Chapter 2
Demand History

2.1 Introduction

Forecasts are necessary for each part in every stocking location as they are used in the stocking decisions on when to replenish the stock and how much. The forecasts are typically revised each month as the new monthly demand entry becomes available. The forecasts are based on the history flow of the demands over the past months. The more accurate the demand history, the better the forecasts. The typical inventory system saves anywhere from 12 to 36 months of demand history.

The data to save is the monthly demand history, denoted as, x(1),…, x(N), where x(t) is the demand in month t, t = 1, the oldest month in the history, and t = N, the most current month. The demand history to save varies depending on the stock keeping facility. Often, the database history includes the monthly demands and monthly lines, where each customer order represents a line and the quantity on the order is the demand. At the end of each month, the total demands and lines are tallied for the month. The database may also include the demand-to-date for the current month as it is progressing. In service parts distribution centers, the demands from dealers are often classified as either regular or emergency. For distribution centers that supply retail stores, the demands may be labeled as new stock and replenish stock. New stock includes the initial delivery to the stores. Some stocking facilities save their demands on a weekly basis rather than monthly. Plants often operate with fiscal (rather than calendar) months that are denoted as 445 or 454 or 544.

Some distribution centers have demands that are classified as either regular demands and as other-requirements. Other requirements are from out-of-the-ordinary customers. In some distribution centers and stores, the demands may be partitioned as regular demands and as promotion demands. Another type of demand is called advance demand and this occurs when a distribution center receives a customer order that is not to be shipped until some future date. When demands are shipped incorrectly, the customer returns the stock to the stocking location and these are labeled as return demands. Sometimes unusual events occur that cause a spike in the demand, or an error happens in order entry where the quantity or part number enters incorrectly, causing the entry on the database to be significantly different from the normal flow

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and is called an outlier demand. This type of demand is damaging to the forecasts and needs to be found and adjusted accordingly, prior to the forecasts. The flow of the demands, called the demand pattern, is mostly of the horizontal, trend or seasonal type. The chapter describes a way to convert fractional forecasts to integer forecasts; and how to compute cumulative forecasts for the future months. An inventory profile section is given that summarizes some revealing statistics on the demand history and forecast results that are taken from an actual large inventory holding system.

2.2 Customer Demand History for a Part

The demand history (for a part) is the main data used to generate a forecast for the future months. The history of customer demands is usually denoted as: x(1), …, x(N) where x(t) is the demand in month t. For newer parts, *N* is also the number of months of history since the part was introduced. For long time parts, *N* may be cut off to 12, or 24 or 36 months depending on the discretion of the management. The number of lines for month t, n(t), is also useful information and the history is here denoted as: n(1),…, n(N). Using the history of the demands and of the lines, it is possible to compute the history of pieces-per-line and this is denoted as: d'(1), …, d'(N) where d'(t) = 0 if n(t) = 0, otherwise, d'(t) = x(t)/n(t). Note also, the number of pieces for line i in month t is here denoted as: $d_{(i,\,t)}$ i = 1, …, n(t). The corresponding demands for month t is tallied from: $x(t) = d_{(1,t)} + \ldots + d_{(n(t),t)}$.

In summary, the notation on the demand history of a part is denoted as follows:

N = number of months history saved
t = 1,…, N = demand history months
x(t) = demands in month t
n(t) = number of lines in month t
d'(t) = x(t) / n(t) = pieces-per-line in month t

2.3 Demand-to-Date

Another piece of demand data that may become useful occurs in the first future month of forecasts as time rolls on. This is the month immediately after month t = N that has been previously labeled as the last month of the history months. In month N + 1, the first subsequent month after t = N, the demands to date for a part, are here labeled as x_w, and the corresponding portion of time of this month, is labeled as *w*. This data is dynamic. the entry x_w will change with each new demand in the month, and *w* will change with each day of the month. This data is useful to determine the integrity of the month-1 forecast, as the month progresses.

A summary of the data for the current month to date is below:

x_w = demand for current-month-to-date
w = portion of current month-to-date $(0 \le w \le 1)$

2.4 Service Part Regular and Emergency Demands

In a service parts distribution center, DC, of finished goods items (autos, trucks, farm equipment, construction equipment) the DC carries service parts to fill the demands that come from its dealers. The demands are of two type, regular and emergency. When a customer orders with a regular line, this is generally to provide stock needed for a subsequent need as it may occur at the dealership, whereas, the quantity on an emergency line is needed to immediately maintain or repair a finished-good-item that is in the dealership in a down status. Typically, the average number of pieces per line for a regular line is larger than for an emergency line.

Some of the data that is useful in this situation is listed below:

x(t) = monthly service part demands from customers
xr(t) = regular service part demands from customers
xe(t) = emergency service part demands from customers

$$x(t) = xr(t) + xe(t)$$

p(r) = probability a line is regular demand
p(e) = probability a line is an emergency demand

$$p(r) + p(e) = 1$$

nr(t) = number of regular lines in month t
ne(t) = number of emergency lines in month t
$n(t) = nr(t) + ne(t)$ = number of lines in month t
$dr_{(i, t)}$ = number of pieces for i-th regular demand in month t
$de_{(i, t)}$ = number of pieces for i-th emergency demand in month t
dr` = average of regular pieces per line
de` = average of emergency pieces per line
$x(t) = \left[dr_{(1,t)} + \ldots + dr_{(nr(t),t)}\right] + \left[de_{(1,t)} + \ldots + de_{(ne(t),t)}\right]$ = demands for month t

2.5 New and Replenish Stock Demands for Retail Items at DC

Consider a distribution center, DC, for finished-good-items that supplies the stock to retail stores. This could be for shoes, sweaters, furniture, and so on. The sales-person for the DC may visit the stores in his/her territory and arrange to provide a quantity of stock to be stocked for sale for the store customers to view and purchase. This stock is not a true demand at this point, but is here called a new stock demand. Subsequently, should sufficient sales of the item be sold to the store customers, the store may reorder more stock from the DC. This new line of demands is here called replenish stock demands. The initial new stock demand is not really a demand until the customer sends in an order to the DC for replenish stock on the item. In the event the store cannot sell the new stock, the store might return part or all of the new stock that it was initially provided by the DC.

Below is a summary of the data for the supplier of this scenario:

xn(t) = new stock demand at month t
xr(t) = replenish stock demand at month t
$x(t) = xn(t) + xr(t)$ = total demand at month t

2.6 Weekly Demands

Most plants and many retailers operate on a weekly basis and thereby cumulate their requirements and demands weekly. The plants typically schedule their production activities weekly and thereby plan accordingly. Many retail stores also schedule their replenish and promotion plans on a weekly basis. Weekly demands tend to fluctuate more than the counterpart monthly demands, and thereby subsequent weekly forecasts are less accurate than monthly forecasts. Weekly demands also are more compatible to horizontal and trend forecast models, whereas, monthly demands are compatible to horizontal, trend and seasonal forecast models.

Some of the data notation for weekly demands is listed below:

w(t) = demand at week t
Nm = 12 = number months in a year
Nw = 52 = number weeks in a year
Nw/Nm = 4.33 = average weeks in a month
w` = forecast for an average week's demand

if w` is the forecast:

$x` = 4.33w`$ = forecast for an average month
σ_w^2 = variance of weekly demands
$\sigma_x^2 = 4.33\sigma_w^2$ = relation between variance for monthly and weekly demands
$\sigma_x = \sqrt{4.33}\sigma_w = 2.08\sigma_w$ = standard deviation for monthly demands
$\sigma_w = 1/2.08\sigma_x = 0.481\sigma_x$ = standard deviation for weekly demands

If x` is the monthly forecast:

$w` = (1/4.33)x`$ = forecast for an average week

2.7 445 Fiscal Months at Plants

Many plants operate on a fiscal monthly basis of the 445 type. This is when the first three fiscal months of the year are defined as follows: the first four weeks of the year represent fiscal January, the next four weeks are fiscal February, and the next five weeks are fiscal March. The pattern repeats for the remaining nine fiscal months of the year. The fiscal months may also be defined as 454 instead of 445, or by 544 instead of 445. In either case of 454 or 544, the fiscal months are defined by the stated number of weeks in the month. This way of defining the fiscal months ensures that there are twelve fiscal months in a year, and also satisfies the plant's desire to end each fiscal month on the same day of the week, e.g. Saturday.

In summary, the months of the year can be of the calendar type or of the fiscal type as described above. The three options for fiscal months are defined as below.

445 is for fiscal January, February and March.
454 is for fiscal January, February and March.
544 is for fiscal January, February and March.

The pattern is repeated for the remaining 9 months of the year

2.8 Regular Demands and Other Requirements at DCs

A distribution center of service parts is structured to house inventory for its regular customers (dealers) so that the stock is available when the customers send in their orders. The demand history is the data that is used to generate the DC forecasts covering the future months of demands. The forecasts are the tools that allow the DC to properly provide stock for the customers accordingly. On some occasions, usually infrequently, an order will come in from a non-regular customer for one or for a variety of parts. This non-regular customer could be from an overseas location, or a government facility or the military. In any event, the demand is not from the regular set of customers that the forecast covers. It also is often a demand for a future time period, perhaps to be delivered in a future month from the date of the order. This demand is here called an: *other requirement*. The demand is not included in the history of demands that are used to forecast the demands of the regular type demands. The demand is added to the forecasts for the future months and becomes part of the total requirements for the part. The inventory replenishment side of the computations needs to provide stock to cover the forecast of regular demands and also for the other requirements.

Below is a summary of the data described for this scenario.

x(t) = regular demands at t
xo(t) = other requirements at t
r(t) = x(t) + xo(t) = total requirements at t
x(t) t = 1 to N is used to forecast the demands from the regular customers for the future months
xo(τ) = other requirement for future month τ and is any demand from a non-regular customer base (overseas customer, military order, etc.)

2.9 Regular and Promotion Demands at DCs and Stores

Promotions of various type occur from time to time at the DC or at the dealers. A common promotion is when the supplier offers a discount on the price for all units sold from day d1 to day d2. This could be for one item or for a line of items. The units sold during the promotion period are recorded and identified as promotion demands. The portion of the months that are included in the promotion period could also be recorded. This data is useful in generating forecasts for the future months, when there is no promotion and when there is a promotion.

One way to capture the demands associated with a promotion is as follows. Let p(t) = portion of month t where a promotion is active. If the promotion runs from d1 = June 14 to d2 = July 14, and June is month t = 6, p(6) = (30-13)/30 = 0.57; and p(7) = 14/31 = 0.45. In any month t with no promotion, p(t) = 0. Further, the demands during the promotion period are saved as xp(6) for June, and xp(7) for July. Subsequent computations allow the forecaster to use this data in generating the forecasts.

A review of the data when promotions are involved is listed below:

p(t) = portion of month t when a promotion is live
xr(t) = regular demands at t
xp(t) = promotion demands at t
$x(t) = xr(t) + xp(t)$ = total demands at t

2.10 Advance Demands

On some occasions, a customer places an order for stock to be delivered in a future date, usually a month or two in the future. This demand is not for the current month and thereby is not recorded as a demand in the current month. Instead, the demand is labeled as *advance demand*, for the future month as the order calls. This advance demand is important information and could be used to adjust the forecast for the named future month.

The data (quantity and month) recorded for this demand is the following:
xa(τ) = advance demand for τ-th future month

2.11 Demand Patterns

There are three basic demand patterns: horizontal, trend and seasonal. Horizontal occurs when the demands are neither rising of falling over time whereby the average is relatively steady. Trend is when the demands are gradually increasing or are decreasing over time. Seasonal is when the demands vary by the months of the year, and the pattern repeats every year. Two versions of the seasonal pattern occur: seasonal multiplicative and seasonal additive. Often, low volume parts are of the horizontal type. Mid to high volume parts could follow any of the three demand patterns.

Letting $\mu(t)$represent the average demand at month t, the demand patterns could be defined as follows:

Horizontal

$$\mu(t) = a \qquad a = \text{level}$$

Trend

$$\mu(t) = a + bt \qquad b = \text{slope}$$

Seasonal multiplicative:

$$\mu(t) = (a + bt)r(t)$$

r(t) = seasonal ratio at month t
r(t) = 1 when month t demand is same as the trend (a+bt)
r(t) > 1 = when month t demand is higher than the trend
r(t) < 1 = when month t demand is lower than the trend

Seasonal additive:

$$\mu(t) = (a + bt) + d(t)$$

d(t) = seasonal increment at month t
d(t) =0 = when month t demand is same as the trend
d(t) >0 = when month t demand is higher than the trend
d(t) <0 = when month t demand is lower that the trend

2.12 Return Demands

As an order comes in to a stocking facility with a line of items, each line lists the part and the quantity to ship to the customer. Should an error occur by incorrectly picking the part, in typing the part number or the quantity, the part and quantity are nevertheless shipped to the customer. When the customer discovers the fault, the pieces are returned to the supplier and are here called *returned demands*. The demand history is in error for the part when this event occurs, and a correction to the demand history should be made accordingly.

Suppose the data associated with a returned demand is the following: $xr(t_o)$ = stock returned at month t_o. With this data in hand, a routine is needed to estimate where this demand came from. One possibility is to scan the demand history of the part, $x(1), \ldots, x(N)$ and find the most recent month, t_1, where $x(t_1) \geq xr(t_o)$ and where $t_1 \leq t_o$, and then adjust the demand entry at t_1 as: $x(t_1) = x(t_1) - xr(t_o)$.

2.13 Outlier Demands

On occasion, the demand history may include a demand entry that is significantly beyond the flow of the normal demands in the history. This demand is here called and *outlier demand*. Outlier demands are mostly above the normal flow of demands. An outlier demand could occur in several ways, one is when the demand is ordered for a wrong part, or the quantity ordered is mistyped. This could also occur due to unusual weather conditions, e.g., windshield wiper demands when an unusual ice storm occurs. Outlier demands are very damaging to the accuracy of the forecasts and as much as possible, prior to generating the forecasts, they should be detected and adjusted accordingly.

A routine is needed to seek if any demand entry, say $x(t_o)$, in the history of demands, x(1),…, x(N), is significantly outside the flow of its neighbor demands. Should a demand entry be found, the entry is adjusted accordingly, ideally to fall in line with the flow of all the demand entries.

2.14 Coefficient of Variation

The coefficient of variation, cov, is a relative way to measure the forecast error associated with a part. This is computed by $cov = (\sigma/a)$, where a is the level, and σ is the standard deviation of the one period ahead forecast error. The level, a, is a measure used in forecasting to represent the average flow of demands in the most current month. When a seasonal demand pattern is in effect, the level represents a measure of the seasonally adjusted demand for the current month. The cov is always positive, and the closer to zero, the more accurate the forecasts.

2.15 Demand Distribution

It is possible to estimate the probability distribution of the forecast errors associated with each part. The one month ahead forecast error for month t would be: $e = (x` - x)$ where x` is the forecast of the month's demand and x is the actual demand. The shape of the probability distribution is important in the subsequent computations where the inventory control on the part takes place, when determining the safety stock that complies with the desired service level specified by the management.

Recall, the level, a, represents the average flow of demands at the current time, and σ is the standard deviation of the one month forecast error. Further, $cov = \sigma/a$ is a relative measure of the forecast error on the part. In the event the forecast errors are shaped like a normal distribution, cov is near or less than 0.33. In the event cov is close to 1.00 or above, the distribution is called lumpy and is more like an exponential distribution, which is far different than a normal distribution.

Very often in forecasting, the cov on a part is not in the neighborhood of 0.33, but is much higher. This is especially true on parts where the demands are of the low volume type. The inventory management is here cautioned not to always assume the normal distribution in the inventory analysis for the parts where the cov is relatively high.

2.16 Cumulative Round Algorithm

The demand forecasting models of this book will generate forecasts that are in fractional form, and are here called raw forecasts. The typical inventory system will convert these fractional forecast to integers. A way to do this is introduced below in pseudo code and is called the *cumulative round algorithm* (CRA). The notation uses f(τ) as the raw forecast for future monthτ (τ = 1 to N`) where N` is the number of future months, and x`(τ) is the associated integer forecast.

```
Start
d = 0
for  τ = 1 to N`
x`( τ) = integer[f(t) + d + 0.5]
d = d + [f(τ)- x`( τ)]
next τ
End
```

2.17 Cumulative Forecasts

Often, forecasts are needed for an accumulation of future months. If the monthly forecast is denoted as x`(τ) for future month τ, and the cumulative forecast for T future months is X`(T), the computations are as below.

$$\text{If} \quad T = 3, \quad X`(3) = x`(1) + x`(2) + x`(3)$$

$$\text{If} \quad T = 1.7, \quad X`(1.7) = x`(1) + 0.7x`(2)$$

$$\text{If} \quad T = 0.6, \quad X`(0.6) = 0.6x`(1)$$

and so forth.

2.18 Inventory Profile

In this section are some statistics from the service parts division of a large automotive corporation with over 100,000 part numbers and annual demand over $ 1 billion. The service part division includes multiple locations in North America.

Table 2.1 lists the percent of part numbers (% *N*), by number-months-of-demand-history (NMH). Note, 7.0 % of the part numbers have 1–12 months of demand

Table 2.1 Percent of part numbers (%*N*), by number-months-of-demand-history (NMH)

NMH	%*N*
1–12	7.0
13–24	6.7
35–36	8.5
37+	77.8
Sum	100

Table 2.2 Percent of part numbers (%*N*), by outlier type (none, low, high)

Outlier Type	%*N*
none	74.7
low	0.1
high	25.2
Sum	100

Table 2.3 Percent of part numbers (%*N*) and percent of annual demand dollars (%$), by forecast type (horizontal, trend, seasonal)

Forecast Type	%*N*	%$
horizontal	52	4
trend	44	69
seasonal	4	27
Sum	100	100

history, while 77.8 % have over 36 months of demand history. Note, the system generates roughly 7 % of new parts each year; and assuming equilibrium, about 7 % of the parts are discarded each year as well. The database holds the most current 36 months of demand history on each part number.

Table 2.2 gives the results of the outlier filtering algorithm, where 74.7 % of the part numbers (%*N*) had no outlier detected, 0.1 % had a low outlier detected, and 25.2 % had a high outlier detected. The filtering process checks all 36 months of demand history in each of the distribution centers. Each of the 36 demands are checked in every distribution center and should any be significantly out-of-the-ordinary, the part is labeled as an outlier.

Table 2.3 lists the type of forecast model by percent of part numbers (%*N*), and percent of annual demand in dollars (%$). The table shows where 52 % of the part numbers had a horizontal forecast model and this amounted to 4 % of the annual demands in dollars. Note also where 4 % of part numbers have a seasonal forecast model and they amount to 27 % of the annual demands in dollars.

Table 2.4 gives the monthly growth rate, g, of the part numbers, by percent of part numbers, (%*N*), and percent of annual demand in dollars, (%$). The monthly growth rate was computed by: $g=(a+b)/a$ where a is the level and b is the slope. Note, 74 % of the part numbers had a monthly growth rate of (0.995–1.005) and these parts amounted to 36 % of the annual demands in dollars.

Table 2.5 gives the monthly coefficient of variation, cov, by percent of part numbers (%*N*), and percent of annual demand dollars (%$). The table shows where 26 %

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