# Contents

1 Introduction, Motivation, and Background .......................................................... 1
   1.1 Models for Geometric Control Systems: Pros and Cons .......................... 2
      1.1.1 Family of Vector Field Models ........................................ 2
      1.1.2 Models with Control as a Parameter ................................... 3
      1.1.3 Fibred Manifold Models ................................................. 7
      1.1.4 Differential Inclusion Models ........................................ 8
      1.1.5 The “Behavioural” Approach .......................................... 9
   1.2 An Introduction to Tautological Control Systems ............................. 9
      1.2.1 Attributes of a Modelling Framework for Geometric
           Control Systems .......................................................... 10
      1.2.2 The “Essentials” of Tautological Control Theory .................... 10
   1.3 An Outline of the Monograph ......................................................... 13
   1.4 Notation, Conventions, and Background ........................................... 14
   References .................................................................................. 17

2 Topologies for Spaces of Vector Fields ............................................................... 21
   2.1 An Overview of Locally Convex Topologies
      for Vector Spaces ........................................................................ 21
      2.1.1 Motivation ........................................................................ 21
      2.1.2 Families of Seminorms and Topologies
           Defined by These ................................................................ 22
   2.2 Seminorms for Locally Convex Spaces of Vector Fields .................... 24
      2.2.1 Fibre Norms for Jet Bundles ............................................ 24
      2.2.2 Seminorms for Spaces of Smooth Vector Fields .................... 26
      2.2.3 Seminorms for Spaces of Finitely Differentiable
           Vector Fields .................................................................... 26
      2.2.4 Seminorms for Spaces of Lipschitz Vector Fields ................. 27
      2.2.5 Seminorms for Spaces of Holomorphic Vector Fields .......... 28
      2.2.6 Seminorms for Spaces of Real Analytic Vector Fields ......... 28
      2.2.7 Summary and Notation ..................................................... 29
   References .................................................................................. 30
3 Time-Varying Vector Fields and Control Systems
   3.1 Time-Varying Vector Fields
      3.1.1 Time-Varying Smooth Vector Fields
      3.1.2 Time-Varying Finitely Differentiable and Lipschitz Vector Fields
      3.1.3 Time-Varying Holomorphic Vector Fields
      3.1.4 Time-Varying Real Analytic Vector Fields
      3.1.5 Topological Characterisations of Spaces of Time-Varying Vector Fields
      3.1.6 Mixing Regularity Hypotheses
   3.2 Parameterised Vector Fields
      3.2.1 The Smooth Case
      3.2.2 The Finitely Differentiable or Lipschitz Case
      3.2.3 The Holomorphic Case
      3.2.4 The Real Analytic Case
      3.2.5 Topological Characterisations of Parameterised Vector Fields
      3.2.6 Mixing Regularity Hypotheses
   3.3 Control Systems
      3.3.1 Control Systems with Locally Essentially Bounded Controls
      3.3.2 Control Systems with Locally Integrable Controls
      3.3.3 Differential Inclusions
   References

4 Presheaves and Sheaves of Sets of Vector Fields
   4.1 Definitions and Examples
   4.2 Sheafification
   4.3 The Étalé Space
   4.4 Stalk Topologies
   References

5 Tautological Control Systems: Definitions and Fundamental Properties
   5.1 Tautological Control Systems
   5.2 Open-Loop Systems
   5.3 Trajectories
   5.4 Attributes that can be Given to Tautological Control Systems
   5.5 Trajectory Correspondences with Other Sorts of Control Systems
   5.6 The Category of Tautological Control Systems
   References
Contents ix

6 Étalé Systems ........................................ 97
  6.1 Sheaves of Time-Varying Vector Fields ................. 98
  6.2 An Alternative Description of Local Sections of Sheaves
        of Time-Varying Vector Fields ....................... 100
  6.3 Étalé Open-Loop Systems and Open-Loop Subfamilies ...... 104
  6.4 Étalé Trajectories ..................................... 105
  References ............................................. 106

7 Ongoing and Future Work .................................. 107
  7.1 Linearisation ......................................... 107
  7.2 Optimal Control Theory ................................ 109
  7.3 Controllability ....................................... 113
  7.4 Feedback and Stabilisation Theory ....................... 115
  7.5 The Category of Tautological Control Systems ............ 116
  References ............................................. 116
Tautological Control Systems
Lewis, A.
2014, XII, 118 p., Softcover
ISBN: 978-3-319-08637-8