Preface

The aim of these notes is to give a brief and elementary introduction to the representation theory of finite-dimensional algebras. The notes originated from an undergraduate course I gave in two occasions at Universidad Nacional Autonóma de México.

The plan of the course was to try to cope with two competing demands: to expect as little as possible and to reach as much as possible: to expect only linear algebra as background and yet to make way to substantial and central ideas and results during its progress.

Therefore some crucial decisions were necessary. We opted for the model case rather than the most general situation, for the most illustrating example rather than the most extravagant one. We sought a guideline through this vast field which conducts to as many important notions, techniques and questions as possible in the limited space of a one-semester course. So, it is a book written from a specific point of view and the title should really be *Introduction to the theory of algebras, which are finite dimensional over some algebraically closed field*.

The book starts with the most difficult chapter: matrix problems. Conceptually there is little to understand in that chapter, but it requires a considerable effort from the reader to follow the argumentation within. However, this chapter is central: it prepares all the main examples which later will guide through the rest. In the following two chapters we consider the main languages of representation theory. Since there are several competing languages in representation theory, a considerable amount of our effort is directed towards mastering and combining all of them. As you will see each of these languages has its own advantages and it therefore not only enables the reader to consult the majority of all research articles in the field, but also enriches the way we may think about the notions themselves.

The rest of the book is devoted to gain structural insight into the categories of modules of a given algebra. In the chapter about module categories some older results are proved, whereas in the next four chapters more recent developments are discussed. The last chapter is more of an open-minded collection concerning indecomposable modules, the building bricks of the module categories, with respect to certain invariants, called dimension vectors.
My thanks go to Jan Schröer, for it was him who suggested to write jointly a book on representation theory but unfortunately gave up on it, to Juraj Hromkovič who invited me to ETH Zürich for a sabbatical in 2010 and made thus possible to finish the manuscript, to Manuela Tschabold who tirelessly read the whole manuscript carefully eliminating thus many errors and misprints, to Karin Baur who took it as a base for a course in 2011 suggesting many improvements, to Christof Geiss who was able to improve the manuscript still further, to Mario Aigner from Springer who tirelessly fought on my side for a good layout and to Angélica Herrera Loyo for her patience.

I apologize to the reader for all the errors which still remain and for the poor English in which it is written.

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