Mathematical Modelling and Simulation for Beaded Pad Automatic Weaving Trajectory

Jianye Yan, Ligang Yao, Dongliang Lin and Zhijun Liu

Abstract Beaded pads utilized as cooling mat and handicraft are popular. Nowadays, all the beaded pads are hand-woven. This paper proposes a beaded pad automatic weaving method to solve the hand-woven problems of inefficiency, poor consistency and higher cost. The single-line direct-threading automatic weaving method is proposed on the basis of the investigation of the hand-woven process. Further, the beaded pad automatic weaving trajectory mathematical modelling and simulation for the proposed single-line direct-threading weaving method is presented.

Keywords Beaded pad · Motion trajectory · Upward-string · Downward-string

1 Introduction

The beaded pad shown in Fig. 1 is a cushion which connects the beads in turn vertically or horizontally. The pad unit consists of four beads (0–3) and a single-line. The constraints between the beads and the single-line make the beaded pad's structure tight and stable. The pad can keep the seat clean and cool in summer.

Although this kind of the beaded pad favored by the majority of consumers, the pad sold in the market is hand-woven so far. There are still no automatic weaving machines for weaving this kind beaded pad, and even the corresponding weaving trajectory analysis and simulation. There are other kinds pads which are weaved with a tool [1–3] based on the wrap/weft weaving method, unfortunately these pads have compact integral structure and poor air permeability. Another kind of pads are made by using lockstitch weaving method [4–6] under the lockstitch machine's stitch and their structures are instability and easy to damage because of the knot in

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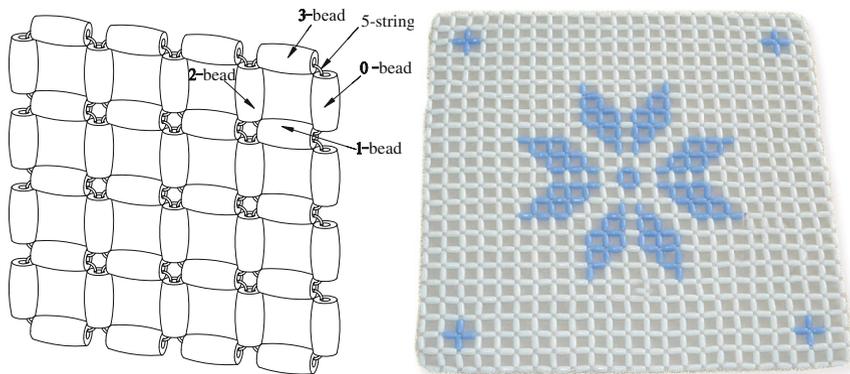e-mail: yanjianye0728@163.com

Fig. 1 3-D model and physical map of the beaded pad

the cord inside the bead hole. The proposed mongline right angle loop [7] and right angle weaving method [8–10] are merely applicable to hand-woven, due to too many turns of the needle weaving.

2 Single-Line Direct-Threading Automatic Weaving Method

Currently, the beaded pad is weaved with method of single-line by hand, which is tedious and inefficient. The human weaving process, shown in Fig. 2, is carried out by stringing four beads (0–3) at the two ends of the single-line and then piercing one string ends through bead 3 to get a four-beads unit. The other units could be weaved in the same way. The two ends of the single string are defined as the
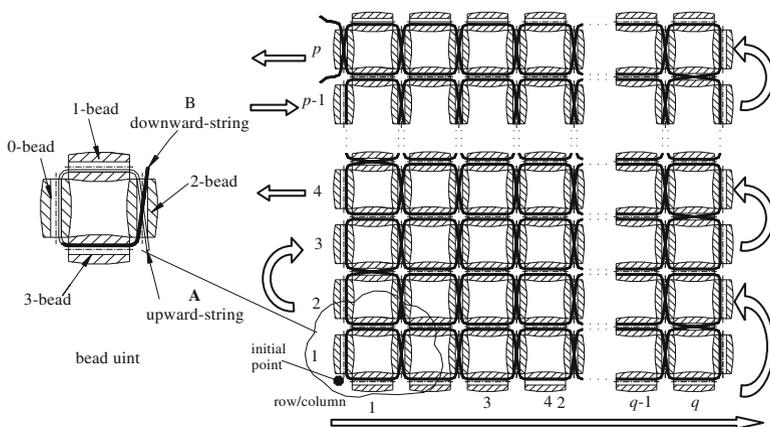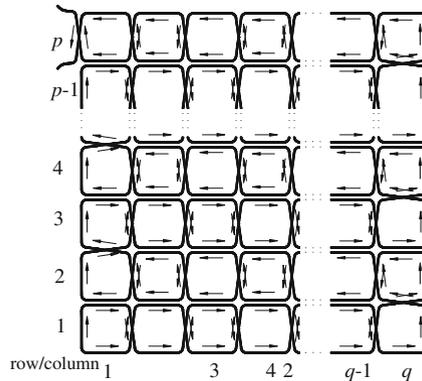

Fig. 2 Beaded pad weaving process

Fig. 3 String g's motion trajectory

downward-string and upward-string [11]. So the weaving trajectory is the movement routes for upward-string and downward-string. The string end whose first step moves upwards/rightwards is defined as upward-string A/downward-string B. A list of pad units connected by a string in the horizontal and vertical direction is called the row and column. The unit numbers in Fig. 2 are termed as row and column respectively. Therefore, the pad is denoted as $LD_{p \times q}$.

According to above mentioned weaving method, the trajectory of the upward-string and downward-string for the $LD_{p \times q}$ pad can be illustrated in Fig. 3 without displaying the beads. The arrows show upward-string and downward-string's movement direction in the process of weaving and the string initial point is located in the lower left corner.

On the basis of the human weaving method and weaving trajectory, the single-line direct-threading automatic weaving method [12] is proposed and shown in Fig. 4. It simplifies the hand-weaving process and avoids the two ends of the string's alternate threading motion. It's easy to be implemented on the machine. The automatic weaving process can be described in the follows. First, thread all needed beads on the upward-string, at this position the upward-string is in a state of straight line. Then, thread a bead on the downward-string B and pierce the downward-string B through the second bead's hole to form a bead unit. The machine can weave single-row pad by repeating the process for downward-string B. The upward-string needs to pierce the bead to complete the weaving process when
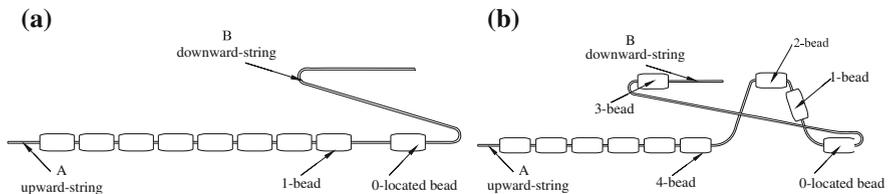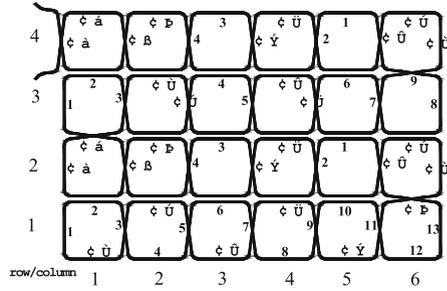

Fig. 4 The weaving process of single-line direct-threading weaving method

Fig. 5 Numbering the beads

it weaves the second-row according to Fig. 2. It has the similar threading movement between the upward-string and downward-string. As long as it controls the woven-bead number, the threading trajectory and when the string passing through the bead, the beaded pad weaved with the above method achieves the same goals that weaved with the method of single-line by hand.

Based on the above weaving method, the number of beads which are pierced on the upward-string is $2q + 1$. While weaving the first row, it needs to thread an additional bead on the downward-string by controlling its movement in weaving a bead unit, so the number of beads pierced on the downward-string is q. The upward-string and downward-string, with the constraints of the first row unit, need to pierce the beads alternately and accomplish the same threading movement, when the machine weaves the next row unit. According to the method and the law of upward-string and downward-string's movement route, the upward-string strings a bead and the downward-string strings another bead. Then the upward-string and downward-string pierce the common bead so as to complete a bead unit. The process of numbering the beads for the beaded pad is shown in Fig. 5. The number of the beads which are on the upward-string and downward-string is shown in Fig. 5. It presents the situation where the row p is 4 and the column q is 6.

3 Mathematical Modeling for the Weaving Trajectory

In order to describe the threading process, it needs to analyze the law of string's movement route. The paper respectively extracts the motion trajectory of upward-string and downward-string in Fig. 6.

It is obvious that the motion trajectories of string in the odd/even row are the same, shown in Fig. 6. That is, weaving trajectory has a period in two rows. The string has the same movement direction every four steps (one arrow stands for one step) in every row, except the last column of the odd row for the downward-string. Therefore, it considers that the string's period of motion is four steps every rows, every four steps has a fixed cycle. So, the downward-string's motion trajectory has a periodical characteristic. It only needs to describe one period of the motion as describe the threading process. Similarly, the downward-string's motion trajectory has the same law.
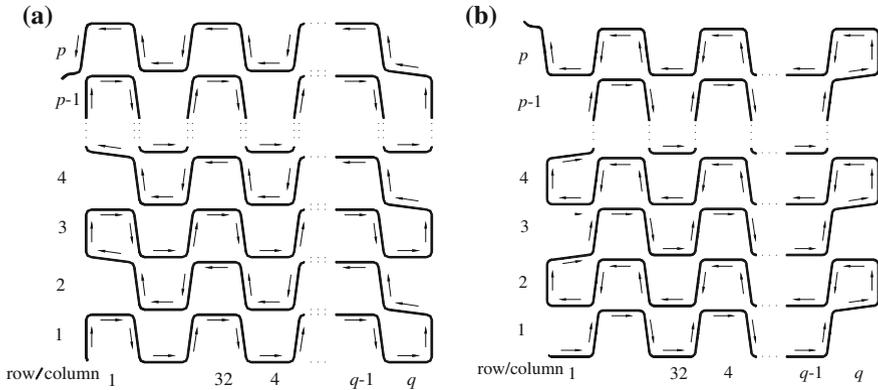
Fig. 6 **a** Upward-string's motion trajectory. **b** Downward-string's motion trajectory

In order to describe the law of motion path of upward-string and downward-string with mathematical method, it needs to simplify the trajectory of string. The string's terminal must be linear motion and pierce through the centre axis of the beads hole, regardless of the position of the string, and the steering angle must be rectangular. The steps of upward-string make the following stipulations: number the steps form 1 in each row, count from left to right (right to left) for the odd (even) row, and start to count form the first horizontal direction of each row. The number of steps in each row is $2q$ after processing. Because the downward-string's motion trajectory has the same law as the upward-string's, this paper only gives the mathematic expression of downward-string.

After simplification, the motion trajectory of i and $i+1$ row for the downward-string is shown in Fig. 7. where i and $i+1$ are odd, the solid line and double dash dot line represent i and $i+1$ row of the downward-string respectively, the number beside the arrow is step number n_1. The relation between step and column number can be deduced from the picture.

The coordinate system is established in Fig. 8. Where the origin of coordinates is the initial point of the first step of the first row in the downward-string and i is odd. The end of each step is the starting point of the next step in the figure and the movement of step can be obtained according to the coordinates of the end of each
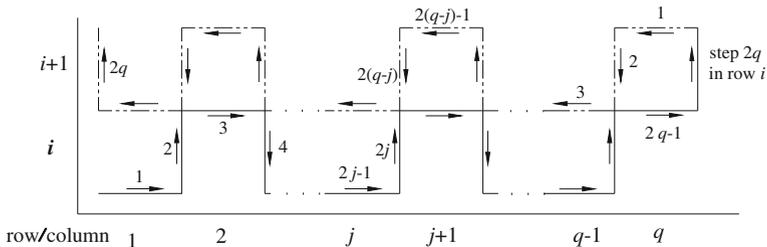

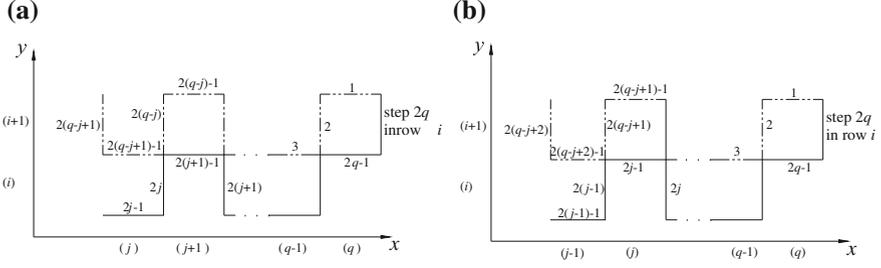**Fig. 7** Relations among motion trajectory, step and row/column number

(a)   **(b)**

Fig. 8 Coordinate representation for the downward-string's motion trajectory **a** column j is odd, **b** column j is even

step. The mathematical expressions of destination coordinates of downward-string can be obtained, if the step-length L (L is the bead length), the step number n_1, row i and column j is given.

The destination coordinates (x, y) of each step at odd rows i of downward-string.

$$\text{For } n_1 = 2j - 1, \quad \begin{cases} x = j \times L \\ y = (i - 1) \times L \end{cases} \tag{1}$$

$$\text{For } n_1 = 2j, \quad \begin{cases} x = j \times L \\ y = i \times L \end{cases} \tag{2}$$

Where j is odd column, as shown in Fig. 8a.

$$\text{For } n_1 = 2j - 1, \quad \begin{cases} x = j \times L \\ y = i \times L \end{cases} \tag{3}$$

$$\text{For } n_1 = 2j, \quad \begin{array}{l} (1) \text{ if } j < p, \begin{cases} x = j \times L \\ y = (i + 1) \times L \end{cases} \\ (2) \text{ if } j = p, \begin{cases} x = j \times L \\ y = (i + 1) \times L \end{cases} \end{array} \tag{4}$$

Where j is even column, as shown in Fig. 8b.

The destination coordinates (x, y) of each step at even rows $i + 1$ for downward-string and each step for upward-string can be obtained with the same method.

4 Simulations of the Weaving Trajectory

The program flow diagram of string's motion trajectory can be obtained according to the relations among string's motion trajectory, step and row and column number. Define the basic parameters as below: $p = 6$ and $q = 6$, which is given a beaded pad of $LD_{6 \times 6}$, and set the step length $L = 10$. The motion trajectories of upward-
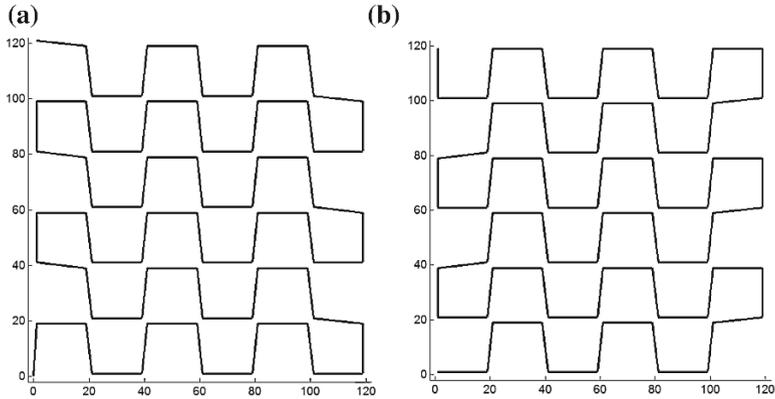
Fig. 9 Simulation result of the motion trajectory **a** uplink's motion trajectory, **b** downlink's motion trajectory
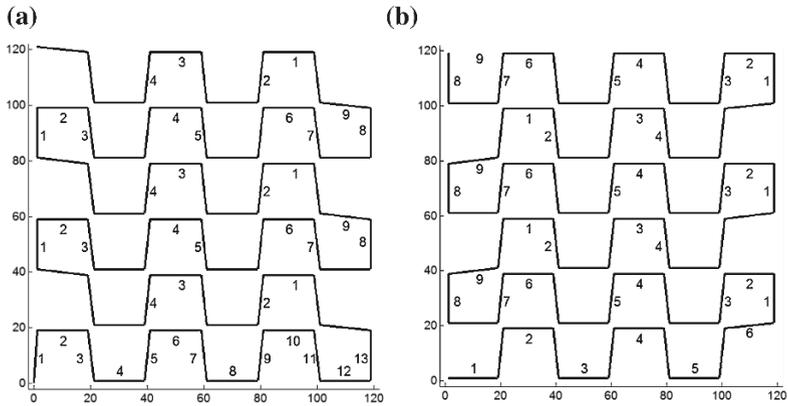

Fig. 10 Numbers of the bead pad **a** numbers for uplink **b** numbers for downlink

string and downward-string in weaving process are solved by computer using the program. The result is shown in Fig. 9. The process of numbering the beads of the beaded pad can be simulated, and the result is shown in Fig. 10. The number of each bead shown in Fig. 10 is one-to-one correspondence to the number shown in Fig. 5.

The motion trajectory shown in Fig. 9 is the simulation result of upward-string and downward-string motion trajectory of beaded pad weaving process shown in Fig. 6. The movement direction of the string can reference Fig. 6, and the initial point is located in the lower left corner.

5 Conclusions

This paper proposed a single-line direct-threading automatic weaving method for beaded pads and classified the motion as the synthesis of upward-string and downward-string. The weaving trajectory has the same period in two rows except the last column of the odd row for the downward-string. The proposed mathematical modelling and simulation methods can illustrate the automatic weaving trajectory directly and correctly.

References

1. Hirai Y (2008) Woven bead fabric and weaving method. Japan patent, JP2008007866A
2. D`Estais M, Cairon O (2004) Bead-weaving system. U.S. patent, US20040069363A1
3. Ogura H (1996) Bead-weaving machine with curved needle. Japan patent, JPH0860490A
4. Zhu T (2001) Bead-braided product and its braiding method. China patent, CN1309032A
5. Zhu T (2001) Device for weaving beads with holes. China patent, CN2458173Y
6. Zhu T (2003) Eye-hole pearl stuff and weaving method. China patent, CN1412011A
7. Gao J (2008) Plane bead weaving handicraft. China patent, CN201172353Y
8. Prussing C (2004) Beading with right angle weave. Interweave Press, UK
9. Fisher GL, Mellor B (2012) Using tiling theory to generate angle weaves with beads. J Math Arts 6(4):141–158
10. http://www.skunkhillstudio.com/projects/right_angle_weave/right_angle_weave.shtml
11. Yao L et al (2011) Method for describing upward-string and downward-string weaving movement rule of bead cool cushion. China patent, CN102206893A
12. Yao L et al (2012) Single-line direct-threading weaving method of bead stringed cooling mat. China patent, CN102587029A