

Contents

Acknowledgements	xi
Part I	
1 Limits	3
1.1 Infinite Decimals	3
1.2 Accumulation Points	6
1.3 Limits of Functions	9
1.4 Calculating with Limits	17
1.5 Variations on Limits	24
1.6 Sequences	28
1.7 Geometric Progression	30
1.8 Steinhaus' Three Distance Conjecture★	31
2 Introduction to Continuity	45
2.1 Definition and Algebra	45
2.2 Removable Discontinuity	47
2.3 One-Sided Continuity	48
3 Sets of Real Numbers	51
3.1 Supremum and Infimum	51
3.2 Intervals	55
3.3 The Nested Interval Theorem	56
3.4 Sets of Continuity★	58
3.5 Roots	60
3.6 Cantor Set	63
4 Counting	69
4.1 Countable Sets	69
4.2 Uncountable Sets★	72

5	Continuity	77
	5.1 Monotone Functions	77
	5.2 The Intermediate Value Theorem	80
	5.3 Continuous Images of Compact Intervals	81
	5.4 Uniform Continuity	83
6	Derivatives and Their Applications	91
	6.1 Definition	91
	6.2 Local Properties	93
	6.3 Calculating with Derivatives	94
	6.4 Global Properties of Derivatives	100
	6.5 Transcendental Numbers★	103
	6.6 Taylor Polynomials	105
	6.7 l'Hôpital's Rule★	109
	6.8 Convexity★	110
7	The Riemann Integral	127
	7.1 Definition of the Integral	127
	7.2 Characterizations of Riemann Integrability	132
	7.3 Examples of Integrable Functions	135
	7.4 Algebra of Integrable Functions	137
	7.5 The Fundamental Theorem of Calculus	141
	7.6 Improper Integrals	146
	7.7 Complex Valued Functions	146
8	The Logarithm and the Exponential Function	157
	8.1 Logarithms	157
	8.2 Exponentials	159
	8.3 The Napier Constant e ★	162
	8.4 Bump Functions★	166
Part II		
9	Convergence of Sequences	177
	9.1 Real and Complex Sequences	177
	9.2 Sequences of Functions	187
	9.3 Convolution	194
	9.4 The Fundamental Theorem of Algebra★	196
10	Series	205
	10.1 Series of Numbers	205
	10.2 Series of Functions	215
	10.3 Space Filling Curves★	216
	10.4 Power Series	218
	10.5 The Weierstrass Approximation Theorem★	221

- 11 Trigonometric Functions and Applications** 229
 - 11.1 Exponential Function 229
 - 11.2 Trigonometric Functions 230
 - 11.3 Arc Length★ 234
 - 11.4 Weierstrass’ Nowhere Differentiable Function★ 237
 - 11.5 The Number Pi is Irrational★ 240

- 12 Fourier Series** 247
 - 12.1 Introduction 247
 - 12.2 Linear Algebra 250
 - 12.3 Partial Sums 257
 - 12.4 Pointwise Convergence★ 260
 - 12.5 Cesàro Summability 263
 - 12.6 Uniform Distribution of Sequences★ 266
 - 12.7 Norm Convergence★ 271

- 13 Topology** 281
 - 13.1 Open Sets 281
 - 13.2 Closed Sets 284
 - 13.3 Compact Sets 285
 - 13.4 Connected Sets 291

- A Logic and Set Theory** 297
 - A.1 Logic 297
 - A.1.1 The Connective “or” 297
 - A.1.2 The Connective “and” 297
 - A.1.3 Implication 297
 - A.1.4 Bi-implication 298
 - A.1.5 Quantifiers 298
 - A.1.5.1 All 298
 - A.1.5.2 Some 298
 - A.1.6 Negation 298
 - A.2 Sets of Numbers 299
 - A.3 Set Theory 300
 - A.3.1 Subset 300
 - A.3.2 Union 300
 - A.3.3 Intersection 300
 - A.3.4 Set Difference 300
 - A.3.5 General Unions and Intersections 301
 - A.4 Functions 301
 - A.4.1 301
 - A.4.2 301
 - A.4.3 302
 - A.4.4 Set Functions 302
 - A.4.4.1 Image 302
 - A.4.4.2 Pre-image 302

B	The Principle of Induction	305
	B.1 Formulations of Induction	305
	B.2 Binomial Theorem	306
C	The Field Axioms	309
	C.1 Statement of the Axioms	309
	C.2 Some Consequences of the Axioms	310
D	Working with Inequalities	313
	D.1 Inequalities	313
	D.2 Absolute Value	316
E	Complex Numbers	319
	E.1 Basic Properties of Complex Numbers	320
	E.2 Balls	322
F	Greek Alphabet	325
G	Credits	327
H	Names	329
	References	335
	Index	337



<http://www.springer.com/978-3-319-13640-0>

From Calculus to Analysis

Pedersen, S.

2015, XIX, 342 p. 48 illus., 14 illus. in color., Hardcover

ISBN: 978-3-319-13640-0