

# Contents

<b>1</b>	<b>Introduction</b> .....	1
1.1	Tasks for Photogrammetric Computer Vision .....	2
1.2	Modelling in Photogrammetric Computer Vision .....	6
1.3	The Book .....	11
1.4	On Notation .....	16
 <b>Part I Statistics and Estimation</b>		
<b>2</b>	<b>Probability Theory and Random Variables</b> .....	21
2.1	Notions of Probability .....	21
2.2	Axiomatic Definition of Probability .....	22
2.3	Random Variables .....	24
2.4	Distributions .....	28
2.5	Moments .....	36
2.6	Quantiles of a Distribution .....	40
2.7	Functions of Random Variables .....	40
2.8	Stochastic Processes .....	48
2.9	Generating Random Numbers .....	55
2.10	Exercises .....	56
<b>3</b>	<b>Testing</b> .....	61
3.1	Principles of Hypothesis Testing .....	61
3.2	Testability of an Alternative Hypothesis .....	65
3.3	Common Tests .....	69
3.4	Exercises .....	72
<b>4</b>	<b>Estimation</b> .....	75
4.1	Estimation Theory .....	75
4.2	The Linear Gauss–Markov Model .....	81
4.3	Gauss–Markov Model with Constraints .....	99
4.4	The Nonlinear Gauss–Markov Model .....	102
4.5	Datum or Gauge Definitions and Transformations .....	108
4.6	Evaluation .....	115
4.7	Robust Estimation and Outlier Detection .....	141
4.8	Estimation with Implicit Functional Models .....	160
4.9	Methods for Closed Form Estimations .....	176
4.10	Estimation in Autoregressive Models .....	183
4.11	Exercises .....	185

## Part II Geometry

<b>5</b>	<b>Homogeneous Representations of Points, Lines and Planes</b> .....	195
5.1	Homogeneous Vectors and Matrices .....	195
5.2	Homogeneous Representations of Points and Lines in 2D .....	205
5.3	Homogeneous Representations in $\mathbb{IP}^n$ .....	209
5.4	Homogeneous Representations of 3D Lines .....	216
5.5	On Plücker Coordinates for Points, Lines and Planes .....	221
5.6	The Principle of Duality .....	229
5.7	Conics and Quadrics .....	236
5.8	Normalizations of Homogeneous Vectors .....	241
5.9	Canonical Elements of Coordinate Systems .....	242
5.10	Exercises .....	245
<b>6</b>	<b>Transformations</b> .....	247
6.1	Structure of Projective Collineations .....	248
6.2	Basic Transformations .....	250
6.3	Concatenation and Inversion of Transformations .....	261
6.4	Invariants of Projective Mappings .....	266
6.5	Perspective Collineations .....	277
6.6	Projective Correlations .....	282
6.7	Hierarchy of Projective Transformations and Their Characteristics .....	284
6.8	Normalizations of Transformations .....	285
6.9	Conditioning .....	286
6.10	Exercises .....	287
<b>7</b>	<b>Geometric Operations</b> .....	291
7.1	Geometric Operations in 2D Space .....	292
7.2	Geometric Operations in 3D Space .....	299
7.3	Vector and Matrix Representations for Geometric Entities .....	311
7.4	Minimal Solutions for Conics and Transformations .....	316
7.5	Exercises .....	322
<b>8</b>	<b>Rotations</b> .....	325
8.1	Rotations in 3D .....	325
8.2	Concatenation of Rotations .....	337
8.3	Relations Between the Representations for Rotations .....	338
8.4	Rotations from Corresponding Vector Pairs .....	339
8.5	Exercises .....	340
<b>9</b>	<b>Oriented Projective Geometry</b> .....	343
9.1	Oriented Entities and Constructions .....	344
9.2	Transformation of Oriented Entities .....	355
9.3	Exercises .....	358
<b>10</b>	<b>Reasoning with Uncertain Geometric Entities</b> .....	359
10.1	Motivation .....	360
10.2	Representing Uncertain Geometric Elements .....	364
10.3	Propagation of the Uncertainty of Homogeneous Entities .....	386
10.4	Evaluating Statistically Uncertain Relations .....	393
10.5	Closed Form Solutions for Estimating Geometric Entities .....	395
10.6	Iterative Solutions for Maximum Likelihood Estimation .....	414
10.7	Exercises .....	432

---

**Part III Orientation and Reconstruction**

<b>11 Overview</b>	441
11.1 Scene, Camera, and Image Models	441
11.2 The Setup of Orientation, Calibration, and Reconstruction	449
11.3 Exercises	453
<b>12 Geometry and Orientation of the Single Image</b>	455
12.1 Geometry of the Single Image	456
12.2 Orientation of the Single Image	489
12.3 Inverse Perspective and 3D Information from a Single Image	523
12.4 Exercises	537
<b>13 Geometry and Orientation of the Image Pair</b>	547
13.1 Motivation	547
13.2 The Geometry of the Image Pair	549
13.3 Relative Orientation of the Image Pair	568
13.4 Triangulation	596
13.5 Absolute Orientation and Spatial Similarity Transformation	607
13.6 Orientation of the Image Pair and Its Quality	608
13.7 Exercises	615
<b>14 Geometry and Orientation of the Image Triplet</b>	621
14.1 Geometry of the Image Triplet	622
14.2 Relative Orientation of the Image Triplet	632
14.3 Exercises	641
<b>15 Bundle Adjustment</b>	643
15.1 Motivation for Bundle Adjustment and Its Tasks	644
15.2 Block Adjustment	645
15.3 Sparsity of Matrices, Free Adjustment and Theoretical Precision	651
15.4 Self-calibrating Bundle Adjustment	674
15.5 Camera Calibration	696
15.6 Outlier Detection and Approximate Values	707
15.7 View Planning	715
15.8 Exercises	722
<b>16 Surface Reconstruction</b>	727
16.1 Introduction	727
16.2 Parametric $2^{1/2}D$ Surfaces	733
16.3 Models for Reconstructing One-Dimensional Surface Profiles	742
16.4 Reconstruction of $2^{1/2}D$ Surfaces from 3D Point Clouds	757
16.5 Examples for Surface Reconstruction	763
16.6 Exercises	765
<b>Appendix: Basics and Useful Relations from Linear Algebra</b>	767
A.1 Inner Product	767
A.2 Determinant	767
A.3 Inverse, Adjugate, and Cofactor Matrix	769
A.4 Skew Symmetric Matrices	770
A.5 Eigenvalues	772
A.6 Idempotent Matrices	774
A.7 Kronecker Product, $\text{vec}(\cdot)$ Operator, $\text{vech}(\cdot)$ Operator	775

---

A.8 Hadamard Product .....	776
A.9 Cholesky and QR Decomposition .....	776
A.10 Singular Value Decomposition .....	777
A.11 The Null Space and the Column Space of a Matrix .....	777
A.12 The Pseudo-inverse .....	779
A.13 Matrix Exponential .....	781
A.14 Tensor Notation .....	782
A.15 Variance Propagation of Spectrally Normalized Matrix .....	783
<b>References</b> .....	785
<b>Index</b> .....	799



<http://www.springer.com/978-3-319-11549-8>

Photogrammetric Computer Vision  
Statistics, Geometry, Orientation and Reconstruction  
Förstner, W.; Wrobel, B.P.  
2016, XVII, 816 p. 281 illus., 59 illus. in color.,  
Hardcover  
ISBN: 978-3-319-11549-8