2.10 Multistage Generalization: The Nested Distance .................... 71
  2.10.1 The Inherited Distance ....................................... 71
  2.10.2 The Nested Distance ......................................... 74
  2.10.3 The Nested Distance for Trees .............................. 79
2.11 Dual Representation of the Nested Distance ......................... 88
  2.11.1 Martingale Representation of the Nested Distance ... 91

3 Risk and Utility Functionals ............................................. 95
  3.1 Single-Period Risk and Utility Functionals ....................... 95
  3.2 Examples of Risk and Utility Functionals ........................ 97
  3.3 Dual Representation of Risk Functionals .......................... 103
    3.3.1 Kusuoka’s Representation ................................... 103
    3.3.2 The Dual Representation ..................................... 105
  3.4 An Alternative Description of Distortion Risk Functionals ...... 110
  3.5 The Impact of the Probability Measure on Risk Functionals ...... 114
    3.5.1 Compound Concavity and Convex-Concavity .................. 114
    3.5.2 Continuity with Respect to the Probability Measure ...... 117
  3.6 Conditional Risk Functionals ...................................... 119
    3.6.1 Properties of Conditional Risk Functionals ............... 122

4 From Data to Models .................................................... 125
  4.1 Approximations of Single-Period Probability Distributions .... 126
    4.1.1 Approximation Quality of the Monte Carlo Generation Method .......................................... 127
    4.1.2 Quasi-Monte Carlo Approximations .......................... 130
    4.1.3 Optimal and Nearly Optimal Single-Period Discretizations ............................................ 132
    4.1.4 The Stochastic Approximation Algorithms for Multidimensional Quantization ....................... 142
    4.1.5 Asymptotic Distribution of Optimal Quantizers ............ 145
  4.2 Approximations of Multiperiod Distributions ...................... 149
  4.3 Construction of Scenario Trees ................................... 154
    4.3.1 Distance Calculation ........................................ 155
    4.3.2 The Construction of Large Trees ............................. 157
  4.4 Scenario Tree Reduction ........................................... 163
  4.5 Improvement of Approximating Trees .............................. 166
    4.5.1 Improvement of the Probability Measure .................... 167
    4.5.2 Improvement of the Paths ................................... 168
  4.6 An Alternative View on Approximations ............................ 172

5 Time Consistency ........................................................ 175
  5.1 Time Consistency in Stochastic Decision Problems ............... 176
  5.2 Time Consistent Risk Functionals ................................ 179
5.3 Time Consistency and Decomposition ........................................ 187
  5.3.1 Composition of Risk Functionals ................................. 187
  5.3.2 Multistage Decomposition of Risk Functionals:
The Decomposition Theorem ........................................... 188
5.4 Martingale Formulations of Time Inconsistent
  Stochastic Programs ..................................................... 196
  5.4.1 Verification Theorems ......................................... 199
  5.4.2 An Algorithm for Sequential Improvement .................... 202
  5.4.3 Numerical Experiments ...................................... 203
5.5 Dualization of Nonanticipativity Constraints .......................... 205
6 Approximations and Bounds ................................................ 209
  6.1 Two-Stage Problems, and Approximation
    in the Wasserstein Distance ....................................... 209
  6.2 Approximation in the Nested Distance Sense ....................... 211
  6.3 Bounds ............................................................. 218
    6.3.1 Lower Bounds by Changing the Probability Measure .... 219
    6.3.2 Lower Bounds for Replacing the Scenario
      Process by Its Expectation .................................... 225
    6.3.3 Bounds for Changing the Filtration ....................... 227
    6.3.4 Upper Bounds by Inserting (Sub)Solutions ................ 227
  6.4 Martingale Properties ............................................ 228
7 The Problem of Ambiguity in Stochastic Optimization .................. 229
  7.1 Single- or Two-Stage Models: Wasserstein Balls ................... 234
  7.2 Solution Methods for the Single- or Two-Stage Case ............... 237
  7.3 The Multistage Case .............................................. 238
    7.3.1 A Minimax Theorem ........................................ 241
    7.3.2 Ambiguity Sets Defined by Nested
      Transportation Kernels ....................................... 245
    7.3.3 Algorithmic Solution ..................................... 247
  7.4 Example: A Multiperiod Production / Inventory Control
    Problem .............................................................. 249
    7.4.1 Mathematical Modeling Summary .......................... 251
    7.4.2 Computational Results ................................... 252
8 Examples ....................................................................... 257
  8.1 Thermal Electricity Production .................................... 257
  8.2 Hydro Electricity Production .................................... 261
  8.3 Budget Management for Risk-Prone Countries ....................... 270
A Risk Functionals: Definitions and Notations ........................... 275
  A.1 Multiperiod Risk Functionals .................................... 279
  A.2 Information Monotonicity ....................................... 280