Chapter 2
The ER in Doping Super Lattices of HD Non-parabolic Semiconductors

2.1 Introduction

The technological importance of super-lattices in general, and specifically doping super-lattices [1–51] has already been stated in the preface and also in the references of this chapter. In Sect. 2.2.1, of the theoretical background, the ER in doping superlattices of HD non-linear optical semiconductors has been investigated. The Sect. 2.2.2 contains the results for doping superlattices of HD III-V, ternary and quaternary semiconductors in accordance with the three and the two band models of Kane together with parabolic energy bands and they form the special cases of Sect. 2.2.1. Sections 2.2.3, 2.2.4 and 2.2.5 contain the study of the ER for doping superlattices of HD II-VI, IV-VI and stressed Kane type semiconductors respectively. Sections 2.3 and 2.4 contain the results and discussion and the open research problems for this chapter.

2.2 Theoretical Background

2.2.1 The ER in Doping Superlattices of HD Non-linear Optical Semiconductors

The dispersion relation of the conduction electrons in doping superlattices of HD nonlinear optical materials can be expressed by using (1.2) and following the method as given in [19–51] as

$$\frac{(n_i + \frac{1}{2})}{\hbar T_{21}(E, \eta_g)} + \frac{\hbar^2 k^2}{2m^*_T_{22}(E, \eta_g)} = 1$$

(2.1)
where \( \omega_{8HD}(E, \eta_g) \equiv \text{Real part of} \left( \frac{n_i |e|^2}{d_0 T_{21}(E, \eta_g)} \right)^{1/2} \), \( n_i \) \((= 0, 1, 2 \ldots)\) is the mini-band index for nipi structures and \( d_0 \) is the mini-band index for nipi structures and \( d_0 \) is the superlattice period.

The EEM in this case assumes the form

\[
m^*(E_{FnHD}, n_i \eta_g) = \text{Real part of} \left( \frac{\hbar^2}{2} \right) G'^{1}_{21HD}(E_{FnHD}, n_i \eta_g) \quad (2.2)
\]

where, \( G_{21HD}(E, \eta_g, n_i) = \frac{2 \omega_{8HD}(E, \eta_g)}{\hbar} \left[ 1 - \frac{(n_i + \frac{1}{2}) \omega_{8HD}(E, \eta_g)}{\hbar T_{21}(E, \eta_g)} \right] \) and \( E_{FnHD} \) is the Fermi energy in the present case as measured from the edge of the conduction band in vertically upward direction in the absence of any quantization.

From (2.2), we observe that the EEM is a function of the Fermi energy, nipi subband index, scattering potential and the other material constants which is the characteristic feature of doping superlattices of HD non-linear optical materials.

The subband energy \((E_{1n,HD})\) can be written as

\[
\frac{(n_i + \frac{1}{2})}{\hbar T_{21}(E_{1n,HD}, \eta_g)} \omega_{8HD}(E_{1n,HD}, \eta_g) = 1 \quad (2.3)
\]

The DOS function for doping superlattices of HD non-linear optical materials can be expressed as

\[
N_{nipiHD}(E, \eta_g) = \frac{g_v}{2 \pi} \sum_{n_i=0}^{n_{\max}} G^{1}_{21HD}(E, \eta_g, n_i) H(E - E_{1n,HD}) \quad (2.4)
\]

The electron concentration, can be written as

\[
n_{2DN} = \frac{g_v}{2 \pi} \text{Real part of} \sum_{n_i=1}^{n_{\max}} [G_{21HD}(E_{FnHD}, \eta_g, n_i) + G_{22HD}(E_{FnHD}, \eta_g, n_i)] \quad (2.5)
\]

where, \( G_{22HD}(E_{FnHD}, \eta_g, n_i) = \sum_{r=1}^{s} L(r) G_{21HD}(E_{FnHD}, \eta_g, n_i) \)

The ER of doping superlattices of HDS can, in general, be expressed as

\[
\frac{D}{\mu} = \frac{n_{2DNO}}{|e|} \text{Real part of} \left[ \frac{\hat{\partial} n_{2DNO}}{\hat{\partial}(E_{FH0HD} - E_{F0HD})} \right]^{-1} \quad (2.6)
\]

where, the aforementioned physical variables are applicable only for electric quantum limit. Thus by combining (2.5) and (2.6) we can study the ER in this case.
The dispersion relation of the conduction electrons in doping superlattices of nonlinear optical materials in the absence of band tails assumes the form

\[ \psi_1(E) = \psi_2(E)k_x^2 + \psi_3(E) \left( n_i + \frac{1}{2} \right) \frac{2m^*}{\hbar} \omega_8(E) \]  \hspace{2cm} (2.7)

where \( \omega_8(E) \equiv \left( \frac{n_0|e|^2}{\hbar^2 c_{\text{vac}} |\psi_1(E)|^2} \right)^{1/2} \) and \( \theta_1(E) \equiv \frac{\hbar}{2} \left\{ \frac{\psi_3(E)|\psi_1(E)|^2 - \psi_1(E)|\psi_3(E)|^2}{|\psi_1(E)|^2} \right\} \).

The EEM in this case can be written as

\[ m^*(E_{Fn}, n_i) = \left( \frac{\hbar^2}{2} \right) R_{81}(E, n_i) \bigg|_{E=E_{Fn}} \] \hspace{2cm} (2.8)

where, \( R_{81}(E, n_i) \equiv [\psi_2(E)]^2 \left[ \psi_2(E) \left\{ \frac{1}{2} \left( \frac{2m^*}{\hbar} \right) [\psi_3(E)]' \right\}\left( n_i + \frac{1}{2} \right) [\omega_8(E)]' - \left( \frac{2m^*}{\hbar} \right) [\psi_3(E)](n_i + \frac{1}{2}) [\omega_8(E)]' \right\} \]

\[ - \left\{ [\psi_1(E)]' \left( \frac{2m^*}{\hbar} \right) [\psi_3(E)](n_i + \frac{1}{2}) [\omega_8(E)]' \right\} \left\{ [\psi_2(E)]' \right\} \]

and \( E_{Fn} \) is the Fermi energy in the present case as measured from the edge of the conduction band in vertically upward direction in the absence of any quantization.

The subband energy \( (E_{1ni}) \) can be written as

\[ \psi_1(E_{1ni}) = \psi_3(E_{1ni}) \left( n_i + \frac{1}{2} \right) \frac{2m^*}{\hbar} \omega_8(E_{1ni}) \] \hspace{2cm} (2.9)

The DOS function for doping superlattices of nonlinear optical materials can be expressed as

\[ N_{ni(pi)}(E) = \frac{g_v}{2\pi} \sum_{n_i=0}^{n_{\text{max}}} R_{81}(E, n_i) H(E - E_{1ni}) \] \hspace{2cm} (2.10)

the electron concentration, can be written as

\[ n_0 = \frac{g_v}{2\pi} \sum_{n_i=0}^{n_{\text{max}}} \left[ T_{81}(E_{Fn}, n_i) + T_{82}(E_{Fn}, n_i) \right] \] \hspace{2cm} (2.11)

where, \( T_{81}(E_{Fn}, n_i) \equiv \left[ \psi_1(E_{Fn}) - \psi_3(E_{Fn}) \left( n_i + \frac{1}{2} \right) \frac{2m^*}{\hbar} \omega_8(E_{Fn}) \right] \left[ \psi_2(E_{Fn}) \right]^{-1} \) and

\[ T_{82}(E_{Fn}, n_i) \equiv \sum_{r=1}^{s} L(r) T_{81}(E_{Fn}, n_i). \]
The electron concentration for the doping superlattices in quantum limit can be expressed as

$$n_0 = \frac{g_v}{2\pi} \left[ \psi_1(E_{F0}) - \psi_3(E_{F0}) \frac{m_\|}{\hbar} \omega_8(E_{F0}) \right]^{-1}$$  \(2.12\)

where, \(E_{F0}\) is the Fermi energy in the present case in the quantum limit and

$$\omega_8(E_{F0}) = \left( \frac{n_0|e|^2}{d_0\varepsilon_{ac}[\theta_1(E_{F0})]} \right)^{1/2}.$$

The ER at the electric quantum limit for doping superlattices in the absence of band tails can be written as

$$\frac{D}{\mu} = \frac{n_0}{e} \left[ \frac{\partial n_0}{\partial (E_{F0} - E_{10})} \right]^{-1}$$  \(2.13\)

where, \(n_0\) is the electron concentration, \(E_{F0}\) is the Fermi energy and \(E_{10}\) is the subband energy at the electric quantum limit respectively.

In this case, \(E_{10}\) can be determined from the equation as given by

$$\psi_1(E_{10}) = \psi_3(E_{10}) \frac{m_\|}{\hbar} \omega_8(E_{10})$$  \(2.14\)

Thus using (2.12), (2.13) and (2.14) we can study the ER in doping superlattices of non-linear optical materials in the absence of band tails.

### 2.2.2 The ER in Doping Superlattices of HD III-V, Ternary and Quaternary Semiconductors

(a) The electron energy spectrum in doping superlattices of HD III-V, ternary and quaternary materials can be expressed from (2.1) under the conditions \(\Delta_\| = \Delta_\perp = \Delta, \delta = 0\) and \(m_\| = m_\perp = m_c\), as

$$\frac{\hbar^2 k_x^2}{2m_c} = \left[ T_{31}(E, \eta_g) + iT_{32}(E, \eta_g) - \left( n_i + \frac{1}{2} \right) \hbar \omega_{gHD}(E, \eta_g) \right]$$  \(2.15\)

where \(\omega_{gHD}(E, \eta_g) \equiv \left( \frac{n_0|e|^2}{d_0\varepsilon_{ac}T_{31}(E, \eta_g)m_c} \right)^{1/2}\)
The EEM in this case assumes the form

\[ m^* (E_{Fn, HD}, n_i, \eta_g) = \text{Real Part of} \left( \frac{h^2}{2} \right) G'_{23, HD} (E_{Fn, HD}, \eta_g, n_i) \] (2.16)

where \( G_{23, HD} (E_{Fn, HD}, \eta_g, n_i) = \frac{2m_e}{\hbar^2} \left[ T_{31} (E_{Fn, HD}, \eta_g) + iT_{32} (E_{Fn, HD}, \eta_g) - \left( n_i + \frac{1}{2} \right) \hbar \omega_{0, HD} (E_{Fn, HD}, \eta_g) \right] \)

The subband energy \( E_{2n, HD} \) can be written as

\[ [T_{31} (E_{2n, HD}, \eta_g) + iT_{32} (E_{2n, HD}, \eta_g) - \left( n_i + \frac{1}{2} \right) \hbar \omega_{0, HD} (E_{2n, HD}, \eta_g)] = 0 \] (2.17)

The DOS function for doping superlattices of HD III-V, ternary and quaternary materials can be expressed as

\[ N_{nipi, HD} (E, \eta_g) = \frac{g_v m_e}{ \pi \hbar^2} \sum_{n_i=0}^{n_{\text{max}}} \text{Real part of} \left[ G_{23, HD} (E_{Fn, HD}, \eta_g, n_i) H (E - E_{2n, HD}) \right] \] (2.18)

The electron concentration, can be written as

\[ n_{2DN} = \frac{g_v m_e}{\pi \hbar^2} \text{Real part of} \sum_{n_i=0}^{n_{\text{max}}} \left[ G_{23, HD} (E_{Fn, HD}, \eta_g, n_i) + G_{24, HD} (E_{Fn, HD}, \eta_g, n_i) \right] \] (2.19)

where, \( G_{24, HD} (E_{Fn, HD}, \eta_g, n_i) = \sum_{r=1}^{s} L(r) G_{23, HD} (E_{Fn, HD}, \eta_g, n_i) \)

Using (2.6) and (2.19) at the electric quantum limit, we can study the ER in this case.

In the absence of band tails, the dispersion relation in this case assumes the form

\[ I_{11} (E) = \left( n_i + \frac{1}{2} \right) \hbar \omega_{19} (E) + \frac{\hbar^2 k_s^2}{2m_c} \] (2.20)

where \( \omega_{19} (E) \equiv \left( \frac{n_{\text{19}} |E|^2}{d_{11} I_{11}' |E|^2} \right) \).

The EEM in this case can be written as

\[ m^* (E_{Fn}, n_i) = m_e R_{82} (E, n_i) \Big|_{E = E_{Fn}} \] (2.21)

in which, \( R_{82} (E, n_i) \equiv \left\{ [I_{11} (E)]' - \left( n_i + \frac{1}{2} \right) \hbar [\omega_{19} (E)]' \right\} \).
From (2.21), we observe that the EEM in this case is a function of the Fermi energy, \( n_{ipi} \) subband index and the other material constants which is the characteristic feature of doping superlattices of III-V, ternary and quaternary compounds whose bulk dispersion relations is defined by the three band model of Kane.

The subband energies \( E_{2ni} \) can be written as

\[
I_{11}(E_{2ni}) = \left( n_i + \frac{1}{2} \right) \hbar \omega_{19}(E_{2ni})
\]  

(2.22)

The DOS function in this case can be expressed as

\[
N_{nipi}(E) = \frac{m_e g_v}{\pi \hbar^2} \sum_{n_i=0}^{n_{\text{max}}} R_{82}(E, n_i) H(E - E_{2ni})
\]  

(2.23)

The use of (2.23) leads to the expression of the electron concentration as

\[
n_0 = \frac{m_e g_v}{\pi \hbar^2} \sum_{n_i=0}^{n_{\text{max}}} \left[ T_{83}(\overline{E}_{Fn}, n_i) + T_{84}(\overline{E}_{Fn}, n_i) \right]
\]  

(2.24)

where \( T_{83}(\overline{E}_{Fn}, n_i) \equiv [I_{11}(\overline{E}_{Fn}) - (n_i + \frac{1}{2}) \hbar \omega_{19}(\overline{E}_{Fn})] \) and \( T_{84}(\overline{E}_{Fn}, n_i) \equiv \sum_{r=1}^{s} L(r) T_{83}(\overline{E}_{Fn}, n_i) \).

Using (2.24), the electron concentration in the electric quantum limit for doping superlattices of III-V, ternary and quaternary materials can be written as

\[
n_0 = \left( \frac{m_e g_v}{\pi \hbar^2} \right) [I_{11}(E_{20}) - \{ (1/2) \hbar \omega_{19}(E_{20}) \}]
\]  

(2.25)

where, \( E_{20} \) is determined from the equation

\[
I_{11}(E_{20}) = \frac{1}{2} \hbar \omega_{19}(E_{20})
\]  

(2.26)

Using (2.13) and (2.26) we can study, the ER in this case.

(b) The electron energy spectrum in doping superlattices of HD III-V, ternary and quaternary materials whose energy band structures in the absence of band tails are described by the two band model of Kane can be expressed from (2.15) under the conditions \( \Delta \gg E_g \) or \( \Delta \ll E_g \), as

\[
\frac{\hbar^2 k^2}{2m_c} = \left[ \gamma_2(E, \eta_g) - \left( n_i + \frac{1}{2} \right) \hbar \omega_{10HD}(E, \eta_g) \right]
\]  

(2.27)

where \( \omega_{10HD}(E) \equiv \left( \frac{m_0|e|}{d_{10\|\gamma_2(E, \eta_g)m_c}} \right)^{1/2} \)
The EEM in this case assumes the form
\[ m^*(E_{Fn,HD}, n_i, \eta_g) = \left( \frac{\hbar^2}{2} \right) G'_{25HD}(E_{Fn,HD}, \eta_g, n_i) \] (2.28)

where \( G_{25HD}(E_{Fn,HD}, \eta_g, n_i) = \frac{2m_e}{\hbar^2} \left[ \gamma_2(E_{Fn,HD}, \eta_g) - \left( n_i + \frac{1}{2} \right) \hbar \omega_{10HD}(E_{Fn,HD}, \eta_g) \right] \)

The subband energy \( E_{3n,HD} \) can be written as
\[ \left[ \gamma_2(E_{3n,HD}, \eta_g) - \left( n_i + \frac{1}{2} \right) \hbar \omega_{9HD}(E_{3n,HD}, \eta_g) \right] = 0 \] (2.29)

The DOS function in this case is given by
\[ N_{nipi,HD}(E, \eta_g) = \frac{g_v m_e}{\pi \hbar^2} \sum_{n_i=0}^{n_{\text{max}}} G'_{25HD}(E, \eta_g, n_i) H(E - E_{3n,HD}) \] (2.30)

The electron concentration, can be written as
\[ n_{2DN} = \frac{g_v m_e}{\pi \hbar^2} \sum_{n_i=0}^{n_{\text{max}}} \left[ G_{25HD}(E_{Fn,HD}, \eta_g, n_i) + G_{26HD}(E_{Fn,HD}, \eta_g, n_i) \right] \] (2.31)

where, \( G_{26HD}(E_{Fn,HD}, \eta_g, n_i) = \sum_{r=1}^{s} L(r) G_{25HD}(E_{Fn,HD}, \eta_g, n_i) \)

Using (2.31) and (2.6) at the electric quantum limit, we can study the ER in this case.

In the absence of band tails, the dispersion relation in this case assumes the form
\[ E(1 + \alpha E) = \left( n_i + \frac{1}{2} \right) \hbar \omega_{20}(E) + \frac{\hbar^2 k_s^2}{2m_e} \] (2.32)

where \( \omega_{20}(E) \equiv \left( \frac{n_0 |e|^2}{m_e (1 + 2\alpha E) m_e} \right)^{1/2} \).

The EEM in this case can be written as
\[ m^*(E_{Fn}, n_i) = m_e R_{182}(E, n_i) \big|_{E=E_{Fn}} \] (2.33)

in which, \( R_{182}(E, n_i) \equiv \left\{ \left[ 1 + 2\alpha E \right] - \left( n_i + \frac{1}{2} \right) \hbar \langle \omega_{19}(E) \rangle \right\} \).

From (2.33), we observe that the EEM in this case is a function of the Fermi energy, nipi subband index and the other material constants which is the characteristic feature of doping superlattices of III-V, ternary and quaternary compounds whose bulk dispersion relations is defined by the three band model of Kane.
The subband energies \(E_{3n_i}\) can be written as

\[
E_{3n_i}(1 + \alpha E_{3n_i}) = \left( n_i + \frac{1}{2} \right) \hbar \omega_{20}(E_{3n_i})
\]  

(2.34)

The DOS function in this case can be expressed as

\[
N_{nipi}(E) = \frac{m_e g_v}{\pi \hbar^2} \sum_{n_i=0}^{n_{\text{max}}} R_{182}(E, n_i) H(E - E_{3n_i})
\]  

(2.35)

The use of (2.35) leads to the expression of the electron concentration as

\[
n_0 = \frac{m_e g_v}{2\pi \hbar^2} \sum_{n_i=0}^{n_{\text{max}}} \left[ T_{85}(E_{Fn}, n_i) + T_{86}(E_{Fn}, n_i) \right]
\]  

(2.36)

where

\[
T_{85}(E_{Fn}, n_i) \equiv \left[ E_{Fn}(1 + \alpha E_{Fn}) - \left( n_i + \frac{1}{2} \right) \hbar \omega_{20}(E_{Fn}) \right]
\]

and

\[
T_{86}(E_{Fn}, n_i) \equiv \sum_{r=1} L(r) T_{85}(E_{Fn}, n_i).
\]

Thus using (2.36) in the electric quantum limit, we can study the ER in this case.

(c) The electron energy spectrum in nipi structures of HD III-V, ternary and quaternary materials whose energy band structures in the absence of band tails are described by the parabolic energy bands can be expressed as

\[
\frac{\hbar^2 k_x^2}{2m_e} = \left[ \gamma_3(E, \eta_g) - \left( n_i + \frac{1}{2} \right) \hbar \omega_{11HD}(E, \eta_g) \right]
\]

(2.37)

where \(\omega_{11HD}(E) \equiv \left( \frac{n_e e^2}{d_{\text{dis}} \gamma_3(E, \eta_g) m_e} \right)^{1/2}\)

The EEM in this case assumes the form

\[
m^*(E_{FnHD}, n_i, \eta_g) = \left( \frac{\hbar^2}{2} \right) G_{27HD}(E_{FnHD}, \eta_g, n_i)
\]

(2.38)

where \(G_{27HD}(E_{FnHD}, \eta_g, n_i) = \frac{2m_e}{\hbar^2} = \left[ \gamma_3(E_{FnHD}, \eta_g) - \left( n_i + \frac{1}{2} \right) \hbar \omega_{11HD}(E_{FnHD}, \eta_g) \right]\)

The subband energy \(E_{4n_{HD}}\) can be expressed as

\[
\left[ \gamma_3(E_{4n_{HD}}, \eta_g) - \left( n_i + \frac{1}{2} \right) \hbar \omega_{11HD}(E_{4n_{HD}}, \eta_g) \right] = 0
\]

(2.39)
The DOS function in this case is given by

\[
N_{nipiHD}(E, \eta_g) = \frac{g_v m_e}{\pi \hbar^2} \sum_{n_i=0}^{n_{\text{max}}} G_{27HD}(E, \eta_g, n_i) H(E - E_{4n,HD}) \tag{2.40}
\]

The electron concentration, can be written as

\[
n_{2DN} = \frac{g_v m_e}{\pi \hbar^2} \sum_{n_i=0}^{n_{\text{max}}} \left[ G_{27HD}(E_{FnHD}, \eta_g, n_i) + G_{28HD}(E_{FnHD}, \eta_g, n_i) \right] \tag{2.41}
\]

where, \( G_{28HD}(E_{FnHD}, \eta_g, n_i) = \sum_{r=1}^{s} L(r) G_{27HD}(E_{FnHD}, \eta_g, n_i) \)

Using (2.41) and (2.6) at the electric quantum limit, we can study the ER in this case.

In the absence of band tails, the dispersion relation in this case assumes the form

\[
E = \left( n_i + \frac{1}{2} \right) \hbar \omega_{21} + \frac{\hbar^2 k_i^2}{2m_c} \tag{2.42}
\]

where \( \omega_{21} \equiv \left( \frac{n_0 \epsilon_i^2}{\frac{\hbar^2}{2m_c}} \right)^{1/2} \).

The EEM in this case can be written as

\[
m^*(E_{Fn}, n_i) = m_c \tag{2.43}
\]

Thus the EEM in this case is a constant quantity.

The subband energies \((E_{4ni})\) can be written as

\[
E_{4ni} = \left( n_i + \frac{1}{2} \right) \hbar \omega_{21} \tag{2.44}
\]

The DOS function in this case can be expressed as

\[
N_{nipi}(E) = \frac{m_c g_v}{\pi \hbar^2} \sum_{n_i=0}^{n_{\text{max}}} H(E - E_{4ni}) \tag{2.45}
\]

The use of (2.45) leads to the expression of the surface electron concentration as

\[
n_0 = \frac{m_c g_v k_B T}{\pi \hbar^2} \sum_{n_i=0}^{n_{\text{max}}} F_0(\eta_{4i}) \tag{2.46}
\]

where \( \eta_{4i} = (E_{Fn} - E_{4ni})/(k_B T) \)

Thus using (2.46) in the electric quantum limit, we can study the ER in this case.
2.2.3 The ER in Doping Superlattices of HD II-VI Semiconductors

The 2D electron dispersion law in doping superlattices of HD II-VI semiconductors can be expressed as

$$
\gamma_3(E, \eta_g) = d_0 k_s^2 + \left( n_i + \frac{1}{2} \right) \hbar \omega_{20}(E, \eta_g) \pm \mathcal{Z}_0 k_s, \ \omega_{30}(E, \eta_g)
$$

$$
\equiv \left( \frac{n_0 |e|^2}{d_0 \gamma'_3(E, \eta_g) \varepsilon_{sc} m^*_n} \right)^{1/2}
$$

(2.47)

The EEM in this case assumes the form as

$$
m^*(E_{FnHD}, n_i, \eta_g) = m^* \left\{ 1 - \mathcal{Z}_0 \left[ (\mathcal{Z}_0)^2 + 4d_0 \gamma'_3(E_{FnHD}, \eta_g) - 4d'_0 \left( n_i + \frac{1}{2} \right) \hbar \omega_{30}(E_{FnHD}, \eta_g) \right] \right\}^{1/2} \gamma'_3(E_{FnHD}, \eta_g)
$$

(2.48)

The subband energy can be written as

$$
\gamma_3(E_{6n,HD}, \eta_g) = \left( n_i + \frac{1}{2} \right) \hbar \omega_{30}(E_{6n,HD}, \eta_g)
$$

(2.49)

The surface electron concentration per unit area in this case is given by

$$
n_{2DN} = \frac{g_v}{4\pi d_0^2} \sum_{n_i=0}^{n_{max}} \left[ G_{30HD}(E_{FnHD}, \eta_g, n_i) + G_{31HD}(E_{FnHD}, \eta_g, n_i) \right]
$$

(2.50)

where

$$
G_{30HD}(E_{FnHD}, \eta_g, n_i) = \left[ (\mathcal{Z}_0)^2 - 2d'_0 \left\{ \left( n_i + \frac{1}{2} \right) \hbar \omega_{30}(E_{FnHD}, \eta_g, n_i) - \gamma_3(E_{FnHD}, \eta_g, n_i) \right\} \right]
$$

and

$$
G_{31HD}(E_{FnHD}, \eta_g, n_i) = \sum_{r=1}^{s} L(r) G_{30HD}(E_{FnHD}, \eta_g, n_i)
$$

Using (2.50) and (2.6) at the electric quantum limit, we can study the ER in this case.

In the absence of band-tails, the carrier dispersion law in doping superlattices of II-VI compounds can be expressed as

$$
E = d'_0 k_s^2 + \left( n_i + \frac{1}{2} \right) \hbar \bar{\omega}_{10} \pm \mathcal{Z}_0 k_s, \quad \bar{\omega}_{10} = \left( \frac{n_0 |e|^2}{d_0 \varepsilon_{sc} m^*_n} \right)^{1/2}
$$

(2.51)
Using (2.51), the EEM in this case can be written as

\[
m'^*_{\text{EEM}}(E_{\text{F}}n_i) = m'^*_{\perp} \left\{ 1 - \frac{1}{\hbar^2} \left[ (\bar{\omega}_0)^2 + 4a'_0 E_{\text{F}}n - 4a'_0 \left( n_i + \frac{1}{2} \right) \hbar \bar{\omega}_0 \right] \right\}^{-1/2}
\]

(2.52)

Thus, the EEM in this case is a function of the Fermi energy, the ni-pi subband index number and the energy spectrum constants due to the only presence of \( \bar{\omega}_0 \).

The subband energies \( E_{\text{ni}} \) assume the form as

\[
E_{\text{ni}} = \left( n_i + \frac{1}{2} \right) \hbar \bar{\omega}_0 \tag{2.53}
\]

The DOS function in this case can be expressed as

\[
N_{\text{ni-pi}}(E) = m'^*_{\text{EEM}} \frac{g^*}{\pi \hbar^2} \sum_{n_i=0}^{n_{\text{max}}} \left[ 1 - \frac{a_{\text{ni}}}{\sqrt{E + b_{\text{ni}}(n_i)}} \right] H(E - E_{\text{ni}}) \tag{2.54}
\]

in which, \( a_{\text{ni}} \equiv \frac{\bar{\omega}_0}{2\sqrt{a'_0}} \) and \( b_{\text{ni}}(n_i) \equiv \left[ \frac{1}{4a'_0} \left( (\bar{\omega}_0)^2 - 4a'_0 \left( n_i + \frac{1}{2} \right) \hbar \bar{\omega}_0 \right) \right]. \)

The use of (2.54) leads to the electron concentration as

\[
n_0 = \frac{m'^*_{\text{EEM}} g^*}{\pi \hbar^2} \sum_{n_i=0}^{n_{\text{max}}} \left[ \frac{a_{\text{ni}}}{\sqrt{E + b_{\text{ni}}(n_i)}} \right] \left[ 2 \left( \sqrt{\eta_{\text{ni}}} + c_{\text{ni}}(n_i) - \sqrt{c_{\text{ni}}(n_i)} \right) \right] + \sum_{r=1}^{\infty} 2(1 - 2^{1-2r}) \frac{(1-2r)^{2r-1}(2r-1)!}{(\eta_{\text{ni}} + c_{\text{ni}}(n_i))^{2r}} \tag{2.55}
\]

where, \( \eta_{\text{ni}} \equiv \frac{E_{\text{F}}n_i - E_{\text{ni}}}{\hbar^2} \) and \( c_{\text{ni}}(n_i) \equiv \frac{b_{\text{ni}}(n_i) + E_{\text{ni}}}{\hbar^2} \).

Using (2.55) in the electric quantum limit we can study the ER in this case.

### 2.2.4 The ER in Doping Superlattices of HD IV-VI Semiconductors

The 2D electron dispersion law in this case is given by

\[
k^2 = \delta_{15}(E, \eta_{g}, n_i) \tag{2.56}
\]

where

\[
\delta_{15}(E, \eta_{g}, n_i) = \left[ 2\delta_{12}(E, \eta_{g}) \right]^{-1} \left[ -\delta_{13}(E, \eta_{g}, n_i) + \sqrt{\delta_{13}^2(E, \eta_{g}, n_i) - 4\delta_{12}(E, \eta_{g})\delta_{14}(E, \eta_{g}, n_i)} \right],
\]

\[
\delta_{12}(E, \eta_{g}) = \frac{\eta_{g} \eta_{g}}{4m'^{\text{EEM}}_{\perp}} \eta_{g}, \delta_{13}(E, \eta_{g}, n_i) = \hbar^2 \left[ \delta_{17}(E, \eta_{g}) \delta_{11}(E, \eta_{g}, n_i) + \lambda_{12}(E, \eta_{g}) \right],
\]

\[
\delta_{14}(E, \eta_{g}, n_i) = \left[ \lambda_{23}(E, \eta_{g}) \delta_{11}^2(E, \eta_{g}, n_i) + \lambda_{14}(E, \eta_{g}) \delta_{12}^2(E, \eta_{g}, n_i) - \lambda_{14}(E, \eta_{g}) \right],
\]

\[
\delta_{11}(E, \eta_{g}, n_i) = \frac{2}{\hbar^2} m'^{\text{EEM}}_{\text{HD}}(0, \eta_{g})(n_i + \frac{1}{2}) \left[ \delta_{0}^{\text{EEM}} m'^{\text{EEM}}_{\text{HD}}(E, \eta_{g}) \right]^{1/2}
\]
and \( m_{HD}^*(E, \eta_g) = \frac{\hbar^2}{4\lambda_{74}^2(E, \eta_g)} \left\{ \frac{\lambda_{74}(E, \eta_g)}{\lambda_{75}(E, \eta_g)} \right\} \) \( \lambda_{74}(E, \eta_g) \lambda_{75}(E, \eta_g) \) 

The EEM in this case assumes the form

\[
m^*(E_{FnHD}, n_i, \eta_g) = \left( \frac{\hbar^2}{2} \right) \delta_{15}(E_{FnHD}, \eta_g, n_i) \tag{2.57}
\]

The sub-band energy \( E_{9n_{HD}} \) can be expressed as in this case as

\[
0 = \delta_{15}(E_{9n_{HD}}, \eta_g, n_i) \tag{2.58}
\]

The surface electron concentration in this case is given by

\[
n_{2DN} = \frac{g_v}{2\pi} \sum_{n_i=0}^{n_{\text{max}}} \left[ G_{32HD}(E_{FnHD}, \eta_g, n_i) + G_{33HD}(E_{FnHD}, \eta_g, n_i) \right] \tag{2.59}
\]

where

\[
G_{32HD}(E_{FnHD}, \eta_g, n_i) = \delta_{15}(E_{FnHD}, \eta_g, n_i) \quad \text{and} \quad G_{33HD}(E_{FnHD}, \eta_g, n_i) = \sum_{r=1}^{s} L(r) G_{32HD}(E_{FnHD}, \eta_g, n_i)
\]

Using (2.59) and (2.6) at the electric quantum limit, we can study the ER in this case.

The carrier energy spectrum in doping superlattices of IV-VI compounds in the absence of band tails can be written as

\[
k_s^2 = (\hbar^2 S_{19})^{-1} \left[ -S_{20}(E, n_i) + \sqrt{S_{20}^2(E, n_i) + 4S_{19}S_{21}(E, n_i)} \right] \tag{2.60}
\]

in which, \( S_{19} \equiv \left( \frac{2}{m_i^* m_r^*} \right), S_{20}(E, n_i) = \left\{ \frac{1}{m_i^*} - \left( \frac{2E}{m_i^*} \right) + \frac{1+2n_i}{2m_i^* m_r^*} \left( n_i + \frac{1}{2} \right) T(E) + \frac{2\hbar^2}{2m_i^* m_r^*} \left( n_i + \frac{1}{2} \right) T(E) \right\} \)
\[ T(E) \equiv \frac{2m^*(0)}{\hbar} \omega_{11}(E), m^*(0) = \left( \frac{m_i^2 m_i^-}{m_i^2 + m_i^-} \right), \quad \omega_{11}(E) = \left( \frac{n_0 |e|^2}{d_0 e \omega m^*(E)} \right) \]

\[ m^*(E) = \frac{1}{4t_1} \left[ -t_2(E)' + \frac{t_2(E)(t_2(E))'}{\sqrt{t_2(E)} + 4E t_1(1 + zE)} \right] \]

\[ t_1 = \left( \frac{\alpha}{4m_i^2 m_i^-} \right), \quad t_2(E) = \frac{1}{2} \left[ \left( \frac{1}{m_i} \right) - \left( \frac{zE}{m_i^-} \right) + \left( \frac{1 + zE}{m_i} \right) \right] \]

\[ \quad \times \left( t_2(E)' = \frac{\alpha}{2} \left( \frac{1}{m_i} \right) - \left( \frac{1}{m_i^-} \right) \right) \]

and

\[ S_{21}(E, n_i) \equiv \left[ E(1 + zE) + \frac{zE \hbar^2}{2m_i^2} \left( n_i + \frac{1}{2} \right) T(E) + \frac{\hbar^2}{2m_i} \left( n_i + \frac{1}{2} \right) T(E)(1 + zE) \right] \]

\[ + \frac{\hbar^4}{4m_i^2 m_i^-} \left( n_i + \frac{1}{2} \right) T(E) + \left( \frac{\hbar^2}{2m_i} \right) T(E) \left( n_i + \frac{1}{2} \right) \]

Using (2.60) the EEM in this case can be written as

\[ m^*(E_{Fn}, n_i) = R_{84}(E, n_i) \big|_{E=E_{Fn}} \quad (2.61) \]

where,

\[ R_{84}(E, n_i) \equiv (2S_{19})^{-1} \left[ -\left( S_{20}(E, n_i) \right)' + \frac{S_{20}(E, n_i)\left[ S_{20}(E, n_i)' + 2S_{19} S_{21}(E, n_i)' \right]}{\left[ \left( S_{20}(E, n_i)' \right)^2 + 4S_{19} S_{21}(E, n_i)' \right]^{1/2}} \right] \]

Thus, one can observe that the EEM in this case is a function of both the Fermi energy and the nipi subband index number together with the spectrum constants of the system due to the presence of band non-parabolicity.

The subband energies \( E_{10ni} \) can be written as

\[ E_{10ni} = \frac{\hbar^2}{2m_i} T(E_{10ni}) \left( n_i + \frac{1}{2} \right) \left[ 1 + zE_{10ni} + + \alpha \frac{\hbar^2}{2m_i} T(E_{10ni}) \left( n_i + \frac{1}{2} \right) \right] \]

\[ = \left( \frac{\hbar^2}{2m_i} T(E_{10ni}) \left( n_i + \frac{1}{2} \right) \right) \quad (2.62) \]

The DOS function in this case assumes the form as

\[ N_{nipi}(E) = \frac{g_v}{\pi \hbar^2} \sum_{n_i=0}^{n_{max}} R_{84}(E, n_i) H(E - E_{10ni}) \quad (2.63) \]
The use of (2.63) leads to the expression of the electron concentration as
\[ n_0 = \frac{g_v}{2\pi \hbar^2 S_{19}} \sum_{n_i=0}^{n_{\text{max}}} T_{85}(E_{F_0}, n_i) + T_{86}(E_{F_0}, n_i) \tag{2.64} \]

where, \( T_{85}(E_{F_0}, n_i) \equiv -S_{20}(E_{F_0}, n_i) + \sqrt{[S_{20}(E_{F_0}, n_i)]^2 + 4S_{19}S_{21}(E_{F_0}, n_i)} \) and
\[ T_{86}(E_{F_0}, n_i) \equiv \sum_{r=1}^{s} L(r) T_{85}(E_{F_0}, n_i). \]

The electron concentration at the quantum limit can be defined through the equation
\[ n_0 = \frac{g_v}{2\pi \hbar^2 S_{19}} \left[ -S_{20}(E_{F_0}, 0) + \sqrt{[S_{20}(E_{F_0}, 0)]^2 + 4S_{19}S_{21}(E_{F_0}, 0)} \right] \tag{2.65} \]

### 2.2.5 The ER in Doping Superlattices of HD Kane Type Semiconductors

The 2D dispersion relation in this case is given by
\[ P_{11}(E, \eta_g)k_x^2 + Q_{11}(E, \eta_g)k_y^2 + S_{11}(E, \eta_g)\delta_{19}(E, \eta_g, n_i) = 1 \tag{2.66} \]

where
\[ \delta_{19}(E, \eta_g, n_i) = \frac{2}{\hbar} m_{zz}^*(0, \eta_g) \left( n_i + \frac{1}{2} \right) \left[ \frac{n_0 e^2}{d_0 \varepsilon_{zz} m_{zz}(E, \eta_g)} \right]^{1/2} \]

and the expression for \( m_{zz}(E, \eta_g) \) has already been given in (1.209) of Chap. 1.

The EEM in this case assumes the form
\[ m^*(E_{F_{0,HD}}, n_i, \eta_g) = \left( \frac{\hbar^2}{2} \right) \delta_{20}(E_{F_{0,HD}}, \eta_g, n_i) \tag{2.67} \]

where \( \delta_{20}(E_{F_{0,HD}}, \eta_g, n_i) = \frac{[1 - S_{11}(E_{F_{0,HD}}, \eta_g)\delta_{19}(E_{F_{0,HD}}, \eta_g, n_i)]}{\sqrt{P_{11}(E_{F_{0,HD}}, \eta_g)Q_{11}(E_{F_{0,HD}}, \eta_g)}} \)

The sub-band energy \( E_{15n_{HD}} \) can be expressed as in this case as
\[ S_{11}(E_{15n_{HD}}, \eta_g)\delta_{19}(E_{15n_{HD}}, \eta_g, n_i) = 1 \tag{2.68} \]
The surface electron concentration in this case is given by

\[
n_{2DN} = \frac{g_v}{2\pi} \sum_{n_i=0}^{n_{\text{max}}} \left[ G_{34\text{HD}}(E_{F_n\text{HD}}, \eta_g, n_i) + G_{35\text{HD}}(E_{F_n\text{HD}}, \eta_g, n_i) \right]
\]

(2.69)

where \( G_{34\text{HD}}(E_{F_n\text{HD}}, \eta_g, n_i) = \delta_20(E_{F_n\text{HD}}, \eta_g, n_i) \) and \( G_{35\text{HD}}(E_{F_n\text{HD}}, \eta_g, n_i) = \sum_{r=1}^{s} L(r) G_{34\text{HD}}(E_{F_n\text{HD}}, \eta_g, n_i) \).

Using (2.69) and (2.6) at the electric quantum limit, we can study the ER in this case.

The electron dispersion law in the doping superlattices of stressed Kane type semiconductors can be written as

\[
k_x^2 \left[ \frac{1}{\tau_0(E)} \right]^2 + k_y^2 \left[ \frac{1}{\tau_0(E)} \right]^2 + \frac{2m^*_{\zeta}(0)}{\hbar} \left( n_i + \frac{1}{2} \right) \omega_{12}(E) = 1
\]

(2.70)

where \( \omega_{12}(E) \equiv \left( \frac{n_0|\eta|^2}{\text{dof}_{\text{m}}} \right)^{\frac{1}{2}} \) and \( m^*_{\zeta}(E) \equiv \hbar^2 \tau_0(E) \frac{\partial}{\partial E} \tau_0(E) \).

The use of (2.70) leads to the expression of the EEM as

\[
m^*(E_{F_n}, n_i) = \left( \frac{\hbar^2}{2} \right) R_{85}(E, n_i) \bigg|_{E=E_{F_n}}
\]

(2.71)

where,

\[
R_{85}(E, n_i) = \left[ \left( \frac{\partial_0(E)}{\text{dof}_0(E)} \right)^2 - \left( \frac{2m^*_{\zeta}(0)}{\hbar} \right) \left( n_i + \frac{1}{2} \right) \omega_{12}(E) \right]
\]

(2.72)

Thus, the EEM is a function of the Fermi energy and the \( n \)ipi subband index due to the presence of stress and band non-parabolicity only.

The subband energies \( (E_{25ni}) \) can be written as

\[
\frac{1}{\left| \tau_0(E_{25ni}) \right|^2} - \frac{2m^*_{\zeta}(0)}{\hbar} \left( n_i + \frac{1}{2} \right) \omega_{12}(E_{25ni}) = 1
\]

(2.73)

The DOS function can be written as

\[
N_{nipi}(E) = \frac{g_v}{\pi \hbar^2} \sum_{n_i=0}^{n_{\text{max}}} R_{85}(E, n_i) H(E - E_{25ni})
\]

(2.74)
Thus, using (2.74), the electron concentration in doping superlattices of stressed compounds can be expressed as

$$n_0 = \frac{g_v}{2\pi} \sum_{n_i=0}^{n_{\text{max}}} C_3(\bar{E}_{Fn}, n_i) + C_4(\bar{E}_{Fn}, n_i)$$  \hspace{1cm} (2.75)$$

where

$$C_3(\bar{E}_{Fn}, n_i) \equiv a_0(\bar{E}_{Fn}) b_0(\bar{E}_{Fn}) \left[ 1 - \frac{2m^*_0(0)}{\hbar} \left( n_i + \frac{1}{2} \right) \frac{\omega_{12}(\bar{E}_{Fn})}{(\tau_0(\bar{E}_{Fn}))^2} \right] \quad \text{and} \quad C_4(\bar{E}_{Fn}, n_i) \equiv \sum_{r=1}^{s} L(r) C_3(\bar{E}_{Fn}, n_i)$$

The use of (2.75) leads to the expression of the electron statistics at the electric quantum limit and at low temperatures as

$$\bar{n}_0 = \frac{g_v}{2\pi} a_0(\bar{E}_{F_0}) b_0(\bar{E}_{F_0}) \left[ 1 - \frac{m^*_z(0)}{(\bar{c}_0(\bar{E}_{F_0}))^2} \frac{\omega_{12}(\bar{E}_{F_0})}{\hbar} \right]$$  \hspace{1cm} (2.76)$$

Using (2.76), we can study the ER in this case.

### 2.3 Result and Discussions

Using the appropriate equations together with the energy band constants as given in Table 1.1, the ER in the quantum limit has been plotted for the doping superlattices of HD tetragonal compounds (taking HD Cd$_3$As$_2$ as an example) as a function of electron concentration as shown in curve (a) of Fig. 2.1. The curve (b) corresponds to $\delta = 0$ and the curve (c) exhibits the dependence of the ER on $n_0$ in accordance with the HD three-band model of Kane, respectively. The plots (d) and (e) correspond to the HD two-band model of Kane and that of HD parabolic energy bands. By comparing the curves (a) and (b) of Fig. 2.1, one can assess the influence of crystal field splitting of the ER in doping superlattices of HD Cd$_3$As$_2$. Figure 2.2 represents all cases of Fig. 2.1 for doping superlattices of HD nonlinear optical materials taking HD CdGeAs$_2$ as an example. It appears from Figs. 2.1 and 2.2 that, the ER in doping superlattices of HD nonlinear optical materials increases with increasing carrier degeneracy as expected for degenerate materials.

Using the appropriate equations one can numerically evaluate the ER in the quantum limit as a function of electron concentration in doping superlattices of HD III-V compounds by using the HD InAs and InSb as shown in Figs. 2.3 and 2.4 by curves (a), (b) and (c) respectively, in accordance with three and two band models of Kane together with the model of parabolic energy bands.

Taking doping superlattices of HD Hg$_{1-x}$Cd$_x$Te as an example of HD ternary compounds, the ER has been plotted for both the structures as a function of electron concentration as shown in Fig. 2.5 for all cases of the Fig. 2.3. It appears
from the Fig. 2.5 that the ER in the quantum limit in both cases of doping superlattices of ternary compounds increases with increasing electron concentration as usual for the degenerate compounds. Taking doping superlattices of HD

\[ \begin{align*}
\text{Fig. 2.1} & \quad \text{The plot of the ER in the quantum limit for doping superlattices of HD Cd}_{3}\text{As}_{2} \text{ as a function of electron concentration in accordance with } a \text{ the generalized band model, } b \ \delta = 0, \\
& \quad c \text{ the three band model of Kane, } d \text{ the two band model of Kane and } e \text{ the parabolic energy bands}
\end{align*} 

\[ \begin{align*}
\text{Fig. 2.2} & \quad \text{The plot of the ER in the quantum limit for doping superlattices of HD CdGeAs}_{2} \text{ as a function of electron concentration in accordance with } a \text{ the generalized band model, } b \ \delta = 0, \\
& \quad c \text{ the three band model of Kane, } d \text{ the two band model of Kane and } e \text{ the parabolic energy bands}
\end{align*} 

In the quantum limit, the ER has been plotted as a function of electron concentration for In\(_{1-x}\)Ga\(_x\)As\(_y\)P\(_{1-y}\) lattice matched to HD InP, as shown in Fig. 2.6 according to the three and two band models of Kane, and the isotropic parabolic energy band model. From Figs. 2.5 and 2.6, one can assess the influence of energy band constants on the ER for doping superlattices of ternary and quaternary materials respectively. Using the appropriate equations, the ER in the quantum limit has been plotted for the doping...
superlattices of CdS, as a function of carrier concentration as shown by curves (a) and (b) in Fig. 2.7 for both \( \lambda_0 \neq 0 \) and \( \lambda_0 = 0 \) respectively. This has been presented for the purpose of assessing the influence of the splitting of the two spin states by the spin-orbit coupling and the crystalline field on the ER for doping superlattices of II-VI materials. In Fig. 2.8, the ER in the quantum limit has been plotted for the HD doping superlattices of (a) PbTe, (b) PbSnTe and (c) Pb\(_{1-x}\)Sn\(_x\)Se

Fig. 2.5 The plot of the ER in the quantum limit for doping superlattices of HD Hg\(_{1-x}\)Cd\(_x\)Te as a function of electron concentration in accordance with a the three band model of Kane, b the two band model of Kane and c the parabolic energy bands

Fig. 2.6 The plot of the ER in the quantum limit for doping superlattices of HD In\(_{1-x}\)Ga\(_x\)As\(_y\)P\(_{1-y}\) lattice matched to InP as a function of electron concentration in accordance with a the three band model of Kane, b the two band model of Kane and c the parabolic energy bands
as a function of electron concentration in accordance with the Dimmock model. For relatively low values of electron concentration, the values of the ER for the three materials exhibit convergence behavior whereas for relatively large values of $n_0$, the numerical values differ widely from each other in this case. In Fig. 2.9, the ER in the quantum limit has been plotted for the doping superlattices of stressed HD InSb as a function of electron concentration.

**Fig. 2.7** The plot of the ER in the quantum limit for doping superlattices of HD CdS as a function of carrier concentration in accordance with $a \overline{\lambda}_0 \neq 0$ and $b \overline{\lambda}_0 = 0$

**Fig. 2.8** The plot of the ER in the quantum limit as a function of electron concentration for the doping superlattices of HD $a$ PbTe, $b$ PbSnTe and $c$ Pb$_{1-x}$Sn$_x$Se
The plot (a) of Fig. 2.9 exhibits the ER for the doping superlattices of stressed HD InSb in the presence of the stress while the plot (b) shows the same in the absence of the stress. In the presence of the stress, the magnitude of the ER is being increased as compared with the same under stress free condition.

2.4 Open Research Problems

R.2.1 Investigate the ER in the presence of an arbitrarily oriented non-quantizing magnetic field for nipi structures of HD nonlinear optical semiconductors by including the electron spin. Study all the special cases for HD III-V, ternary and quaternary materials in this context.

R.2.2 Investigate the ERs in nipi structures of HD IV-VI, II-VI and stressed Kane type compounds in the presence of an arbitrarily oriented non-quantizing magnetic field by including the electron spin.

R.2.3 Investigate the ER for nipi structures of all the materials as stated in R.2.1.

R.2.4 Investigate the ER for all the problems from R.2.1 to R.2.3 in the presence of an additional arbitrarily oriented electric field.

R.2.5 Investigate the ER for all the problems from R.2.1 to R.2.3 in the presence of arbitrarily oriented crossed electric and magnetic fields.
References

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