In this book, several topics of Control theory are presented as a means to solving the synchronization and secure communication problems. Some analytic, algebraic, geometric, and asymptotic concepts are assembled as design tools for a wide variety of chaotic systems. Concepts from differential geometry and differential algebra reveal important structural properties of chaotic systems. The control community has attacked the synchronization concept as an observation problem. In this book, however, we have conceived synchronization theory as a tracking control problem under the master–slave configuration.

The presentation is organized as follows. The basic differential-algebraic and differential-geometric concepts are presented in Chaps. 1–3 in a novel way as design tools. In addition, some experimental results are presented. Most of the more recent results appear in Chaps. 4–14. The first three chapters present the basic concepts of differential algebra and differential geometry. Chapter 1 is introductory. It presents several concepts and examples in nonlinear control theory and synchronization. In Chap. 2, we deal with the synchronization problem using a proportional reduced-order observer. Chapter 3 is concerned with a sliding-mode observer, which is proposed for the synchronization problem. Chapter 4 shows the experimental synchronization of a Colpitts oscillator in real time. Chapter 5 is devoted to the synchronization and parameter estimations of an uncertain Rikitake system. Chapter 6 treats an aspect of chaotic communications and synchronization via a sliding-mode observer. Chap. 7 introduces synchronization and antisynchronization problems in chaotic systems by means of an observer. Chapter 8 expands our investigation to the synchronization of chaotic oscillators with Liouvillian properties and offers some experimental results. Chapter 9 extends the property called the algebraic observability condition (AOC) to the property known as fractional algebraic observability (FAO) to treat the synchronization problem of fractional-order systems. In Chapter 10, we expand our results to generalized synchronization by means of a differential primitive element, which is a linear combination of the known states and the inputs of the system. Chapter 11 studies generalized synchronization for a class of nondifferentially flat and Liouvillian chaotic system. Chapter 12 extends our results to generalized multisynchronization through a family of dynamical
feedbacks. Chapter 13 introduces fractional generalized synchronization via dynamical feedback. The book concludes with Chapter 14, which treats synchronization theory for a certain class of incommensurate fractional-order systems in which a new incommensurate fractional algebraic observability property has been introduced.

The book is written for an audience of graduate students, control engineers, and applied mathematicians interested in synchronization of chaotic systems that are by nature commensurate or incommensurate and in secure communications. It is self-contained and accessible to those with a basic knowledge of integer- and fractional-order differential equations for synchronization theory. For clarity, most of the concepts are introduced and explained by means of examples. Design applications are illustrated on several physical models of practical interest. The book can be used for a first-level graduate course on synchronization theory or as collateral reading for an advanced or specialized course on synchronization theory. Chapters 4–14 can be incorporated into a more advanced course on dynamical nonlinear feedback design.

The authors have attempted to write in such a way that this book can be read not only by mathematicians and physicists, but also by students in engineering (control, systems, electrical, mechanical, aerospace, chemical) who need more background than is provided in the basic mathematics courses and Chap. 1 of this book. We hope that the material presented here will also be useful in the study of secure communications and incommensurate systems.

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