Discrete geometry is the study of the geometric properties of discrete objects including lines, triangles, rectangles, circles, cubes, and spheres. These shapes are usually subsets of Euclidean space. On the other hand, digital geometry has two meanings: (1) The objects are formed by digital or integer points, more narrow digital geometry; and (2) The objects are computerized formations of geometric data. Sometimes we can view digital geometry as a subcategory of discrete geometry.

While discrete geometry has a long history, it has recently garnered much attention due to its large role in fulfilling computer vision and image processing needs. Such a need is the motivation behind the creation of digital geometry. The subject provides tremendous new research areas within discrete geometry. In the past, geometric tiling and counting were the primary research topics in discrete geometry.

Digital geometry mainly comes from two areas: image processing and computer graphics. A digital image in 2D is in the form of digital grid points; it is a natural treatment of using geometry in image processing including segmentation, recognition, and reconstruction. On the other hand, computer graphics use geometric design, object dynamics, and modification.

Computerized geometry must deal with efficient algorithms for many applications including classifications of digital objects, which also uses topological properties and geometry processing. It can be applied to a vast number of areas including biomathematics, medical imaging, the film industry, etc.

Digital geometry is also highly related to algorithmic geometry (computational geometry), which is more focused on algorithm design for discrete objects in Euclidean space. However, digital geometry has its own set of problems and challenges including those involving distance measure and the formatting of digital objects, which are different than that of discrete objects. Digital geometry also has some advantages since sampling the data can usually be directly applied in its digital form. There is no need to do a conversion from discrete forms.

This book provides detailed methods and algorithms in discrete geometry, especially digital geometry. We also provide the necessary knowledge in its connections to other types of geometry such as differential geometry and algebraic topology. In addition, there is much discussion on the recent development of applications in variety of methods of image processing, computer vision, and computer graphics.
This book is intended to offer comprehensive coverage of the modern methods for
geometric problems in the computing sciences. We also discuss concurrent topics in
BigData and data science as well.

This book is written to be suitable to different groups of readers. Chapters 1–6 are for junior and senior college students in computer science and mathematics; Chaps. 7–12 are for graduate students. Chapters 13–15 are written for researchers or
students with advanced knowledge in geometry and topology.

This book can also be categorized into three parts: (a) Chaps. 1–9 are introduc-
tions to digital and discrete geometry, (b) Chaps. 10–12 mainly deal with geometric
processing for readers interested in applications, and (c) Chaps. 13–15 present topics
in high level mathematics that are related to discrete geometry. The sections marked
with “*” may require some advanced knowledge. The book is also self-contained.

Acknowledgments: Many thanks to my daughter Boxi Cera Chen who helped me
correct my grammar for the whole book.

Many thanks to my wife Lan Zhang and my son Kyle Chen for their patience
and support while I was working on this book. I never thought that I had to put
increasingly more effort into completing this book. Many thanks to my colleagues
Professors Feng Luo, Petra Wiederhold, and Sherali Zeadally for their continued
support and encouragement. Many thanks to my colleagues in digital topology for
their help and support, Professors Reinhard Klette, Reneta Barneva, Jacques-Olivier
Coeurjolly, Tae Yung Kong, and Konrad Polthier, just name a few. Thanks also to
UDC for giving me one semester of sabbatical to work on this project.

My goal was set to write a complete introductory and comprehensive book to
digital and discrete geometry. As I was reviewing my writing today, I found that
it is still too far from reaching this initial vision. I hope that this book has laid a
good foundation for learning digital and discrete geometry, as well as linking to
various topics as a stepping stone to future research in this relatively new discipline
of computer science and mathematics.

Aug., 2014
Li M. Chen
Washington, DC
Digital and Discrete Geometry
Theory and Algorithms
Chen, L.
2014, XVII, 322 p. 130 illus. in color., Hardcover
ISBN: 978-3-319-12098-0