Contents

1 Introduction ......................................................... 1
  1.1 A Brief History of Quantum Mechanics .............. 1
  1.2 Revolutionary Concepts of Quantum Mechanics .... 5
  1.3 Quantum Information ........................................... 7
  1.4 Content of the Book ........................................... 10
  1.5 Suggested Paths ................................................ 13
  1.6 Conventions on Notation ..................................... 14
References ............................................................. 16

Part I Fundamentals

2 Vector and Hilbert Spaces ...................................... 21
  2.1 Introduction ..................................................... 21
  2.2 Vector Spaces .................................................... 22
  2.3 Inner-Product Vector Spaces ................................. 25
  2.4 Definition of Hilbert Space .................................... 29
  2.5 Linear Operators ................................................ 33
  2.6 Eigenvalues and Eigenvectors ............................... 38
  2.7 Outer Product. Elementary Operators .................... 40
  2.8 Hermitian and Unitary Operators ......................... 44
  2.9 Projectors ........................................................ 47
  2.10 Spectral Decomposition Theorem (EID) ................. 54
  2.11 The Eigendecomposition (EID) as Diagonalization ...... 60
  2.12 Functional Calculus .......................................... 62
  2.13 Tensor Product ................................................ 67
  2.14 Other Fundamentals Developed Throughout the Book ... 74
References ............................................................. 75
## Contents

### 3 Elements of Quantum Mechanics

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1</td>
<td>Introduction</td>
<td>77</td>
</tr>
<tr>
<td>3.2</td>
<td>The Environment of Quantum Mechanics</td>
<td>78</td>
</tr>
<tr>
<td>3.3</td>
<td>On the Statistical Description of a Closed Quantum System</td>
<td>81</td>
</tr>
<tr>
<td>3.4</td>
<td>Dynamical Evolution of a Quantum System</td>
<td>86</td>
</tr>
<tr>
<td>3.5</td>
<td>Quantum Measurements</td>
<td>91</td>
</tr>
<tr>
<td>3.6</td>
<td>Measurements with Observables</td>
<td>98</td>
</tr>
<tr>
<td>3.7</td>
<td>Generalized Quantum Measurements (POVM)</td>
<td>102</td>
</tr>
<tr>
<td>3.8</td>
<td>Summary of Quantum Measurements</td>
<td>105</td>
</tr>
<tr>
<td>3.9</td>
<td>Combined Measurements</td>
<td>106</td>
</tr>
<tr>
<td>3.10</td>
<td>Composite Quantum Systems</td>
<td>111</td>
</tr>
<tr>
<td>3.11</td>
<td>Nonuniqueness of the Density Operator Decomposition</td>
<td>117</td>
</tr>
<tr>
<td>3.12</td>
<td>Revisiting the Qubit and Its Description</td>
<td>121</td>
</tr>
</tbody>
</table>

### References | 129 |

### Part II Quantum Communications

#### 4 Introduction to Part II: Quantum Communications

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.1</td>
<td>A General Scheme of a Telecommunications System</td>
<td>135</td>
</tr>
<tr>
<td>4.2</td>
<td>Essential Performances of a Communication System</td>
<td>137</td>
</tr>
<tr>
<td>4.3</td>
<td>Classical and Quantum Communications Systems</td>
<td>143</td>
</tr>
<tr>
<td>4.4</td>
<td>Scenarios of Classical Optical Communications</td>
<td>146</td>
</tr>
<tr>
<td>4.5</td>
<td>Poisson Processes</td>
<td>155</td>
</tr>
<tr>
<td>4.6</td>
<td>Filtered Poisson Processes</td>
<td>158</td>
</tr>
<tr>
<td>4.7</td>
<td>Optical Detection: Semiclassical Model</td>
<td>165</td>
</tr>
<tr>
<td>4.8</td>
<td>Simplified Theory of Photon Counting and Implementation</td>
<td>175</td>
</tr>
</tbody>
</table>

### References | 181 |

#### 5 Quantum Decision Theory: Analysis and Optimization

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.1</td>
<td>Introduction</td>
<td>183</td>
</tr>
<tr>
<td>5.2</td>
<td>Analysis of a Quantum Communications System</td>
<td>186</td>
</tr>
<tr>
<td>5.3</td>
<td>Analysis and Optimization of Quantum Binary Systems</td>
<td>192</td>
</tr>
<tr>
<td>5.4</td>
<td>Binary Optimization with Pure States</td>
<td>195</td>
</tr>
<tr>
<td>5.5</td>
<td>System Specification in Quantum Decision Theory</td>
<td>203</td>
</tr>
<tr>
<td>5.6</td>
<td>State and Measurement Matrices with Pure States</td>
<td>204</td>
</tr>
<tr>
<td>5.7</td>
<td>State and Measurement Matrices with Mixed States</td>
<td>204</td>
</tr>
<tr>
<td>5.8</td>
<td>Formulation of Optimal Quantum Decision</td>
<td>209</td>
</tr>
<tr>
<td>5.9</td>
<td>Holevo’s Theorem</td>
<td>211</td>
</tr>
<tr>
<td>5.10</td>
<td>Numerical Methods for the Search for Optimal Operators</td>
<td>213</td>
</tr>
<tr>
<td>5.11</td>
<td>Kennedy’s Theorem</td>
<td>216</td>
</tr>
</tbody>
</table>
Part III Quantum Information

10 Introduction to Quantum Information ........................................... 451
  10.1 Introduction ................................................................. 451
  10.2 Partial Trace and Reduced Density Operators ......................... 454
  10.3 Overview of Entanglement .................................................. 457
  10.4 Purification of Mixed States .............................................. 461
References .............................................................................. 462

11 Fundamentals of Continuous Variables ........................................ 463
  11.1 Introduction ................................................................. 464
  11.2 From Discrete to Continuous in Quantum Mechanics .................. 466
  11.3 The Harmonic Oscillator ................................................... 473
  11.4 Coherent States ............................................................ 479
  11.5 Abstract Formulation of Continuous Quantum Variables ............... 481
  11.6 Phase Space Representation: Preliminaries .............................. 484
  11.7 Phase Space Representation: Definitions for the N-Mode ............... 491
  11.8 Phase Space Representations in the Single Mode ....................... 499
  11.9 Examples of Continuous States in the Single Mode .................... 503
  11.10 Gaussian Transformations and Gaussian Unitaries ...................... 508
  11.11 Gaussian Transformations in the N-Mode ................................ 512
  11.12 N-Mode Gaussian States .................................................. 519
  11.13 Normal Ordering of Gaussian Unitaries ................................ 522
  11.14 Gaussian Transformations in the Single Mode ........................ 525
  11.15 Single-Mode Gaussian States and Their Statistics .................... 529
  11.16 More on Single-Mode Gaussian States .................................. 535
12 Classical and Quantum Information Theory 
12.1 Introduction .......................... 573
12.2 Messages of Classical Information .............................. 577
12.3 Measure of Information and Classical Entropy .................. 580
12.4 Quantum Entropy ........................................... 585
12.5 Classical Data Compression (Source Coding) .................... 595
12.6 Quantum Data Compression ..................................... 600
12.7 Classical Channels and Channel Encoding ....................... 605
12.8 Quantum Channels and Open Systems .......................... 614
12.9 Accessible Information and Holevo Bound ........................ 620
12.10 Transmission Through a Noisy Quantum Channel .............. 625
References ........................................................................ 636

13 Applications of Quantum Information ............................ 639
13.1 Introduction ................................................. 639
13.2 Quantum Random Number Generation ........................ 640
13.3 Introduction to Quantum Cryptography ........................ 645
13.4 Quantum Key Distribution (QKD) ............................... 646
13.5 Teleportation ................................................. 659
References ........................................................................ 662

Index ............................................................. 665