap. 12, *Testing*.

The drag coefficient incorporates all of the opposing drag force components mentioned: friction (c_{F}), pressure (c_S), and ventilation (c_{V}). Referring each component to $A_D q$ gives

$$c_{\mathrm{D}} = c_{\mathrm{F}} + c_S + c_{\mathrm{V}}. \tag{2.4}$$

We shall now explain in more detail why these components arise.

[2] It is the relative motion that counts; you could also blow on a stationary car and create drag, as in a wind tunnel.

2.5 Friction Drag

Boundary Layer Suppose that the relative airflow approaching the car is smooth and at zero yaw angle ($\beta = 0$ in Fig. 2.1). Like the water–pepper mixture in the cup, the air at the car's surface moves at zero speed relative to that surface. However, air farther from the surface moves nearer to the relative speed of the surrounding air, as shown in Fig. 2.2 (in which the *n*-axis is the local vertical). The air layer over which the local relative flow speed changes from zero to 99 % of that of the surrounding air is defined as the *boundary layer*.

The boundary layer thickens as the distance from the front of the car increases. A velocity gradient now exists in a viscous fluid. Hence, the air applies a retarding frictional force tangent to the surface of the car. As the car increases speed, the gradient becomes steeper, and the friction force at the surface increases. Figure 2.2 shows the gradient at the surface as the slope ($\Delta V_R/\Delta n$) of the tangent to the velocity distribution at that point. The symbol τ_0 represents the friction force per unit surface area. The *streamlines* shown in Fig. 2.2 are imaginary lines tangent to the local flow velocity.

Viscosity As we expect from the cup experiment, the proportionality factor between friction force and the velocity gradient is the viscosity of the air *(μ)*.[3] If the car were moving through water, the viscosity of which is about 48 times that of air at 25 °C, the frictional drag would be much larger at a given speed (remember the honey). Near atmospheric pressure, the viscosity of air shows a weak tendency to

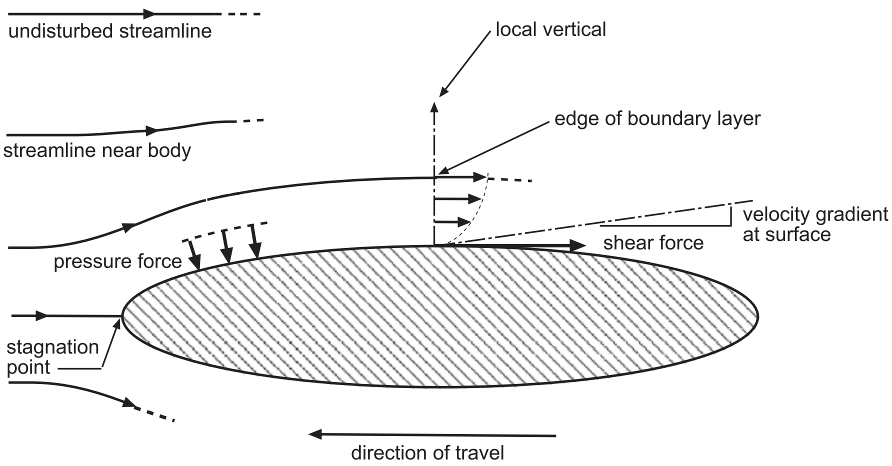

Fig. 2.2 Boundary layer and aerodynamic forces

[3] Many fluids obey this relation between the surface shear force and the velocity gradient, air and water, for instance. Such fluids are called newtonian, after Sir Isaac Newton, who first proposed this linear model.

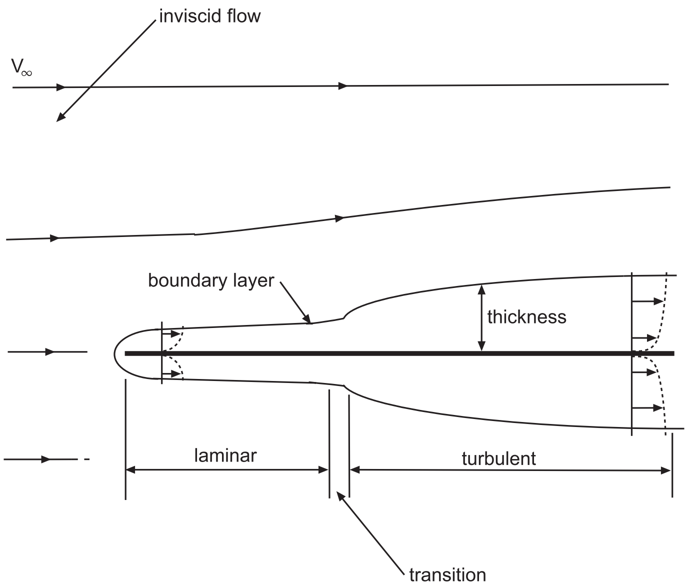

Fig. 2.3 Laminar and turbulent boundary layers

increase with pressure and a strong tendency to increase with temperature. Thus, the frictional drag increases with increasing temperature.

Is this the whole story of friction drag? No, unfortunately; we are headed for more trouble as we increase speed.

Laminar and Turbulent Flow Figure 2.3 shows a smooth, flat plate traversed by initially smooth air flow.

The flow contacts the plate, and the boundary layer forms. The flow in the boundary layer at this early stage is still smooth. We can visualize this as very thin air layers, *lamina,* moving relative to each other with no velocity components transverse to their motion. The flow is called *laminar* because of this characteristic. A particle of air striking, say, a small bump (no actual surface can be exactly smooth) may deflect up a bit, but the viscous friction of the other particles drags it back into line, keeping the flow laminar.

An impulsive force was applied between the fluid and the surface when the element glanced off the little bump and thereby gained momentum away from the surface. Further on, at the next bump (or other disturbance, maybe even loud rock music), the thickening of the boundary layer has magnified the destabilizing impulse force relative to the stabilizing viscous force because the velocity gradient is reduced. So, as the boundary layer thickens, small waves appear in it. The waves grow into chaotic eddies and the boundary layer makes a transition to *turbulence,* accompanied by additional thickening.

Besides the torque need to overcome the viscous friction, the engine of the car must now exert extra torque on the driving wheel or wheels to cause the eddies to circulate. Consequently, the friction drag in turbulent flow is higher than in laminar flow.

Reynolds Number The foregoing discussion implies that a number correlated with the ratio of the impulsive to viscous forces in the boundary layer would also correlate strongly with the transition to turbulence. Because force is proportional to the rate of change of the momentum, we expect the impulsive force will be correlated with the momentum flow rate of the air external to the boundary layer. This is ρV_R^2, when expressed as force per unit area perpendicular to the flow, or $\rho V_R^2 A_{flow}$ in force units, where A_{flow} is a conveniently chosen reference area perpendicular to the flow. The friction drag per unit surface area is proportional to $\mu V_R/\ell$, where ℓ is the local boundary layer thickness and A_{fric} is a reference area on the car's surface. Since $\ell A_{flow}/A_{fric}$ has units of length and A_{flow} and A_{fric} are arbitrary, the ratio of interest is:

$$\mathrm{Re}_\lambda = \frac{\rho V_R \lambda}{\mu}, \tag{2.5}$$

where λ stands for a conveniently chosen reference length. Equation (2.5) defines the *Reynolds number*. For the present discussion, we choose the distance x from the nose of the car to a point in the boundary layer measured along the surface as the characteristic length because the thickening of the boundary layer depends on the distance from the nose.[4] The number, now called the *local* Reynolds number because it depends upon the location, is:

$$\mathrm{Re}_x = \frac{\rho V_R x}{\mu}. \tag{2.6}$$

The local Reynolds number at which the transition to turbulence begins on the surface is called the *critical* local Reynolds number. This number is usually found by experiment. The transition to turbulence is affected by the roughness of the surface: the rougher the plate, the lower the critical Reynolds number at which it begins. On the other hand, as Eq. 2.6 implies, for a given fluid and surface, the critical Reynolds number will be reached at a shorter distance from the nose when the flow is faster.

Thickness Compared to the characteristic dimension of the body in the flow direction, say the length of the plate in Fig. 2.3, the boundary layer is quite thin, even in turbulent flow. Suppose the length of the plate were 2 m and the latter portion of its surface were in turbulent flow, as shown, the boundary layer thickness would be only of the order of 4 cm at the trailing edge. (Its dimensions have been exaggerated in the figures.) Nevertheless, all of the viscous interaction of the airflow with a body takes place in the boundary layer. Compared to the boundary layer, the flow external to this layer may be treated as if it had no viscosity.

Total Friction Drag Because the local frictional force discussed above is expressed as a force per unit area, the total frictional force on the car is proportional to the surface area of the car. The larger this area, the larger the force will be. However, the

[4] The Reynolds number is important in other contexts. So, other reference lengths more appropriate for the context are defined for these cases.

line of action of the drag force is antiparallel to the direction of motion. Therefore, the friction force at a particular location contributes to the drag in proportion as the surface upon which it acts is parallel to the direction of motion. The flow over the upstream face of a rear-view mirror is nearly perpendicular to the direction of motion and therefore contributes little to the total friction drag force, for example. It contributes to the pressure drag, however, as the following discussion demonstrates.

2.6 Pressure Drag

Frictionless Flow We now return to the effect of shape on pressure. Figure 2.4 shows a cross-sectional view of the steady flow of air over two smooth cylinders. Both are very long compared to their diameters, so the complicating effect of flow near their ends may be neglected. Consider first the flow over the upper cylinder, for which we imagine the viscosity of the air to be zero, so that the flow is frictionless.[5]

Since a streamline is an imaginary line tangent to the local flow velocity, if the flow is undisturbed, all the streamlines are parallel and flat. Note that, by definition, flow cannot cross a streamline. Now we can imagine that the upper half of the cylinder is in a channel.[6] The upper "wall" of this channel is a surface formed by the streamlines of air far enough from the cylinder to be undisturbed by its presence. These bounding streamlines, taken together, could be called a *stream surface*. The lower wall is formed by the stream surface that hits the front of the cylinder and then follows its surface.

Bernoulli's Equation Consider any streamline between two vertical planes, such as those marked 1 and 2 in the figure. The flow is steady, there is no friction and, we assume, no heating of the air. Then it is true that for any two points along the streamline

$$\frac{p_1}{\rho_1} + \frac{V_1^2}{2} + gZ_1 - \left(\frac{p_2}{\rho_2} + \frac{V_2^2}{2} + gZ_2 \right) = 0. \tag{2.7}$$

Equation (2.7), called *Bernoulli's equation,* shows that

$$\frac{p}{\rho} + \frac{V^2}{2} + gZ = const. \tag{2.8}$$

along a streamline for the conditions assumed. Now, to unclutter things even more, we observe that the gravitational potential energy term, gZ, may be neglected for height changes on the order of the height of an automobile or truck. Also, if the flow were incompressible, the density, ρ, would be constant and then the quantity

[5] This apparently oversimplified scenario will still yield valid insights, believe it or not.

[6] Actually, we could equally well imagine the entire cylinder in a channel, but the drawing of a half cylinder takes up less space.

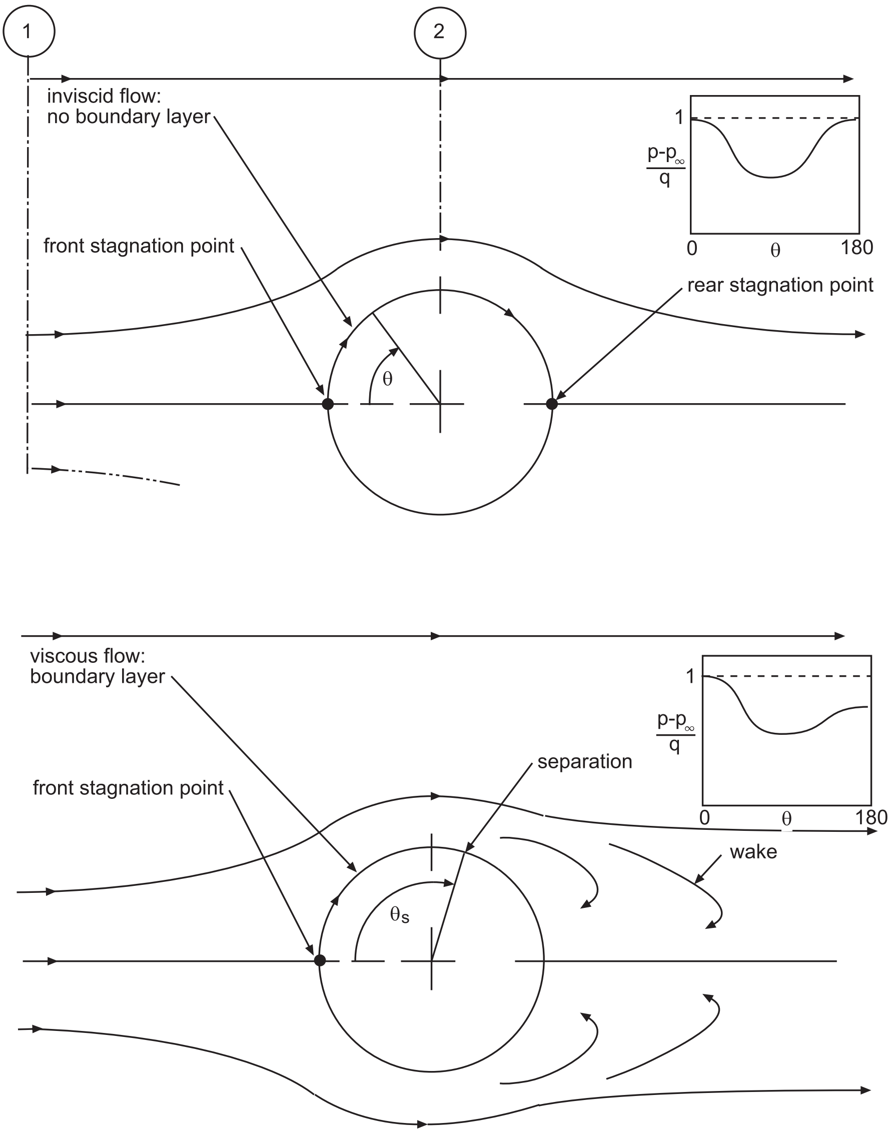

Fig. 2.4 Origin of pressure drag

$p+\rho V^2/2$, the *total pressure*, p_0, would be constant along a streamline. The pressure changes typical of external air flows produce only small changes in density. We will model such flows as incompressible.

Using our simple model, let us investigate the static pressure distribution on the upper cylinder of Fig. 2.4. Upstream of the disturbance of the cylinder, the velocity is uniform (all streamlines flat, parallel, and evenly spaced). Therefore, the pressure is uniform in the flow. At the forward-most point ($\theta=0°$ in the figure), the air speed is momentarily zero, so it is a stagnation point (even though it is really a line) and $p = p_0$. The flow then turns and moves up, tangent to the cylinder's surface. Hence, because the mass flow rate is steady and the density cannot change, the air speed increases to a maximum, and, as required by Bernoulli's equation, the static pressure decreases to a minimum as the point of minimum channel cross-sectional area at the top of the cylinder ($\theta = 90°$). Beyond this point, the air speed decreases and the static pressure increases. At the downstream location opposite to the front stagnation point ($\theta = 180°$), the tangential air speed component becomes zero. A rear stagnation point forms at which $p = p_0$ once again.

Figure 2.4 shows the pressure variation around the cylinder. Clearly the shape of the cylinder strongly influences the pressure distribution over it. But notice, there is no net pressure change across it in the flow direction for the ideal, frictionless conditions assumed. Therefore, there is no pressure drag.

Flow with Friction Pressure drag on objects immersed in a real, viscous fluid arises because of *boundary layer separation*. Consider the lower cylinder of Fig. 2.4, which is immersed in a real, viscous, approximately incompressible fluid such as air. A boundary layer now forms on the cylinder. Bernoulli's equation is invalid inside the boundary layer. But because, as we observed earlier, the flow external to the boundary layer is approximately frictionless and the boundary layer is quite thin, the pressure imposed on the boundary layer approximately obeys Bernoulli's law. The pressure increase on the downstream surface of the lower cylinder of Fig. 2.4 opposes the flow in the layer. The more sharply the surface curves down, the more rapid will be the opposing pressure increase predicted by Bernoulli's equation. At some position angle, this causes the velocity gradient at the surface to be zero.[7] At that point, the main flow ceases to follow the curved surface, and the boundary layer is said to *separate* from that surface. The flow then forms a turbulent wake, as shown. This causes the air pressure on the rear surface downstream of the separation area to drop below that near the front stagnation point and perhaps even below that of the ambient air. There is now a pressure force difference, high in front, low in back. This net opposing force is called *pressure drag* (or sometimes *profile drag*). Pressure or profile drag is reduced by making the shape less blunt.

Streamlining Figure 2.5 shows a cylindrically shaped body and a streamlined air-foil-shaped body, both with circular cross sections and having the same profile area. The drawing of the cylindrically shaped body shows that separation can occur at locations upstream of the trailing surface, such as at the forward corners of the box shape. Downstream of these locations, the flow may reattach to the car and some pressure loss be recovered.[8] Compare the cylindrical shape to the streamlined shape which minimizes separation and thus pressure drag, by avoiding rapid changes in the slope of its surface.

Qualitative pressure and friction force profiles for the two shapes are shown at the bottom of the figure (tear drop: dashed line). These curves were constructed using the exchange of pressure and velocity expressed in the Bernoulli equation.

[7] There will even be back flow downstream of the separation point.

[8] These local zones of separated flow are called separation bubbles.

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